



India's Number 1 Education App

## MATHS

### BOOKS - CENGAGE PUBLICATION

#### LIMITS AND DERIVATIVES

##### Others

1. Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$



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2. If  $\lim_{x \rightarrow 1} \frac{a \sin(x-1) + b \cos(x-1) + 4}{x^2 - 1} = -2$ , then  $|a+b|$  is \_\_\_\_\_.



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3. Evaluate the limit  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$



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4. The integer  $n$  for which  $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$  is a finite non-zero number is :



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5. Evaluate the limit:  $(\lim)_{n \rightarrow \infty} \frac{(1^2 - 2^2 + 3^2 - 4^2 + 5^2 + n \text{ terms})}{n^2}$



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6. Let  $(\lim)_{x \rightarrow 1} \frac{x^a - ax + a - 1}{(x - 1)^2} = f(a)$ . Then the value of  $f(4)$  is \_\_\_\_\_



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7. Evaluate the limit:  $\lim_{x \rightarrow a} \frac{\sqrt{3x - a} - \sqrt{x + a}}{x - a}$



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8.  $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$  and  $\lim_{x \rightarrow -2} f(x)$  exists. Then the value of  $(a - 4)$  is \_\_\_\_\_



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9. Evaluate the limit:  $(\lim)_{x \rightarrow \infty} \left[ \sqrt{a^2 x^2 + ax + 1} - \sqrt{a^2 x^2 + 1} \right]$



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10.  $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos 2x} \cdot \sqrt[3]{\cos 3x} \dots \sqrt[n]{\cos nx}}{x^2}$  has value 10 then value of  $n$  equal to



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11. Evaluate the limit:  $\lim_{x \rightarrow 1} \frac{\sum_{k=1}^{100} x^k - 100}{x - 1}$



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12. Let  $S_n = 1 + 2 + 3 + \dots + n$  and

$$P_n = \frac{S_2}{S_2 - 1} \cdot \frac{S_3}{S_3 - 1} \cdot \frac{S_4}{S_4 - 1} \cdots \cdot \frac{S_n}{S_n - 1} \quad \text{Where}$$

$$n \in N, (n \geq 2). \text{ Then } \lim_{n \rightarrow \infty} P_n = \underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$$



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13. If  $a_1 = 1$  and

$$a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n}, n \geq 1 \text{ and if } \lim_{n \rightarrow \infty} a_n = a, \text{ then find the value of } a.$$



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14. If  $L = \lim_{x \rightarrow \infty} \left\{ x - x^2 (\log_e \left( 1 + \frac{1}{x} \right)) \right\}$ , then the value of  $8L$  is \_\_\_\_\_



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**15.** Evaluate the limit:  $(\lim)_{n \rightarrow \infty} \cos(\pi \sqrt{n^2 + n})$  when  $n$  is an integer



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**16.** Evaluate:  $\lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$ , ( $a \neq 0$ ).



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**17.** Evaluate the limit:  $\lim_{x \rightarrow 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$



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**18.** Let  $f''(x)$  be continuous at  $x = 0$

If  $\lim_{x \rightarrow 0} \frac{2f(x) - 3af(2x) + bf(8x)}{\sin^2 x}$  exists and  $f(0) \neq 0, f'(0) \neq 0$ ,

then find the value of  $3a/b$  is \_\_\_\_\_.



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19. Evaluate the limit:  $(\lim)_{h \rightarrow 0} \left[ \frac{1}{h(8+h)^{\frac{1}{3}}} - \frac{1}{2h} \right]$

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20. Evaluate:  $\lim_{x \rightarrow 0} \frac{e - (1+x)^{\frac{1}{x}}}{x}$

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21. Using  $\lim_{\theta \rightarrow 0} \left( \frac{\sin \theta}{\theta} \right) = 1$  prove that the area of circle of radius  $R$  is  $\pi R^2$

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22. Evaluate:  $\lim_{x \rightarrow 1} \sec\left(\frac{\pi}{2x}\right) \log x.$

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**23.** Evaluate :  $\left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} \right]$ , where  $[ \cdot ]$  represents the greatest integer function.



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**24.** Let  $f(x) = \lim_{m \rightarrow \infty} \left\{ \lim_{n \rightarrow \infty} \cos^{2m}(n! \pi x) \right\}$ , where  $x \in R$ . Then prove that  $f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$



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**25.** Evaluate:  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$



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**26.**

Evaluate

$$\lim_{n \rightarrow \infty} n^{-n^2} [(n + 2^0)(n + 2^{-1})(n + 2^{-2}) \dots (n + 2^{-n+1})]^n.$$



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27. Solve:  $\lim_{x \rightarrow \infty} 2^{x-1} \tan\left(\frac{a}{2^x}\right)$



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28. Evaluate  $\lim_{x \rightarrow \pi/2} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \log_e \sin x}.$



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29. Evaluate:  $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 2x - \sin(x-2)}$



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30. Evaluate:  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$



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**31.** Evaluate:  $\lim_{x \rightarrow \infty} x \left( \tan^{-1} \left( \frac{x+1}{x+4} \right) - \frac{\pi}{4} \right)$



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**32.**

Evaluate

$$\lim_{x \rightarrow \frac{\pi}{2}} \tan^2 x \left[ \sqrt{2 \sin^2 x + 3 \sin x + 4} - \sqrt{\sin^2 x + 6 \sin x + 2} \right]$$



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**33.** Evaluate the limit:  $\lim_{x \rightarrow 1} \frac{\sin(\log x)}{\log x}$



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**34.** Let if then one of the possible value of is:



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**35.** Evaluate:  $\lim_{x \rightarrow 0} \frac{1}{x} \sin^{-1} \left( \frac{2x}{1+x^2} \right)$



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**36.** At the endpoint and midpoint of a circular arc AB, tangent lines are drawn, and the points, A and B are jointed with a chord. Prove that the ratio of the areas of the triangles thus formed tends to 4 as the arc AB decreases infinitely.



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**37.**  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = ?$



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**38.** Evaluate  $(\lim)_{x \rightarrow 0} \frac{x - \sin x}{x^3}$ . (Do not use either L'Hopital's rule or series expansion for  $\sin x$ ). Hence, evaluate

$$(\lim)_{x \rightarrow 0} \frac{\sin x - x \cos x + x^2 \cot x}{x^5}$$



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**39.** Evaluate :

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$



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**40.** The value of  $\lim_{n \rightarrow \infty} \left[ \frac{1}{n} + \frac{e^{1/n}}{n} + \frac{e^{2/n}}{n} + \dots + \frac{e^{(n-1)/n}}{n} \right]$  is



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**41.** Find the values of a and b in order that

$$\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1 \text{ [using L' Hospital's rule].}$$



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42.  $(\lim)_{x \rightarrow 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$ , where  $n = 100$ , is equal to :  
 $\frac{5050}{\pi e}$  (b)  $\frac{100}{\pi e}$  (c)  $-\frac{5050}{\pi e}$  (d)  $-\frac{4950}{\pi e}$



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43. Find the integral value of  $n$  for which  
 $\lim_{x \rightarrow 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$  is a finite nonzero number



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44. If  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$ , then



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45. Evaluate:  $\lim_{x \rightarrow 0} \frac{\log \cos x}{x}$



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46. Let  $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$ ,  $a > 0$ . If  $L$  is finite, then



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47. Evaluate:  $\lim_{x \rightarrow 0} \frac{2^x - 1}{(1 + x)^{\frac{1}{2}} - 1}$



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48. The largest value of non negative integer for which

$$\lim_{x \rightarrow 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$



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49. Evaluate:  $\lim_{x \rightarrow 0} x^x$



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50. Let  $m$  and  $n$  be two positive integers greater than 1. if

$$\lim_{\alpha \rightarrow 0} \left( \frac{e^{\cos(\alpha^n)} - e}{\alpha^m} \right) = -\left(\frac{e}{2}\right), \text{then the value of } m/n \text{ is}$$



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51. Evaluate:  $(dy/dx)$  of  $\log \sin x + \tan x$



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52. The integer  $n$  for which  $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$  is a finite non-zero number is :



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53. If  $m, n \in I_0$  and  $\lim_{x \rightarrow 0} \frac{\tan 2x - n \sin x}{x^3} = \text{some integer}$ , then find the value of  $n$  and also the value of limit.



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54. If  $\lim_{x \rightarrow 0} \frac{(a - n)nx - \tan x) \sin nx}{x^2} = 0$ , where  $n$  is non zero real number, then  $a$  is equal to:



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55. If  $(\lim)_{x \rightarrow 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$  is finite, find  $a$  and  $b$  using expansion formula.



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56. The value of  $\lim_{x \rightarrow 0} \left( (\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x} \right)$ , where  $x > 0$ , is (a) 0 (b)  $-1$  (c) 1 (d) 2



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57. If  $\lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$  and  $a > 0$ , then find the value of  $a$ .



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58. If  $\lim_{x \rightarrow 0} [1 + x \ln(1 + b^2)]^{\frac{1}{x}} = 2b \sin^2 \theta$ ,  $b > 0$  and  $\theta \in (-\pi, \pi]$ . then the value of  $\theta$  is



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59. If  $L = \lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3}$  is finite, then find the value of  $a$



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60. Evaluate:  $(\lim)_{x \rightarrow \infty} \left( \frac{a_1^{\frac{1}{x}} + a_2^{\frac{1}{x}} + \dots + a_n^{\frac{1}{x}}}{n} \right)^{nx}$



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61.  $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x} = ?$



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62. Evaluate:  $(\lim)_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$ .



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63. Evaluate:  $\left[ (\lim)_{x \rightarrow 0} \frac{\tan^{-1} x}{x} \right]$ , where [.] represent the greatest integer function



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64. Let  $f(x) = \begin{cases} x + 1, & x > 0, \\ 2 - x, & x \leq 0 \end{cases}$  and  
 $g(x) = \begin{cases} x + 3, & x < 1, \\ x^2 - 2x - 2, & 1 \leq x < 2, \\ x - 5, & x \geq 2 \end{cases}$  Find the LHL and RHL of  $g(f(x))$  at  $x = 0$  and, hence, find  $\lim_{x \rightarrow 0} g(f(x))$ .



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65. Evaluate:  $(\lim)_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1}$



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66.  $\lim_{x \rightarrow 0} \left[ (1 - e^x) \frac{\sin x}{|x|} \right]$  is (where  $[.]$  represents the greatest integer function )



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67. Evaluate:  $\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}$



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68.  $\lim_{x \rightarrow \infty} \frac{\cot^{-1}(x^{-a} \log_a x)}{\sec^{-1}(a^x \log_x a)}$ , ( $a > 1$ ), is equal to



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69. Evaluate:  $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$



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70. The value of  $(\lim)_{x \rightarrow a} \sqrt{a^2 - x^2} \cot\left(\frac{\pi}{2} \sqrt{\frac{a-x}{a+x}}\right)$  is (a)  $\frac{2a}{\pi}$  (b)  
-  $\frac{2a}{\pi}$  (c)  $\frac{4a}{\pi}$  (d)  $-\frac{4a}{\pi}$



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71. Evaluate:  $\lim_{x \rightarrow 0} \frac{\cot 2x - \cos ec 2x}{x}$



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72.  $\lim_{x \rightarrow 0} \frac{\log(1 + x + x^2) + \log(1 - x + x^2)}{\sec x - \cos x} =$

A. -1

B. 1

C. 0

D. 2



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73. Evaluate:  $\lim_{n \rightarrow \infty} n \cos\left(\frac{\pi}{4n}\right) \sin\left(\frac{\pi}{4n}\right)$



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74. The value of

$$\lim_{n \rightarrow \infty} \left[ \frac{2n}{2n^2 - 1} \cos\left(\frac{n+1}{2n-1}\right) - \frac{n}{1-2n} \frac{n(-1)^n}{n^2 + 1} \right]$$

is (a) 1 (b) -1 (c)

0 (d) none of these



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75. Evaluate:  $(\lim)_{h \rightarrow 0} \frac{2\left[\sqrt{3} \sin\left(\frac{\pi}{6} + h\right) - \cos\left(\frac{\pi}{6} + h\right)\right]}{\sqrt{3}h\left(\sqrt{3} \cosh - \sinh\right)}$



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76.

Evaluate:

$$\lim_{x \rightarrow 0} \frac{8}{x^8} \left\{ 1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right) \right\}$$



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77. Evaluate:  $\lim_{x \rightarrow 0} \frac{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)}{\sin^{-1}x}$



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78.

Evaluate:

$$\lim_{n \rightarrow \infty} n^2 \left\{ \sqrt{\left(1 - \cos\left(\frac{1}{n}\right)\right)} \sqrt{\left(1 - \cos\left(\frac{1}{n}\right)\right)} \sqrt{\left(1 - \cos\left(\frac{1}{n}\right)\right)} \dots \infty \right\}$$



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79. Evaluate:  $(\lim)_{x \rightarrow 0, y \rightarrow 0} \frac{y^2 + \sin x}{x^2 + \sin y^2}$  where  $(x, y) \rightarrow (0, 0)$  along the curve  $x = y^2$



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80. Evaluate  $\lim_{n \rightarrow \infty} \left\{ \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right) \cos\left(\frac{x}{8}\right) \dots \cos\left(\frac{x}{2^n}\right) \right\}$ .



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81. Find the value of  $\alpha$  so that  $\lim_{x \rightarrow 0} \frac{1}{x^2} (e^{\alpha x} - e^x - x) = \frac{3}{2}$



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82. If  $x_1$  and  $x_2$  are the real and distinct roots of  $ax^2 + bx + c = 0$  then

prove that  $\lim_{x \rightarrow x_1} (1 + \sin(ax^2 + bx + c))^{\frac{1}{x - x_1}}$ . equals to



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83. If  $\lim_{x \rightarrow 0} (1 + ax + bx^2)^{2/x} = e^3$ , then the value of a and b, is :



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**84.** Evaluate:  $(\lim)_{n \rightarrow \infty} x \left[ \tan^{-1} \left( \frac{x+1}{x+2} \right) - \tan^{-1} \left( \frac{x}{x+2} \right) \right]$

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**85.** If  $(\lim)_{x \rightarrow \infty} \left\{ \frac{x^2 - 1}{x + 1} - (ax + b) \right\} = 2$ , then find the value of  $a$  and  $b$ .

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**86.** Evaluate:  $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$

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**87.** If  $(\lim)_{x \rightarrow 0} \frac{ae^x - b}{x} = 2$ , then find the value of  $a$  and  $b$ .

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**88.** Evaluate:  $(\lim)_{x \rightarrow 1} \frac{x \sin \{x\}}{\{x - 1\}}$  if exists, where  $\{x\}$  is the fractional part of  $x$ .



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**89.** Evaluate:  $\lim_{x \rightarrow 2} \frac{x^2 - 1}{2x + 4}$



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**90.** Evaluate:  $(\lim)_{x \rightarrow 0} \left( 1^{1/\sin^2 x} + 2^{1/(\sin^2 x)} + \dots + n^{1/\sin^2 x} \right)^{\sin^2 x}$



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**91.** Let  $f(x) = \begin{cases} \cos[x], & x \leq 0 \\ |x| + a, & x > 0 \end{cases}$ . Then find the value of  $a$ , so that  $\lim_{x \rightarrow 0} f(x)$  exists, where  $[x]$  denotes the greatest integer function less than or equal to  $x$ .



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**92.** If  $f(x) = |x - 1| - [x]$ , where  $[x]$  is the greatest integer less than or equal to  $x$ , then

(A)  $f(1 + 0) = -1, f(1 - 0) = 0$

(B)  $f(1 + 0) = 0 = f(1 - 0)$

(C)  $(\lim)_{x \rightarrow 1} f(x)$  exists

(D)  $(\lim)_{x \rightarrow 1} f(x)$  does not exist



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**93.** If  $y = 2^{-2^{\left(\frac{1}{1-x}\right)}}$ , then find  $\lim_{x \rightarrow 1^+} y$



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**94.** Let  $f(x) = \begin{cases} 1 + \frac{2x}{a}, & 0 \leq x < 1 \\ ax, & 1 \leq x < 2 \end{cases}$  If  $\lim_{x \rightarrow 1} f(x)$  exists, then  $a$  is

(a) 1

(b) -1

(c) 2

(d) -2



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95. Evaluate  $(\lim)_{x \rightarrow 0} \frac{\sin x - 2}{\cos x - 1}$



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96.  $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$  is equal to :

A.  $-\pi$

B.  $\pi$

C.  $\frac{\pi}{2}$

D. 1



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97. Evaluate  $\lim_{x \rightarrow 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$



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98. For  $x \in R$ ,  $\lim_{x \rightarrow \infty} \left( \frac{x - 3}{x + 2} \right)^x$  is equal to

A.  $e$

B.  $e^{-1}$

C.  $e^{-5}$

D.  $e^5$



99. Evaluate  $\lim_{n \rightarrow \infty} \left[ \sum_{r=1}^n \frac{1}{2^r} \right]$ , where  $[.]$  denotes the greatest integer function.



100.  $\lim_{n \rightarrow \infty} \left\{ \frac{1}{1 - n^2} + \frac{2}{1 - n^2} + \dots + \frac{n}{1 - n^2} \right\}$  is equal to  $\rightarrow$

(a) 0

(b)  $-\frac{1}{2}$

(c)  $\frac{1}{2}$

(d) none of these



**101.** Evaluate  $\lim_{x \rightarrow \frac{5\pi}{4}} [\sin x + \cos x]$ , where  $[.]$  denotes the greatest integer function.



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**102.** If  $G(x) = -\sqrt{25 - x^2}$  then  $\lim_{x \rightarrow 1} \frac{G(x) - G(1)}{x - 1} = ?$



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**103.** Evaluate the left-and right-hand limits of the function defined by  
 $f(x) = \begin{cases} 1 + x^2 & 0 \leq x < 1 \\ 2 - x & x > 1 \end{cases}$  at  $x = 1$  Also, show that  $\lim_{x \rightarrow 1} f(x)$  does not exist



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**104.** If  $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$ , then  $\lim_{x \rightarrow \infty} f(x)$  is



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105. Evaluate the right hand limit and left hand limit of the function

$$f(x) = \begin{cases} \frac{|x-4|}{x-4} & x \neq 4 \\ 0 & x = 4 \end{cases}$$



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106. If  $\lim_{x \rightarrow a} [f(x)g(x)]$  exists, then both  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist.



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107. If  $\alpha_1, \alpha_2, \dots, \alpha_n$  are the roots of equation  $x^n + nax - b = 0$ , show that  $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\dots(\alpha_1 - \alpha_n) = n(\alpha_1^{n-1} + a)$



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**108.** Evaluate :  $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$



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**109.** Evaluate  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$ .



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**110.**  $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$

a. exists and its equals  $\sqrt{2}$

b. exists and its equals  $\sqrt{-2}$

c. does not exist because  $x - 1 \rightarrow 0$

d. L.H.L not equal R.H.L



$$111. \lim_{x \rightarrow 0} \sin^2 \left( \frac{\pi}{2 - px} \right)^{\sec^2 \left( \left( \frac{\pi}{2 - px} \right) \right)}$$



$$112. \text{The value of } \lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x} \text{ is}$$

(a) 1

(b) -1

(c) 0

(d) none of these



**113.** Evaluate:  $\lim_{x \rightarrow \frac{7}{2}} (2x^2 - 9x + 8)^{\cot(2x-7)}$



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**114.** If  $f(x) = \begin{cases} \sin[x], & \text{for } [x] \neq 0 \\ 0, & \text{for } [x] = 0 \end{cases}$  where  $[x]$  denotes the greatest integer less than or equal to  $x$ . Then find  $\lim_{x \rightarrow 0} f(x)$ .



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**115.** Let  $f(a) = g(a) = k$  and their  $n$ th derivatives exist and are not equal for some  $n$ . If.  $(\lim)_{x \rightarrow a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$ .

Find the value of  $k$ .



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**116.** If  $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x^2}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ , ( $n \in I$ ), then (a)  $\lim_{x \rightarrow 0} f(x)$  exists

for  $n > 1$  (b)  $\lim_{x \rightarrow 0} f(x)$  exists for  $n < 0$  (c)  $\lim_{x \rightarrow 0} f(x)$  does not exist

for any value of  $n$  (d)  $\lim_{x \rightarrow 0} f(x)$  cannot be determined



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117. Let  $f(x)$  be a twice-differentiable function and  $f''(0)=2$ . Then evaluate:

$$\lim_{x \rightarrow 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$$



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118. The value of  $\lim_{x \rightarrow 1} \left( \frac{p}{1-x^p} - \frac{q}{1-x^q} \right)$ ,  $p, q, \in N$ , equal (a)  $\frac{p+q}{2}$

(b)  $\frac{pq}{2}$  (c)  $\frac{p-q}{2}$  (d)  $\sqrt{\frac{p}{q}}$



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119. Evaluate :  $\lim_{x \rightarrow 0} (\log)_{\tan^2 x} (\tan^2 2x)$ .



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120.  $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{\ln(\cos(2x^2 - x))}$  is equal to

- (a) 2
- (b) -2
- (c) 1
- (d) -1



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121. If the graph of the function  $y = f(x)$  has a unique tangent at the point  $(a, 0)$  through which the graph passes, then evaluate

$$\lim_{x \rightarrow a} \frac{\log_e\{1 + 6f(x)\}}{3f(x)}$$



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122.  $\lim_{x \rightarrow -1} \frac{1}{\sqrt{|x| - \{-x\}}}$  (where  $\{x\}$  denotes the fractional part of  $x$ )

is equal to



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**123.** Evaluate:  $\lim_{x \rightarrow 0} x^m (\log x)^n$ ,  $m, n \in N$ .



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**124.** Let  $f(x) = (\lim)_{n \rightarrow \infty} \frac{1}{\left(\frac{3}{\pi} \tan^{-1} 2x\right)^{2n}} + 5$ . Then the set of values of  $x$  for which  $f(x) = 0$  is (a)  $|2x| > \sqrt{3}$  (b)  $|2x|$



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**125.** Evaluate  $\left( \lim_{x \rightarrow \infty} \right) \left( x (\log)_e \left\{ \frac{\sin(a + \frac{1}{x})}{\sin a} \right\} \right)$ ,  $0 < a < \frac{\pi}{2}$



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**126.**  $\lim_{x \rightarrow 0} \left\{ (1+x)^{\frac{2}{x}} \right\}$  (where  $\{.\}$  denotes the fractional part of  $x$ )

(a)  $e^2 - 7$

(b)  $e^2 - 8$

(c)  $e^2 - 6$

(d) none of these



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**127.** Evaluate:  $\lim_{x \rightarrow 0} \frac{2^x - 1}{(1+x)^{\frac{1}{2}} - 1}$



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**128.** If  $f(n+1) = \frac{1}{2} \left\{ f(n) + \frac{9}{f(n)} \right\}$ ,  $n \in N$ , and  $f(n) > 0$  for all  $n \in N$ , then find  $\lim_{n \rightarrow \infty} f(n)$



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129. Evaluate  $\lim_{x \rightarrow 1} \frac{a^{x-1} - 1}{\sin \pi x}$ .



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130. Find  $\lim_{x \rightarrow 0} \frac{5x + 2 \cos x}{3x + 14}$  using sandwitch theorem



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131. Evaluate :  $\left[ \lim_{x \rightarrow 0} \frac{\tan x}{x} \right]$ , where  $[ \cdot ]$  represents the greatest integer function.



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132. If  $\lim_{n \rightarrow \infty} \frac{1}{(\sin^{-1} x)^n + 1} = 1$ , then find the range of x.



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**133.** Evaluate :  $\left[ \lim_{x \rightarrow 0} \frac{\tan x}{x} \right]$ , where  $[ \cdot ]$  represents the greatest integer function.



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**134.** Evaluate  $\lim_{x \rightarrow 0} \frac{e^x - e^{x \cos x}}{x + \sin x}$ .



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**135.** Evaluate:  $\lim_{x \rightarrow 2} \frac{x - 2}{(\log)_a(x - 1)}$



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**136.** Evaluate:  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - (1 + \sin x)}{\{\tan^{-1}(\sin x)\}^2}$



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**137.** Evaluate:  $\lim_{x \rightarrow a} \frac{\log x - \log a}{x - a}$



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**138.** Evaluate:  $(\lim)_{x \rightarrow \frac{3\pi}{4}} \frac{1 + (\tan x)^{\frac{1}{3}}}{1 - 2 \cos^2 x}$



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**139.** Evaluate:  $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x}$



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**140.**  $\lim_{x \rightarrow -1} \left( \frac{x^4 + x^2 + x + 1}{x^2 - x + 1} \right)^{\frac{1 - \cos(x+1)}{(x+1)^2}}$  is equal to (a) 1 (b)  $\left(\frac{2}{3}\right)^{\frac{1}{2}}$  (c)  $\left(\frac{3}{2}\right)^{\frac{1}{2}}$  (d)  $e^{\frac{1}{2}}$



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$$141. \text{ Evaluate: } \lim_{x \rightarrow 0} \frac{3^{2x} - 2^{3x}}{x}$$



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142.

$$\lim_{x \rightarrow 2} \left( \left( \frac{x^3 - 4x}{x^3 - 8} \right)^{-1} - \left( \frac{x + \sqrt{2x}}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right) \text{ is equal to } \rightarrow$$

$\frac{1}{2}$

2

1

none of these



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**143.** Evaluate:  $\lim_{x \rightarrow 0} \frac{\log(5+x) - \log(5-x)}{x}$



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**144.** Each question contains statements given in two columns which have to be matched. Statements (a,b,c,d) in column I have to be matched with statements (p,q,r,s) in column II. If the correct match are a-p, s, b-r c-p, q. and d-s, then the correctly bubbled  $4 \times 4$  matrix should be as follows:

figure Column I, a) If  $\int \frac{2^x}{\sqrt{1-4^x}} dx = k \sin^{-1}(f(x)) + C$ , then  $k$  is greater than, b) If  $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx = a \frac{\ln(x^k)}{x^k + 1} + c$ , then  $a$  is less than, c) If  $\int \frac{x^4 + 1}{x(x^2 + 1)^2} dx = k \ln|x| + \frac{m}{1 + x^2} + n$ , where  $n$  is the constant of integration, then  $m$  is greater than, d) If  $\int \frac{dx}{5 + 4 \cos x} = k \tan^{-1}\left(m \tan \frac{x}{2}\right) + C$ , then  $k/m$  is greater than,  
COLUMN II p) 0 q) 1 r) 3 s) 4



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**145.** Let  $P_n = a^{P_{n-1}} - 1$ ,  $\forall n = 2, 3, \dots$ , and let  $P_1 = a^x - 1$ , where  $a \in R^+$ . Then evaluate  $\lim_{x \rightarrow 0} \frac{P_n}{x}$ .



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**146.** Column I ([.] denotes the greatest integer function), Column II

$$(\lim)_{x \rightarrow 0} \left( \left[ 100 \frac{\sin x}{x} \right] + \left[ 100 \frac{\tan x}{x} \right] \right) , \quad p. \quad 198$$

$$(\lim)_{x \rightarrow 0} \left( \left[ 100 \frac{x}{\sin x} \right] + \left[ 100 \frac{\tan x}{x} \right] \right) , \quad q. \quad 199$$

$$(\lim)_{x \rightarrow 0} \left( \left[ 100 \frac{\sin^{-1} x}{x} \right] + \left[ 100 \frac{\tan^{-1} x}{x} \right] \right) , \quad r. \quad 200$$

$$(\lim)_{x \rightarrow 0} \left( \left[ 100 \frac{x}{\sin^{-1} x} \right] + \left[ 100 \frac{\tan^{-1} x}{x} \right] \right) , s. 201$$



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**147.** Let  $f(x) = \begin{cases} x+1, & x > 0 \\ 2-x, & x \leq 0 \end{cases}$  and

$g(x) = \begin{cases} x+3, & x < 1 \\ x^2 - 2x - 2, & 1 \leq x < 2 \\ x-5, & x \geq 2 \end{cases}$  Find the

LHL and RHL of  $g(f(x))$  at  $x = 0$  and, hence, find  $\lim_{x \rightarrow 0} g(f(x))$ .



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148. Evaluate:  $\lim_{x \rightarrow \infty} \frac{x + 7 \sin x}{-2x + 13}$



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149. Find the values of b for which the points  $(2b + 3, b^2)$  lies above of the line  $3x - 4y - a(a-2) = 0 \quad \forall a \in R$ .



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150. If  $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ , show that  $\lim_{x \rightarrow 0} f(x)$  does not exist.



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151. The reciprocal of the value of:

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \left(1 - \frac{1}{n^2}\right) \text{ is}$$



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152. Show that  $(\lim)_{x \rightarrow 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$  does not exist



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153. If  $f(x) = \begin{cases} x^2 + 2 & x \geq 2 \\ 1 - x & x < 2 \end{cases}$ ;  $g(x) = \begin{cases} 2x & x > 1 \\ 3 - x & x \leq 1 \end{cases}$  then

the value of  $\lim_{x \rightarrow 1} f(g(x))$  is



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154. Evaluate  $\lim_{x \rightarrow 2^+} \frac{[x - 2]}{\log(x - 2)}$ , where  $[.]$  represents the greatest integer function.



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155. Find the value of the limit  $(\lim)_{x \rightarrow 0} \left( \frac{(\sqrt{a+x}) - (\sqrt{a-x})}{x} \right)$



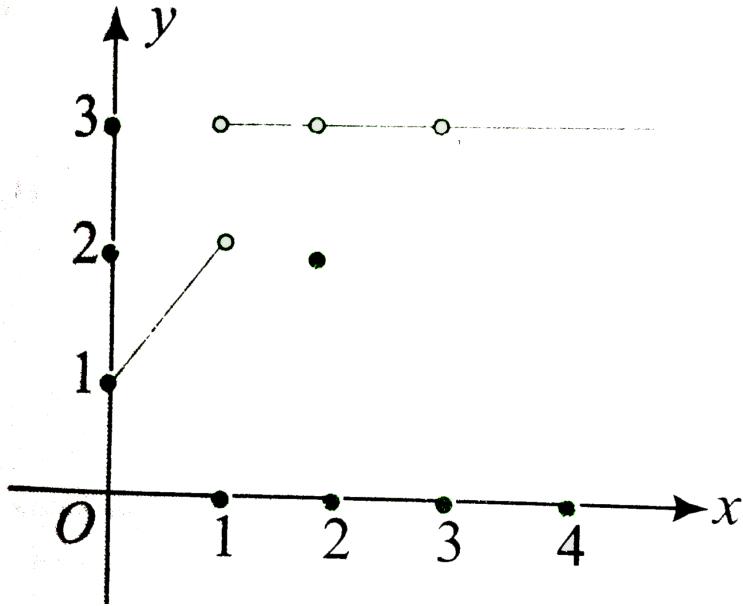
156. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin[\cos x]}{1 + [\cos x]}$  ( $[.]$  denotes the greatest integer function).



157.  $\lim_{x \rightarrow 0} \left( \frac{1^x + 2^x + \dots + n^x}{n} \right)^{\frac{1}{x}}$  is equal to



158. Consider the following graph of the function  $y=f(x)$ . Which of the following is//are correct?



- (a)  $\lim_{x \rightarrow 1} f(x)$  does not exist.
- (b)  $\lim_{x \rightarrow 2} f(x)$  does not exist.
- (c)  $\lim_{x \rightarrow 3} f(x) = 3$ .
- (d)  $\lim_{x \rightarrow 1.99} f(x)$  exists.



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159. If  $\lim_{x \rightarrow 0} \frac{x^n - \sin x^n}{x - \sin^n x}$  is non-zero finite, then  $n$  must be equal to 4 (b)  
 1 (c) 2 (d) 3



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**160.** If  $\lim_{x \rightarrow a} [f(x) + g(x)] = 2$  and  $\lim_{x \rightarrow a} [f(x) - g(x)] = 1$ , then find the value of  $\lim_{x \rightarrow a} f(x)g(x)$ .



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**161.** Among (i)  $\lim_{x \rightarrow \infty} \sec^{-1}\left(\frac{x}{\sin x}\right)$  and (ii)  $\lim_{x \rightarrow \infty} \sec^{-1}\left(\frac{\sin x}{x}\right)$ .



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**162.** Evaluate  $\lim_{x \rightarrow 0} \frac{3x + |x|}{7x - 5|x|}$



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**163.** The value of  $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$  is

(a)  $\frac{1}{2}$

(b)  $-\frac{1}{2}$

(c) 0

(d) none of these



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164.

$$f(x) = \begin{cases} x, & x \leq 0 \\ 1, & x = 0 \end{cases}, \text{ then } f \in d(\lim_{x \rightarrow 0} f(x)x^2, x > 0) \text{ if exists}$$



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165.  $\lim_{x \rightarrow \infty} \frac{e^{1/x^2} - 1}{2 \tan^{-1}(x^2) - \pi}$  is equal to



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166. Evaluate:  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{a + bx}\right)^{c+dx}$ , where  $a, b, c$  and  $d$  are positive



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167. If  $x_1 = 3$  and  $x_{n+1} = \sqrt{2 + x_n}$ ,  $n \geq 1$ , then  $(\lim)_{x \rightarrow \infty} x_n$  is  
(a) -1 (b) 2 (c)  $\sqrt{5}$  (d) 3



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168. Evaluate:  $(\lim)_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$



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169.  $\lim_{x \rightarrow 0} \frac{\cos(\tan x) - \cos x}{x^4}$  is equal to (a)  $\frac{1}{6}$  (b)  $-\frac{1}{3}$  (c)  $\frac{1}{2}$  (d) 1



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170. Evaluate:  $(\lim)_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x}{3} \right)^{\frac{2}{x}}$ ; ( $a, b, c > 0$ )



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171.  $\lim_{x \rightarrow \infty} \{(x + 5)\tan^{-1}(x + 5) - (x + 1)\tan^{-1}(x + 1)\}$  is equal to



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172. If  $f(n) = \lim_{x \rightarrow 0} \left\{ \left(1 + \sin\frac{x}{2}\right) \left(1 + \sin\frac{x}{2^2}\right) \dots \left(1 + \sin\frac{x}{2^n}\right) \right\}^{\frac{1}{x}}$   
then find  $\lim_{n \rightarrow \infty} f(n)$ .



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173. If  $\lim_{x \rightarrow -2^-} \frac{ae^{1/|x+2|} - 1}{2 - e^{1/|x+2|}} = \lim_{x \rightarrow -2^+} \sin\left(\frac{x^4 - 16}{x^5 + 32}\right)$ , then a is



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174. The population of a country increases by 2% every year. If it increases k times in a century, then prove that  $[k] = 7$ , where  $[.]$

represents the greatest integer function.



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175. Evaluate  $\lim_{x \rightarrow -\infty} \left[ \frac{x^4 \sin\left(\frac{1}{x}\right) + x^2}{(1 + |x|^3)} \right].$



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176. Evaluate the limit:  $\lim_{x \rightarrow \infty} \left( \frac{x+2}{x+1} \right)^{x+3}$



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177. ABC is an isosceles triangle inscribed in a circle of radius  $r$ . If  $AB = AC$  and  $h$  is the altitude from A to BC, then triangle ABC has perimeter  $P = 2\left(\sqrt{2hr - h^2} + \sqrt{2hr}\right)$  and area  $A = \text{_____}$  and  $= \text{_____}$  and also  $(\lim)_{h \rightarrow 0} \frac{A}{P^3} = \text{_____}$



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**178.** Evaluate:  $\lim_{x \rightarrow \frac{\pi}{2}} (\cos x)^{\cos x}$



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**179.**  $\lim_{x \rightarrow 0} \left( x^4 \frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1} \right)$  is equal to (a) 1 (b) 0 (c) 2 (d) none  
of these



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**180.** Evaluate:  $\lim_{x \rightarrow 0} (1 + x)^{\cos ex}$



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**181.**  $\lim_{x \rightarrow \infty} \left( \frac{1}{e} - \frac{x}{1+x} \right)^x$  is equal to (a)  $\frac{e}{1-e}$  (b) 0 (c)  $\frac{e}{e^{1-e}}$  (d) does not exist



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**182.** Evaluate:  $\lim_{x \rightarrow 0} (\cos x)^{\cot x}$



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**183.**  $\lim_{x \rightarrow 1} \frac{1 - x^2}{\sin 2\pi x}$  is equal to

(a)  $\frac{1}{2\pi}$

(b)  $-\frac{1}{\pi}$

(c)  $\frac{-2}{\pi}$

(d) none of these



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**184.** Evaluate  $(\lim)_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\left( \frac{\sin x}{x - \sin x} \right)}$



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**185.**  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{2x-1}}$  is equal to (a) 0 (b)  $\infty$  (c)  $\frac{1}{2}$  (d) none of these



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**186.** Evaluate  $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e + \frac{1}{2}ex}{x^2}$ .



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**187.** If  $\lim_{x \rightarrow 0} \frac{x^a \sin^b x}{\sin(x^c)}$ , where  $a, b, c \in \mathbb{R} \sim \{0\}$ , exists and has non-zero value. Then,



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**188.** Evaluate  $\lim_{x \rightarrow 0} \frac{5 \sin x - 7 \sin 2x + 3 \sin 3x}{x^2 \sin x}$ .



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**189.**  $(\lim)_{x \rightarrow \infty} \left\{ \frac{x^3 + 1}{x^2 + 1} - (ax + b) \right\} = 2$ , then (a)  $a = 1, b = 1$  (b)  $a = 1, b = 2$  (c)  $a = 1, b = -2$  (d) none of these



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**190.** Evaluate  $\lim_{x \rightarrow \infty} \frac{\log_e x}{x}$



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**191.** The value of  $\lim_{x \rightarrow 1} (2 - x)^{\tan(\frac{\pi x}{2})}$  is

(a)  $e^{-\frac{2}{\pi}}$

(b)  $e^{\frac{1}{\pi}}$

(c)  $e^{\frac{2}{\pi}}$

(d)  $e^{-\frac{1}{\pi}}$



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192. Evaluate the following limits using sandwich theorem:

$$\lim_{x \rightarrow \infty} \frac{[x]}{x}, \text{ where } [.] \text{ represents greatest integer function.}$$



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193.  $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m}$ , ( $m < n$ ), is equal to (a) 1 (b) 0 (c)  $n/m$  (d) none of these



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**194.** If  $\frac{x^2 + x - 2}{x + 3} \leq \frac{f(x)}{x^2} \leq \frac{x^2 + 2x - 1}{x + 3}$  holds for a certain interval containing the value of  $\lim_{x \rightarrow -1} f(x)$ .



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**195.**  $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + 3\sqrt[3]{x} + 4\sqrt[4]{x} + \dots + n\sqrt[n]{x}}{\sqrt{(2x - 3)} + \sqrt[3]{2x - 3} + \dots + \sqrt[n]{2x - 3}}$  is equal to (a) 1  
(b)  $\infty$  (c)  $\sqrt{2}$  (d) none of these



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**196.** Let  $f: (1, 2) \xrightarrow{R}$  satisfies the inequality  $\frac{\cos(2x - 4) - 33}{2} < f(x) < \frac{x^2|4x - 8|}{x - 2} \forall x \in (1, 2)$ . Then find  $\lim_{x \rightarrow 2^-} f(x)$ .



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197.  $\lim_{y \rightarrow 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$  is equal to (a)  $\sec x(x \tan x + 1)$   
(b)  $x \tan x + \sec x$  (c)  $x \sec x + \tan x$  (d) none of these



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198. Evaluate  $\lim_{n \rightarrow \infty} \frac{1}{1+n^2} + \frac{2}{2+n^2} + \dots + \frac{n}{n+n^2}.$



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199. If  $L = \lim_{x \rightarrow 2} \frac{(10-x)^{\frac{1}{3}} - 2}{x-2}$ , then the value of  $\left| \frac{1}{4L} \right|$  is



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200. Suppose that  $f$  is a function such that  $2x^2 \leq f(x) \leq x(x^2 + 1)$  for all  $x$  that are near to 1 but not equal to 1. Show that this fact contains enough information for us to find  $(\lim)_{x \rightarrow 1} f(x)$ . Also, find this limit.



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201. If  $L = \lim_{x \rightarrow 0} \frac{e^{-x^2/2} - \cos x}{x^3 \sin x}$ , then find the value of  $1/(3L)$  is \_\_\_\_\_.



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202. If  $[.]$  denotes the greatest integer function, then find the value of

$$\lim_{x \rightarrow 0} \frac{[x] + [2x] + \dots + [nx]}{n^2}$$



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203. If  $\lim_{x \rightarrow \infty} f(x)$  exists and is finite and nonzero and if  $\lim_{x \rightarrow \infty} \left\{ f(x) + \frac{3f(x) - 1}{f^2(x)} \right\} = 3$ , then find the value of  $\lim_{x \rightarrow \infty} f(x)$



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**204.** If  $3 - \left(\frac{x^2}{12}\right) \leq f(x) \leq 3 + \left(\frac{x^3}{9}\right)$  in the neighborhood of  $x=0$ , then find the value of  $\lim_{x \rightarrow 0} f(x)$ .



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**205.** If  $f(x) = \begin{cases} x - 1, & x \geq 1 \\ 2x^2 - 2, & x < 1 \end{cases}$ ,  $g(x) = \begin{cases} x + 1, & x > 0 \\ -x^2 + 1, & x \leq 0 \end{cases}$ , and  $h(x) = |x|$ , then  $\lim_{x \rightarrow 0} f(g(h(x)))$  is \_\_\_\_\_.



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**206.** Evaluate  $\lim_{x \rightarrow 0} \left\{ \frac{\sin x - x + \frac{x^3}{6}}{x^5} \right\}$ .



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**207.**  $\lim_{x \rightarrow \infty} f(x)$ , where  $\frac{2x - 3}{x} < f(x) < \frac{2x^2 + 5x}{x^2} \forall x > 0$ , is \_\_\_\_\_.



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208. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x + \log(1 - x)}{x^2}$ .



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209. If  $\lim_{x \rightarrow 0} \left[ 1 + x + \frac{f(x)}{x} \right]^{1/x} = e^3$ , then the value of  $\ln \left( \lim_{x \rightarrow 0} \left[ 1 + \frac{f(x)}{x} \right]^{1/x} \right)$  is \_\_\_\_\_.



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210. Evaluate the limits using the expansion formula of functions

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$$



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211. The value of  $(\lim)_{n \rightarrow \infty} \left[ (n+1)^2 3 - (n-1)^2 3 \right]$  is \_\_\_\_\_



212. Evaluate :  $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4}$



213. If:  $(\lim)_{x \rightarrow 1} (1 + ax + bx^2)^{\frac{c}{(x-1)}} = e^3$ , then Find the conditions of a,b,c



214. Evaluate :  $\lim_{x \rightarrow 1} \left( \frac{2}{1 - x^2} + \frac{1}{x - 1} \right)$



215.  $\lim_{x \rightarrow 0} \left( \frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2} = - -$



216. Evaluate:  $\lim_{x \rightarrow 1} \frac{x^2 + x(\log)_e x - (\log)_e x - 1}{(x^2 - 1)}$



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217. If  $L = \lim_{n \rightarrow \infty} (2 \times 3^2 \times 2^3 \times 3^4 \dots \times 2^{n-1} \times 3^n)^{\frac{1}{(n^2+1)}}$ , and n is even then the value of  $L^4$  is \_\_\_\_\_.



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218. Evaluate  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \sin 2x}{1 + \cos 4x}$



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219. The value of  $\lim_{x \rightarrow \infty} \frac{\log_e (\log_e x)}{e^{\sqrt{x}}}$  is \_\_\_\_\_.



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220. Evaluate :  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$



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221.  $\lim_{y \rightarrow 0} \frac{(x + y)\sec(x + y) - x \sec x}{y}$  is equal to (a)  $\sec x(x \tan x + 1)$   
(b)  $x \tan x + \sec x$  (c)  $x \sec x + \tan x$  (d) none of these



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222. Evaluate  $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$



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223. The value of  $\lim_{m \rightarrow \infty} \left( \cos\left(\frac{x}{m}\right) \right)^m$  is 1 (b) e (c)  $e^{-1}$  (d) none of these



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**224.** Find the value of  $\lim_{x \rightarrow 0} \frac{\sin x + \log_e \left( \sqrt{1 + \sin^2 x} - \sin x \right)}{\sin^3 x}$ .



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**225.** The value of  $\lim_{h \rightarrow 0} \frac{\ln(1 + 2h) - 2\ln(1 + h)}{h^2}$ , is



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**226.** Evaluate :  $\lim_{x \rightarrow \infty} \frac{ax^2 + bx + c}{dx^2 + ex + f}$ .



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**227.** The value of  $\lim_{x \rightarrow 1} (2 - x)^{\tan \left( \frac{\pi x}{2} \right)}$  is

(a)  $e^{-\frac{2}{\pi}}$

(b)  $e^{\frac{1}{\pi}}$

(c)  $e^{\frac{2}{\pi}}$

(d)  $e^{-\frac{1}{\pi}}$



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228. Evaluate  $\lim_{x \rightarrow 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)} - 2\sqrt[3]{(3x-5)}}$ .



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229.  $\lim_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m}$ , ( $m < n$ ), is equal to (a) 1 (b) 0 (c)  $n/m$  (d) none

of these



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230. Evaluate:  $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$



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231.  $\lim_{x \rightarrow 0} \left( x^4 \frac{\cot^4 x - \cot^2 x + 1}{\tan^4 x - \tan^2 x + 1} \right)$  is equal to (a) 1 (b) 0 (c) 2 (d) none  
of these



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232. Evaluate  $\lim_{n \rightarrow \infty} \sin^n \left( \frac{2\pi n}{3n+1} \right)$ ,  $n \in N$ .



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233.  $\lim_{x \rightarrow \infty} \left( \frac{1}{e} - \frac{x}{1+x} \right)^x$  is equal to (a)  $\frac{e}{1-e}$  (b) 0 (c)  $\frac{e}{e^{1-e}}$  (d) does not exist



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234. Evaluate  $\lim_{x \rightarrow 1} \frac{\sqrt{x} + \sqrt{\sqrt{x}} + \sqrt{\sqrt{\sqrt{x}}} + \sqrt{\sqrt{\sqrt{\sqrt{x}}}-4}}{x-1}$ .



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235.  $\lim_{x \rightarrow 1} \frac{1 - x^2}{\sin 2\pi x}$  is equal to

(a)  $\frac{1}{2\pi}$

(b)  $-\frac{1}{\pi}$

(c)  $-\frac{2}{\pi}$

(d) none of these



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236. Evaluate  $\lim_{x \rightarrow 2} \frac{x^{10} - 1024}{x^5 - 32}$



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**237.**  $(\lim)_{x \rightarrow 0} \frac{1}{x} \cos^{-1} \left( \frac{1 - x^2}{1 + x^2} \right)$  is equals to (a) 1 (b) 0 (c) 2 (d) none of these

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**238.** If  $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$  and  $n \in N$ , then find the value of n.

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**239.**  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{2x-1}}$  is equal to (a) 0 (b)  $\infty$  (c)  $\frac{1}{2}$  (d) none of these

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**240.** Evaluate:  $\lim_{x \rightarrow a} \frac{(x + 2)^{\frac{5}{3}} - (a + 2)^{\frac{5}{3}}}{x - a}$

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**241.**  $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + 3x^3 + 4x^4 + \dots + nx^n}{\sqrt{(2x - 3)} + (2x - 3)^3 + \dots + (2x - 3)^n}$  is equal to

- (a) 1 (b)  $\infty$  (c)  $\sqrt{2}$  (d) none of these



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**242.** Evaluate:  $\lim_{x \rightarrow \infty} \left( \frac{\sqrt{x^2 + 1} - 3\sqrt{x^2 + 1}}{4\sqrt{x^4 + 1} - 5\sqrt{x^4 + 1}} \right)$



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**243.** The value of  $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$  is

(a) 4

(b)  $\frac{1}{2}$

(c) 2

(d)  $\frac{1}{4}$



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244. Evaluate  $\lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+c} - \sqrt{x})$ .



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245.  $\lim_{x \rightarrow \infty} \frac{(x+1)^{10} + (x+2)^{10} + \dots + (x+100)^{10}}{x^{10} + 10^{10}}$  is equal to

(a) 0

(b) 1

(c) 10

(d) 100



246. Evaluate:  $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$



247.  $\lim_{x \rightarrow \infty} \left[ \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right]$  is equal to

(a) 0

(b)  $\frac{1}{2}$

(c)  $\log 2$

(d)  $e^4$



**248.** Evaluate:  $\lim_{x \rightarrow 2} \frac{\sin(e^{x-2} - 1)}{\log(x-1)}$



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**249.**  $\lim_{n \rightarrow \infty} \frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$  is equal to

(a) 0

(b) 2

(c) 4

(d)  $\infty$



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**250.** Evaluate:  $\lim_{x \rightarrow 0} \frac{x2^x - x}{1 - \cos x}$



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251.  $\lim_{x \rightarrow 0} \frac{x(e^x - 1)}{1 - \cos x}$  is equal to



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252. Evaluate:  $\lim_{x \rightarrow \infty} [x(a^{\frac{1}{x}} - 1)], a > 1$



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253. If  $f(x) = \frac{2}{x-3}$ ,  $g(x) = \frac{x-3}{x+4}$  , and  $h(x) = -\frac{2(2x+1)}{x^2+x-12}$

then  $\lim_{x \rightarrow 3} [f(x) + g(x) + h(x)]$  is

(a) -2

(b) -1

(c)  $-\frac{2}{7}$

(d)0



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254. Evaluate:  $\lim_{x \rightarrow 0} \frac{(1 - 3^x - 4^x + 12^x)}{\sqrt{(2 \cos x + 7)} - 3}$



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255.  $\lim_{x \rightarrow \infty} \left( \frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)$  is equal to



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256. Evaluate:  $\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}, a > 0$



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**257.**  $\lim_{x \rightarrow \infty} \frac{(2x+1)^{40}(4x-1)^5}{(2x+3)^{45}}$  is equal to

(a) 16

(b) 24

(c) 32

(d) 8



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**258.** Evaluate:  $\lim_{x \rightarrow a} \frac{\log(x-a)}{\log(e^x - e^a)}$



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**259.** The value of  $\lim_{x \rightarrow 2} \frac{\sqrt{1 + \sqrt{2+x}} - \sqrt{3}}{x-2}$  is



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260. Evaluate:  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{x^2}$



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261.  $\lim_{n \rightarrow \infty} n^2 \left( x^{\frac{1}{n}} - x^{\frac{1}{(n+1)}} \right)$ ,  $x > 0$  , is equal to (a)0 (b)  $e^x$  (c)  $(\log)_e x$   
(d) none of these



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262. Evaluate:  $(\lim)_{x \rightarrow \infty} x^{\frac{1}{x}}$



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263. The value of  $\lim_{x \rightarrow \pi} \frac{1 + \cos^3 x}{\sin^2 x}$  is

(a)  $\frac{1}{3}$

(b)  $\frac{2}{3}$

(c)  $-\frac{1}{4}$

(d)  $\frac{3}{2}$



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**264.** Evaluate:  $\lim_{x \rightarrow 0} \frac{(729)^x - (243)^x - (81)^x + 9^x + 3^x - 1}{x^3}$



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**265.** If  $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$ , then which of the following can be correct

- (a)  $(\lim)_{x \rightarrow 1} f(x) f$  or  $a = -2$  (b)  $(\lim)(x \rightarrow -2) f(x) f$  or  $a = 13$   
(c)  $(\lim)(x \rightarrow 1) f(x) = \frac{4}{3}$  (d)  $(\lim)(x \rightarrow -2) f(x) = -\frac{1}{3}$



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266. Evaluate  $\lim_{n \rightarrow \infty} (-1)^{n-1} \sin(\pi\sqrt{n^2 + 0.5n + 1})$ , where  $n \in N$



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267.  $(\lim)_{n \rightarrow \infty} \frac{1}{1 + n \sin^2 nx}$  is equal to (a) -1 (b) 0 (c) 1 (d)  $\infty$



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268. Let the sequence  $\langle b_n \rangle$  of real numbers satisfy the recurrence relation  $b_{n+1} = \frac{1}{3} \left( 2b_n + \frac{125}{b_n^2} \right)$ ,  $b_n \neq 0$ . Then find  $\lim_{n \rightarrow \infty} b_n$ .



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269. Which of the following true ( $\{.\}$  denotes the fractional part of the function)?  $(\lim)_{x \rightarrow \infty} \frac{(\log_e x)}{\{x\}} = \infty$  (b)  $(\lim)_{x \rightarrow 2^+} \frac{x}{x^2 - x - 2} = \infty$

$$(\lim)_{x \rightarrow 1^-} \frac{x}{x^2 - x - 2} = -\infty \text{ (d)} (\lim)_{x \rightarrow \infty} \frac{(\log)_{0.5} x}{\{x\}} = \infty$$



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270. Evaluate:  $\lim_{n \rightarrow \infty} (4^n + 5^n)^{\frac{1}{n}}$



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271. If  $(\lim)_{x \rightarrow 1}(2 - x + a[x - 1] + b[1 + x])$  exists, then  $a$  and  $b$  can take the values of (where  $[.]$  denotes the greatest integer function). (a)  $a = \frac{1}{3}, b = 1$  (b)  $a = 1, b = -1$  (c)  $a = 9, b = -9$  (d)  $a = 2, b = \frac{2}{3}$



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272. Evaluate  $\lim_{n \rightarrow \infty} \frac{n^p \sin^2(n!)}{n + 1}$ , where  $0 < p < 1$ .



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273. If  $\lim_{n \rightarrow \infty} \left( an - \frac{1+n^2}{1+n} \right) = b$  a finite number then



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274. Evaluate  $\lim_{x \rightarrow \infty} x^3 \left\{ \sqrt{x^2 + \sqrt{1+x^4}} - x\sqrt{2} \right\}.$



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275. prove that, If  $m, n \in N$ ,  $(\lim)_{x \rightarrow 0} \frac{\sin x^n}{(\sin x)^m}$  is (a) 1, if  $n=m$  (b) 0, if  $n>m$



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276. For any two complex numbers  $z_1, z_2$  and any real numbers  $a$  and  $b$ ,  $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$



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277.  $\lim_{n \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$  is equal to a.  $-\frac{3}{4}$  b. 0 if n is even c.  $-\frac{3}{4}$  if n is odd d. none of these



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278. Evaluate  $\lim_{x \rightarrow \infty} \left( \sqrt{25x^2 - 3x} + 5x \right)$ .



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279. Given a real-valued function  $f$  such that

$$f(x) = \begin{cases} \tan^2\{x\} & \text{Where } [x] \text{ is} \\ (x^2 - [x]^2) \sqrt{\{x\}\cot\{x\}}, f \text{ or } x < 0, fx > 0 & \end{cases}$$

the integral part and  $\{x\}$  is the fractional part of  $x$ , then

$$(\lim)_{x \rightarrow 0^+} f(x) = 1 , \quad (\lim)_{x \rightarrow 0^-} f(x) = \cot 1,$$

$$\cot^{-1} \left( (\lim)_{x \rightarrow 0^-} f(x) \right)^2 = 1, \tan^{-1} \left( (\lim)_{x \rightarrow 0^+} f(x) \right) = \frac{\pi}{4}$$



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**280.** Evaluate  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + x - 1}{3x^2 + 2x + 4} \right)^{\frac{3x^2 + x}{x - 2}}$



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**281.**  $L = \lim_{x \rightarrow a} \frac{|2 \sin x - 1|}{2 \sin x - 1}$ . Then

- (a) limit does not exist when  $a = \frac{\pi}{6}$
- (b)  $L = -1$  when  $a = \pi$
- (c)  $L = 1$  when  $a = \frac{\pi}{2}$
- (d)  $L = 1$  when  $a = 0$



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**282.** If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then evaluate  $\lim_{n \rightarrow \infty} \frac{1}{n^3} ([1^2 x] + [2^2 x] + [3^2 x] + \dots + [n^2 x]).$



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**283.**  $f(x) = (\lim)_{n \rightarrow \infty} \frac{x}{x^{2n} + 1}$  Then, A.  $f(1^+) + f(1^{-1}) = 0$  B.  $f(1^+) + f(1^-) + f(1) = \frac{3}{2}$  C.  $f(-1^+) + f(-1^-) = -1$  D.  $f(1^+) + f(-1^-) = 0$



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**284.** Evaluate the limit:  $\lim_{x \rightarrow 1} \frac{(2x - 3)(\sqrt{x} - 1)}{2x^2}$



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**285.**  $(\lim)(n \rightarrow \infty) \sum_{x=1}^{20} \cos^{2n}(x - 10)$  is equal to



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**286.**  $\lim_{x \rightarrow 1} \frac{1 + \sin \pi \left( \frac{3x}{1+x^2} \right)}{1 + \cos \pi x}$  is equal to



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**287.**  $f(x) = \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}$ . Then  $\lim_{x \rightarrow \infty} f(x)$  is equal to

(a) 1

(b)  $\frac{1}{2}$

(c) 2

(d) none of these



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**288.**  $\lim_{n \rightarrow \infty} \left\{ \left( \frac{n}{n+1} \right)^\alpha + \sin \frac{1}{n} \right\}^n$  (where  $\alpha \in Q$ ) is equal to



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**289.** The value of  $\lim_{x \rightarrow 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$  is



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**290.** If  $\lim_{n \rightarrow \infty} \frac{n \cdot 3^n}{n(x-2)^n + n \cdot 3^{n+1} - 3^n} = \frac{1}{3}$ , then the range of  $x$  is  
(where  $n \in N$ )



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**291.** If  $(\lim)_{x \rightarrow a}[f(x)g(x)]$  exists, then both  $(\lim)_{x \rightarrow a}f(x)$  and  $(\lim)_{x \rightarrow a}g(x)$  exist.



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**292.** If  $f(x) = \lim_{n \rightarrow \infty} n(x^{1/n} - 1)$ , then for  $x > 0, y > 0$ ,  $f(xy)$  is equal to



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293.  $\lim_{n \rightarrow \infty} \left( \frac{n^2 - n + 1}{n^2 - n - 1} \right)^{n(n-1)}$  is



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294.  $\lim_{x \rightarrow 1} \left[ \cos ec \frac{\pi x}{2} \right]^{1/(1-x)}$  (where  $[.]$  represents the greatest integer function) is equal to



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295. Given  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 2$ , where  $[.]$  denotes the greatest integer function, then which options are correct?



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296. Let  $f(x) = \frac{x^2 - 9x + 20}{x - [x]}$  (where  $[x]$  is the greatest integer not greater than  $x$ ). Then (A)  $(\lim)_{x \rightarrow 5} f(x) = 1$  (B)  $(\lim)_{x \rightarrow 5} f(x) = 0$  (C)

$(\lim)_{x \rightarrow 5} f(x)$  does not exist. (D)none of these



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**297.** Use formula  $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log(a)$  to find  $\lim_{x \rightarrow 0} \frac{2^x - 1}{(1 + x)^{\frac{1}{2}} - 1}$



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**298.** Evaluate:  $\lim_{x \rightarrow 0} \left\{ \tan\left(\frac{\pi}{4} + x\right) \right\}^{\frac{1}{x}}$



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**299.**  $f(x)$  is the integral of  $\frac{2 \sin x - \sin 2x}{x^3}$ ,  $x \neq 0$ . Find  $\lim_{x \rightarrow 0} f'(x)$  [where  $f'(x) = \frac{df}{dx}$ ]



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300. Evaluate  $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$ .



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301.  $\lim_{x \rightarrow \infty} \frac{x(\log x)^3}{1+x+x^2}$  equals



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302.  $\lim_{x \rightarrow 0} \frac{(2^m + x)^{1/m} - (2^n + x)^{1/n}}{x}$  is equal to



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303.  $\lim_{x \rightarrow 1} (1-x)\tan\left(\frac{\pi x}{2}\right)$



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**304.** If  $f(x) = \{\sin x, x \neq n\pi, n \in \mathbb{Z}; \text{otherwise}\}$

$g(x) = \{x^2 + 1, x \neq 0, 4, x = 05, x = 2 \text{ then } (\lim_{x \rightarrow 0}) g\{f(x)\} \text{ is } =$



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**305.**  $(\lim_{x \rightarrow 0}) \left[ \min (y^2 - 4y + 11) \frac{\sin x}{x} \right]$  (where  $\lfloor \cdot \rfloor$  denotes the greatest integer function is 5 (b) 6 (c) 7 (d) does not exist



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**306.**  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin(x \cos x)}{\cos(x \sin x)}$  is equal to (a) 0 (b)  $\frac{\pi}{2}$  (c)  $\pi$  (d)  $2\pi$



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**307.** If  $\lim_{x \rightarrow 0} (x^{-3} \sin 3x + ax^{-2} + b)$  exists and is equal to 0, then

(a)  $a = -3$  and  $b = \frac{9}{2}$  (b)  $a = 3$  and  $b = \frac{9}{2}$  (c)

$a = -3$  and  $b = -\frac{9}{2}$  (d)  $a = 3$  and  $b = -\frac{9}{2}$



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308. If  $\lim_{x \rightarrow 0} \frac{x^n - \sin x^n}{x - \sin^n x}$  is non-zero finite, then  $n$  must be equal to 4 (b)  
1 (c) 2 (d) 3



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309.  $\lim_{x \rightarrow 1} \frac{(1-x)(1-x^2)\dots(1-x^{2n})}{\{(1-x)(1-x^2)\dots(1-x^n)\}^2}$ ,  $n$  in  $N$ , equals`



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310. The value of  $\lim_{x \rightarrow 0} \left( \left[ \frac{100x}{\sin x} \right] + \left[ \frac{99 \sin x}{x} \right] \right)$  (where  $[.]$  represents the greatest integral function) is

(a) 199

(b) 198

(c) 0

(d) none of these



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311. The value of  $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \left( \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)} \right)$  is

(a)  $-\frac{1}{\sqrt{2}}$

(b)  $\frac{1}{\sqrt{2}}$

(c)  $\sqrt{2}$

(d)  $-\sqrt{2}$



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312. The value of  $\lim_{x \rightarrow \infty} \frac{(2^{x^n})e^{\frac{1}{x}} - (3^{x^n})e^{\frac{1}{x}}}{x^n}$  (where  $n \in N$ ) is

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313. Let  $\lim_{x \rightarrow 0} \frac{[x]^2}{x^2} = m$ , where  $[.]$  denotes greatest integer. Then, m equals to

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314.  $\lim_{x \rightarrow 1} \frac{x \sin(x - [x])}{x - 1}$ , where  $[.]$  denotes the greatest integer function, is equal to

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315.  $\lim_{x \rightarrow 0} \left[ \frac{\sin(sgn(x))}{(sgn(x))} \right]$ , where  $[.]$  denotes the greatest integer function, is equal to

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316.  $\lim_{x \rightarrow \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$  is equal to



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317. If  $f(x) = \frac{\cos x}{(1 - \sin x)^{\frac{1}{3}}}$  then (a)  $(\lim)_{x \rightarrow \frac{\pi}{2}} f(x) = -\infty$  (b)  $(\lim)_{x \rightarrow \frac{\pi}{2}} f(x) = \infty$  (c)  $(\lim)_{x \rightarrow \frac{\pi}{2}} f(x) = 0$  (d) none of these



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318.  $\lim_{x \rightarrow -\infty} \frac{x^2 \cdot \tan\left(\frac{1}{x}\right)}{\sqrt{8x^2 + 7x + 1}}$  is



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319.  $T_1$  is an isosceles triangle in circle C. Let  $T_2$  be another isosceles triangle inscribed in C whose base is one of the equal sides of  $T_1$  and

which overlaps the interior of  $T_1$ . Similarly, create isosceles triangle  $T_3$  from  $T_2$ ;  $T_4$  and  $T_5$ , and so on. Prove that the triangle  $T_n$ , approaches an equilateral triangle as  $n \rightarrow \infty$ ,

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320. If  $f(x) = 0$  is a quadratic equation such that  $f(-\pi) = f(\pi) = 0$  and  $f\left(\frac{\pi}{2}\right) = -\frac{3\pi^2}{4}$ , then  $\lim_{x \rightarrow -\pi^+} \frac{f(x)}{\sin(\sin x)}$  is equal to

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321.  $\lim_{x \rightarrow \infty} \left[ \left( \frac{e}{1-e} \right) \left( \frac{1}{e} - \frac{x}{1+x} \right) \right]^x$  is :

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322.  $(\lim)_{x \rightarrow 0} \left( \frac{1 + \tan x}{1 + \sin x} \right)^{\cos ex}$  is equal to

(a)e

(b)  $\frac{1}{e}$

(c) 1

(d) none of these



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323.  $\lim_{x \rightarrow \infty} \frac{\sin^4 x - \sin^2 x + 1}{\cos^4 x - \cos^2 x + 1}$  is equal to

(a) 0

(b) 1

(c)  $\frac{1}{3}$

(d)  $\frac{1}{2}$



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**324.** Find  $\frac{dy}{dx}$  if  $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$



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**325.** Find  $\frac{dy}{dx}$  if  $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ , where  $x \neq 0$



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**326. (c)** answer any three questions: 1. if  $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ , find  $\frac{dy}{dx}$ .



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**327.** If  $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ ,  $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ , then find  $\frac{dy}{dx}$



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**328.** Find  $\frac{dy}{dx}$  for the functions:  $y = \frac{x + \sin x}{x + \cos x}$

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**329.** Find the derivative of the function given by  
 $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$  and, hence, find  $f'(1)$ .

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**330.** Find  $\frac{dy}{dx}$  if  $y = \sec^{-1} \left( \frac{\sqrt{x} + 1}{\sqrt{x} - 1} \right) + \sin^{-1} \left( \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \right)$

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**331.** Let  $f(x) = x \sin \pi x$ ,  $x > 0$ . Then for all natural numbers  $n$ ,  $f'(x)$  vanishes at (a) A unique point in the interval  $\left(n, n + \frac{1}{2}\right)$  (b) a unique point in the interval  $\left(n + \frac{1}{2}, n + 1\right)$  (c) a unique point in the interval  $(n, n + 1)$  (d) two points in the interval  $(n, n + 1)$



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332. Find  $\frac{dy}{dx}$  if  $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$



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333. If  $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ ;  $0 < x < (\sqrt{2})$ , then find  $\frac{dy}{dx}$



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334.  $\frac{d^2x}{dy^2}$  equals: (1)  $\left(\frac{d^2y}{dx^2}\right)^{-1}$  (2)  $-\left(\frac{d^2y}{dx^2}\right)^{-1} \left(\frac{dy}{dx}\right)^{-3}$  (3)  
 $\left(\frac{d^2y}{dx^2}\right)^{-1} \left(\frac{dy}{dx}\right)^{-2}$  (4)  $-\left(\frac{d^2y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-3}$



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335. Find  $\frac{dy}{dx}$  if  $y = \log\left\{e^x \left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$



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336. let  $f(x) = 2 + \cos x$  for all real  $x$ . Statement 1: For each real  $t$ , there exists a point  $c$  in  $[t, t + \pi]$  such that  $f'(c) = 0$ . Statement 2:  $f(t) = f(t + 2\pi)$  for each real  $t$

- A. Only statement 1 is correct
- B. Only statement 2 is correct
- C. Both statements are correct
- D. None of these



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337. Differentiate the function with respect to  $x$  using the first principle :  
 $\tan x$



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**338.** Differentiate the function with respect to  $x$  using the first principle :

$$\cos^2 x$$



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**339.** Differentiate the following functions with respect to  $x$  from first

principles:  $\sqrt{\sin x}$



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**340.** If  $f(x)$  is differentiable and strictly increasing function, then the value

of  $\lim_{x \rightarrow 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$  is



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**341.** If  $f(x) = [2x]\sin 3\pi x$  then prove that  $f'(k^+) = 6k\pi(-1)^k$ , (where  $[.]$  denotes the greatest integer function and  $k \in N$ ).



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**342.**  $\lim_{h \rightarrow 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$  given that  $f'(2) = 6$  and  $f'(1) = 4$   
then (a) limit does not exist (b) is equal to  $-\frac{3}{2}$  (c) is equal to  $\frac{3}{2}$  (d) is  
equal to 3



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**343.** Evaluate  $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin^{-1}(a+h) - a^2 \sin^{-1} a}{h}$



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**344.** If  $x^2 + y^2 = 1$ , then

(a)  $yy'' - 2(y')^2 + 1 = 0$

(b)  $yy'' + (y')^2 + 1 = 0$

(c)  $yy'' + (y')^{-2} - 1 = 0$

(d)  $yy'' + 2(y')^2 + 1 = 0$



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345. Using the first principle, prove that  $\frac{d}{dx}(1 - x^2) = -2x$



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346. If  $y$  is function of  $x$  and  $\log(x+y)=2xy$ , then find the value of  $y'(0)$ .



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**347.** Find the derivative of  $\sqrt{4 - x}$  w.r.t.  $x$  using the first principle.



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**348.** The slope of the tangent to the curve  $(y - x^5)^2 = x(1 + x^2)^2$  at the point (1,3) is



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**349.** If  $f(x) = x \tan^{-1} x$ , find  $f'(\sqrt{3})$ .



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**350.** Let  $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin \theta}{\sqrt{\cos 2\theta}}\right)\right)$ , where  $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$  then the value of  $\frac{d}{d(\tan \theta)} f(\theta)$  is



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351. Find the derivative of  $e^{\sqrt{x}}$  w.r.t.  $x$  using the first principle.



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352.  $f(x) = \tan^{-1} \left\{ \frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)} \right\} + \tan^{-1} \left( \frac{3 + 2 \log x}{1 - 6 \log x} \right)$  then find  $\frac{d^n y}{dx^n}$



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353. If

$$f\left(\frac{x+2y}{3}\right) = \frac{f(x) + 2f(y)}{3} \quad \forall x, y \in R \text{ and } f'(0) = 1, f(0) = 2, \text{ then}$$

find  $f(x)$ .



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354. If  $f(x) = |x^2 - 5x + 6|$ , then  $f'(x)$  equals '2x-5 for 2 < x < 3'



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**355.** If graph of  $y = f(x)$  is symmetrical about the point  $(5, 0)$  and  $f'(7) = 3$ , then the value of  $f'(3)$  is \_\_\_\_\_



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**356.** Let  $f(x^m y^n) = mf(x) + nf(y)$  for all  $x, y \in R^+$  and for all  $m, n \in R$ . If  $f'(x)$  exists and has the value  $\frac{e}{x}$ , then find  $\lim_{x \rightarrow 0} \frac{f(1+x)}{x}$



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**357.** Let  $g: \overrightarrow{RR}$  be a differentiable function satisfying  $g(x) = g(y)g(x-y) \forall x, y \in R$  and  $g'(0) = a$  and  $g'(3) = b$ . Then find the value of  $g'(-3)$ .



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**358.** If  $f$ ,  $g$ , and  $h$  are differentiable functions of  $x$  and

$$d(x) = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2f)'' & (x^2g)'' & (x^2h)'' \end{vmatrix}$$

prove that

$$d'(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$$



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**359.** Let  $f$  be a function such that  $f(x + y) = f(x) + f(y)$  for all  $x$  and  $y$  and  $f(x) = (2x^2 + 3x)g(x)$  for all  $x$ , where  $g(x)$  is continuous and  $g(0) = 3$ . Then find  $f'(x)$ .



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**360.** If  $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$  for all  $x, y$   $f'(2) = 2$  then find  $f(x)$



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**361.** Prove that  $\lim_{x \rightarrow 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x)$

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**362.** Let  $f(x+y) = f(x) + f(y) + 2xy - 1$  for all real  $x$  and  $y$  and  $f(x)$  be a differentiable function. If  $f'(0) = \cos \alpha$ , then prove that  $f(x) > 0 \forall x \in R$ .

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**363.** If  $x = e^{y+e^{y+e^y+\dots}}$ , where  $x > 0$ , then find  $\frac{dy}{dx}$

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**364.** Let  $f: R \xrightarrow{R}$  be a function satisfying condition  $f(x+y^3) = f(x) + [f(y)]^3 f$  or all  $x, y \in R$ . If  $f'(0) \geq 0$ , find  $f(10)$ .

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365. If  $xy = e^{(x-y)}$ , then find  $\frac{dy}{dx}$



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366.  $\frac{dy}{dx}$  for  $y = \tan^{-1} \left\{ \sqrt{\frac{1 + \cos x}{1 - \cos x}} \right\}$ , where  $0 < x < \pi$ , is



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367. If  $y^x = x^y$ , then find  $\frac{dy}{dx}$ .



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368. Differentiate  $(x \cos x)^x$  with respect to  $x$ .



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**369.** If  $f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$  holds for all real  $x$  and  $y$  greater than 0 and  $f(x)$  is a differentiable function for all  $x > 0$  such that  $f(e) = \frac{1}{e}$ , then find  $f(x)$



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**370.** Find  $\frac{dy}{dx}$  for  $y = x^x$ .



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**371.** If  $P_n$  is the sum of a  $GP$  upto  $n$  terms ( $n \geq 3$ ), then prove that  $(1 - r) \frac{dP_n}{dr} = (1 - n)P_n + nP_{n-1}$ , where  $r$  is the common ratio of  $GP$ .



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**372.** If  $y = f(a^x)$  and  $f'(\sin x) = (\log_e x)$ , then find  $\left(\frac{dy}{dx}\right)$ , if it exists,

where  $\pi/2 < x < \pi$



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**373.** if  $x < 1$  then  $\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \dots \dots \dots \infty$



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**374.** Find the derivative of  $\frac{\sqrt{x}(x+4)^{\frac{3}{2}}}{(4x-3)^{\frac{4}{3}}}$



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**375.** Find  $(\lim)_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$  if  $f(2)=4$  and  $f'(2)=1$



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**376.** If  $y = x^{x^{x^{\dots \infty}}}$ , find  $\frac{dy}{dx}$

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**377.** If  $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$ , then  $\frac{dy}{dx}$  is equal to (a)  $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$  (b)  $\frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}}$   
(c)  $\frac{1}{2\sqrt{x}}\sqrt{y^2 - 4}$  (d)  $\frac{1}{2\sqrt{x}}\sqrt{y^2 + 4}$

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**378.** Differentiate  $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$  with respect to  $x$

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**379.** If  $f(x) = |\sin x - |\cos x||$ , then the value of  $f'(x)$  at  $x = \frac{7\pi}{6}$  is  
(a) positive (b)  $\frac{1 - \sqrt{3}}{2}$  (c) 0 (d) none of these

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**380.** If  $y = (\tan x)^{(\tan x)^{(\tan x) \dots \infty}}$ , then find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$



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**381.** If graph of  $y = f(x)$  is symmetrical about the y-axis and that of  $y = g(x)$  is symmetrical about the origin and if  $h(x) = f(x)g(x)$ , then  $\frac{d^3h(x)}{dx^3}$  at  $x=0$  is (a) cannot be determined (b)  $f(0)g(0)$  (c) 0 (d) none of these



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**382.** Differentiate the function with respect to  $x$  using the first principle :  
 $(\log)_e x$



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**383.** If  $x = t^2$ ,  $y = t^3$ , then  $\frac{d^2y}{dx^2}$  is (a)  $\frac{3}{2}$  (b)  $\frac{3}{4t}$  (c)  $\frac{3}{2t}$  (d)  $\frac{3t}{2}$



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384. Using the first principle, prove that:

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$



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385. If  $y = x + e^x$ , then  $\frac{d^2y}{dx^2}$  is

(a)  $e^x$

(b)  $-\frac{e^x}{(1 + e^x)^3}$

(c)  $-\frac{e^x}{(1 + e^x)^2}$

(d)  $\frac{-1}{(1 + e^x)^3}$



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**386.** If  $y = \left(1 + x^{\frac{1}{4}}\right)\left(1 + x^{\frac{1}{2}}\right)\left(1 - x^{\frac{1}{4}}\right)$ , then find  $\frac{dy}{dx}$ .



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**387.** Let  $f(x) = (\lim)_{h \rightarrow 0} \frac{(\sin(x+h))^{1n(x+h)} - (\sin x)^{1nx}}{h}$ . Then  $f\left(\frac{\pi}{2}\right)$  equal to (a) 0 (b) equal to 1 (c)  $\ln \frac{\pi}{2}$  (d) non-existent



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**388.** If  $f(x) = x|x|$ , then prove that  $f'(x) = 2|x|$



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**389.** A function  $f: R \rightarrow R$  satisfies  $\sin x \cos y(f(2x+2y) - f(2x-2y) = \cos x \sin y(f(2x+2y) + f(2x-2y))$

If  $f'(0) = \frac{1}{2}$ , then (a)  $f''(x) = f(x) = 0$  (b)  $4f''(x) + f(x) = 0$  (c)  $f''(x) + f(x) = 0$  (d)  $4f''(x) - f(x) = 0$



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390. If  $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ ,  $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$ , then find  $\frac{dy}{dx}$ .



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391. If  $x = \log p$  and  $y = \frac{1}{p}$ , then

(a)  $\frac{d^2y}{dx^2} - 2p = 0$

(b)  $\frac{d^2y}{dx^2} + y = 0$

(c)  $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(d)  $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$



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392. If  $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$ , show that  $\frac{dy}{dx} - y + \frac{x^n}{n!} = 0$ .



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393. Let  $y = \ln(1 + \cos x)^2$ . Then the value of  $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$  equal

(a) 0

(b)  $\frac{2}{1 + \cos x}$

(c)  $\frac{4}{1 + \cos x}$

(d)  $\frac{-4}{(1 + \cos x)^2}$



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394. Find  $\frac{dy}{dx}$  for  $y = x \sin x \log x$ .



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395. If the function  $f(x) = -4e^{\frac{1-x}{2}} + 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$  and  $g(x) = f^{-1}(x)$ , then the reciprocal of  $g'\left(\frac{-7}{6}\right)$  is \_\_\_\_\_



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396. Differentiate  $y = \frac{e^x}{1 + \sin x}$



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397. Suppose that  $f(0) = 0$  and  $f'(0) = 2$ , and let  $g(x) = f(-x + f(f(x)))$ . The value of  $g'(0)$  is equal to \_\_\_\_\_



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398. If  $y = \sqrt{\frac{1-x}{1+x}}$  find  $\frac{dy}{dx}$  and prove that  $(1-x^2)\frac{dy}{dx} + y = 0$



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399. If  $g(x) = \frac{f(x)}{(x-a)(x-b)(x-c)}$ , where  $f(x)$  is a polynomial of degree  $< 3$ , then

$$\int g(x) dx = \begin{vmatrix} 1 & a & f(a)\log|x-a| \\ 1 & b & f(b)\log|x-b| \\ 1 & c & f(c)\log|x-c| \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + k$$
$$\frac{dg(x)}{dx} = \begin{vmatrix} 1 & a & -f(a)(x-a)^{-2} \\ 1 & b & -f(b)(x-b)^{-2} \\ 1 & c & -f(c)(x-c)^{-2} \end{vmatrix} : \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$



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400. If  $f(x) = \cos x \cdot \cos 2x \cdot \cos 4x \cdot \cos 8x \cdot \cos 16x$ , then  $f'(\pi/4)$  is :



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**401.**  $f(x) = e^{-\frac{1}{x}}$ , where  $x > 0$ . Let for each positive integer  $n$ ,  $P_n$  be the polynomial such that  $\frac{d^n f(x)}{dx^n} = P_n\left(\frac{1}{x}\right)e^{-\frac{1}{x}}$  for all  $x > 0$ . Show that  $P_{n+1}(x) = x^2 \left[ P_n(x) - \frac{d}{dx} P_n(x) \right]$



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**402.** If  $a_i, b_i \in N$  for  $i = 1, 2, 3$ , then coefficient of  $x$  in the determinant;

$$\begin{vmatrix} (1+x)^{a_1 b_1} & (1+x)^{a_1 b_2} & (1+x)^{a_1 b_3} \\ (1+x)^{a_2 b_1} & (1+x)^{a_2 b_2} & (1+x)^{a_2 b_3} \\ (1+x)^{a_3 b_1} & (1+x)^{a_3 b_2} & (1+x)^{a_3 b_3} \end{vmatrix}$$



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**403.**

$$\frac{d}{dx} [(x^m - A_1 x^{m-1} + A_2 x^{m-2} - \dots + (-1)^m A_m) e^x] = x^m e^x,$$

find the value of  $A_r$



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**404.** If  $y = \begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$  find  $\frac{dy}{dx}$ .



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**405.** Let  $f(x)$  and  $g(x)$  be two function having finite nonzero third-order derivatives  $f'''(x)$  and  $g'''(x)$  for all  $x \in R$ . If  $f(x) \cdot g(x) = 1$  for all  $x \in R$ , then prove that  $\frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right)$ .



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**406.** Differentiate  $\log \sin x$  w.r.t.  $\sqrt{\cos x}$ .



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**407.** If  $\frac{\cos x}{2} \frac{\cos x}{2^2} \frac{\cos x}{2^3} \dots \infty = \frac{\sin x}{x}$ , then find the value of  $\frac{1}{2^2} \frac{\sec^2 x}{2} + \frac{1}{2^4} \frac{\sec^2 x}{2^2} + \frac{1}{2^6} \frac{\sec^2 x}{2^3} + \dots \infty$



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408. Differentiate  $\frac{\tan^{-1}(\sqrt{1+x^2} - 1)}{x}$  w.r.t.  $\tan^{-1} x$



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409. If  $0 < x < 1$ , then="" prove="" that=""

$$(1 - 2x) = (1 - x - x^2) + \frac{2x - 4x^3}{1 - x^2 + x^4} + \frac{4x^3 - 8x^7}{1 - x^4 + x^8} + \dots \infty = \frac{1}{1 + }$$



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410. Find the derivative of  $f(\tan x)$  w.r.t.  $g(\sec x)$  at  $x = \frac{\pi}{4}$ , where  $f'(1)=2$

and  $g'(\sqrt{2}) = 4$ .



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411. If  $f(x) = \cos^{-1}\left(\frac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right)$   
+  $\sin^{-1}\left(\frac{1}{\sqrt{13}}(2\cos x + 3\sin x)\right)$ , then find  $\frac{df(x)}{dx}$  at  $x = \frac{3}{4}$ .



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412. Find the derivative of  $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$  w.r.t.  $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ .



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413. If  $|a_1 \sin x + a_2 \sin 2x + \dots + a_n \sin nx| \leq |\sin x|$  for  $x \in R$ , then prove that  $|a_1 + 2a_2 + 3a_3 + na_n| \leq 1$



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414. Find the derivative of  $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$  w.r.t  $\sqrt{1 - x^2}$  at  $x = \frac{1}{2}$



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415. If  $y = \left(\frac{1}{2}\right)^{n-1} \cos(n \cos^{-1} x)$ , then prove that  $y$  satisfies the differential equation  $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + n^2 y = 0$



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416. If  $y = f(x^3)$ ,  $z = g(x^5)$ ,  $f'(x) = \tan x$ , and  $g'(x) = \sec x$ , then

find the value of  $\lim_{x \rightarrow 0} \frac{\left(\frac{dy}{dz}\right)}{x}$



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417. If  $x \in \left(0, \frac{\pi}{2}\right)$ , then show that  
 $\cos^{-1}\left(\frac{7}{2}(1 + \cos 2x) + \sqrt{(\sin^2 x - 48 \cos^2 x)} \sin x\right) = x - \cos^{-1}(7 \cos x)$



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**418.** If  $f(x) = \begin{vmatrix} x + a^2 & ab & ac \\ ab & x + b^2 & bc \\ ac & bc & x + c^2 \end{vmatrix}$ , then prove that

$$f'(x) = 3x^2 + 2x(a^2 + b^2 + c^2).$$



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**419.** If  $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ , then  $\frac{dy}{dx}$  is (a)  $\frac{-2}{1+x^2}$  for all  $x$  (b)  $\frac{-2}{1+x^2}$  for all  $|x| < 1$  (c)  $\frac{2}{1+x^2}$  for  $|x| > 1$  (d) none of these



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**420.** If  $f(x)$ ,  $g(x)$  and  $h(x)$  are three polynomials of degree 2, then prove

that  $\phi(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$  is a constant polynomial.



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**421.** If  $f(x - y)$ ,  $f(x)f(y)$  and  $f(x + y)$  are in A.P. for all  $x, y$ , and  $f(0) \neq 0$ , then (a)  $f(4) = f(-4)$  (b)  $f(2) + f(-2) = 0$  (c)  $f'(4) + f'(-4) = 0$  (d)  $f'(2) = f'(-2)$



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**422.** If  $x^3 + y^3 + 3axy = 0$ , find  $\frac{dy}{dx}$



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**423.** If 1 is the twice repeated root of the equation  $ax^3 + bx^2 + bx + d = 0$ , then (a)  $a = b = d$  (b)  $a + b = 0$  (c)  $b + d = 0$  (d)  $a = d$



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**424.** If  $\log(x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$ , show that  $\frac{dy}{dx} = \frac{x+y}{x-y}$



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425. If  $y = b \tan^{-1} \left( \frac{x}{a} + \frac{\tan^{-1} y}{x} \right)$ , find  $\frac{dy}{dx}$ .



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426. Let  $y = \sqrt{x + \sqrt{x + \sqrt{x + \infty}}}$ ,  $\frac{dy}{dx}$  is equal to

(a)  $\frac{1}{2y - 1}$

(b)  $\frac{x}{x + 2y}$

(c)  $\frac{1}{\sqrt{1 + 4x}}$

(d)  $\frac{y}{2x + y}$



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**427.** If  $y = \sqrt{\sin x + y}$ , then find  $\frac{dy}{dx}$



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**428.**  $f(x) = |x^2 - 3|x| + 2|$ . Then which of the following is/are true

$f'(x) = 2x - 3$  for  $x$  in  $(0, 1) \cup (2, \infty)$

$f'(x) = 2x + 3$  for  $x$  in  $(-\infty, -2) \cup (-1, 0)$

$f'(x) = -2x - 3$  for  $x$  in  $(-2, -1)$

None of these



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**429.** If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ , prove that  $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$



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**430.** If  $x^3 - 2x^2y^2 + 5x + y - 5 = 0$  and  $y(1) = 1$ , then (a)  $y'(1) = \frac{4}{3}$

(b)  $y'(1) = -\frac{4}{3}$  (c)  $y''(1) = -8\frac{22}{27}$  (d)  $y'(1) = \frac{2}{3}$



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431. If  $x^y = e^{x-y}$ , Prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$



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432. Let  $f(x) = \frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}x$ . Then (a)  $f'(10) = 1$  (b)  $f'\left(\frac{3}{2}\right) = -1$  (c) domain of  $f(x)$  is  $x \geq 1$  (d) range of  $f(x)$  is  $(-2, -1) \cup (2, \infty)$



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433. If  $\sqrt{x} + \sqrt{y} = 4$ , find  $\left(\frac{dx}{dy}\right)$  at  $(y = 1)$ .



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**434.** If  $y = \frac{x^4 - x^2 + 1}{x^2 + \sqrt{3}x + 1}$  and  $\frac{dy}{dx} = ax + b$ , then the value of  $a+b$  is  
(a)  $\cot\left(\frac{\pi}{8}\right)$  (b)  $\cot\left(\frac{5\pi}{12}\right)$  (c)  $\tan\left(\frac{5\pi}{12}\right)$  (d)  $\tan\left(\frac{5\pi}{8}\right)$



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**435.** If  $xy + y^2 = \tan x + y$ , then find  $\frac{dy}{dx}$



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**436.** Find  $\frac{dy}{dx}$  for  $y = \sin^{-1}(\cos x)$



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**437.** If  $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots \infty}}}}$  then  $\frac{dy}{dx}$  equal to



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**438.** If  $y = x^{\log x} \wedge ((\log(\log x)))$ , then  $\frac{dy}{dx}$  is

$$\frac{y}{x}(1nx^{\infty x - 1}) + 21nx1n(1nx) \quad \frac{y}{x}(\log x)^{\log(\log x)}(2\log(\log x) + 1)$$
$$\frac{y}{x1nx}[(1nx)^2 + 21n(1nx)] \quad \frac{y}{x} \frac{\log y}{\log x}[2\log(\log x) + 1]$$



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**439.** If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots}}}$ , prove that  $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$



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**440.** The  $n$ th derivative of  $xe^x$  vanishes when (a)  $x = 0$  (b)  $x = -1$  (c)  $x = -n$  (d)  $x = n$



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**441.** If  $x = a \left( \cos t + \frac{1}{2} \log \tan^2 t \right)$  and  $y = a \sin t$  then find  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$



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**442.** or, find the derivatives of  $\tan^{-1} \left( \frac{\sqrt{1+x^2} - 1}{x} \right)$  with respect to  $\tan^{-1} \left( \frac{2x\sqrt{1-x^2}}{1-2x^2} \right)$  at  $x=0$ .



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**443.** If  $x^m y^n = (x+y)^{m+n}$ , prove that  $\frac{dy}{dx} = \frac{y}{x}$ .



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**444.** Let  $g(x) = f(x) \sin x$ , where  $f(x)$  is a twice differentiable function on  $(-\infty, \infty)$  such that  $f'(-\pi) = 1$ . The value of  $|g''(-\pi)|$  equals



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**445.** If  $x = a \cos^3 \theta$ ,  $y = a \sin^3 \theta$ , then find the value of  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{6}$



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**446.** If  $x = \sqrt{a^{\sin^{-1}t}}$  and  $y = \sqrt{a^{\cos^{-1}t}}$  show that  $\frac{dy}{dx} = -\frac{y}{x}$ .



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**447.** If  $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$  touches the ellipses  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then find the eccentricity angle  $\theta$  of point of contact.



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**448.** If  $y = f\left(\frac{2x - 1}{x^2 + 1}\right)$  and  $f'(x) = \sin x^2$ , then find  $\frac{dy}{dx}$



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**449.**

Let

$y = x^3 - 8x + 7$  and  $x = f(t)$ . If  $\frac{dy}{dt} = 2$  and  $x = 3$  at  $t = 0$ , then  $\frac{dx}{dt}$  at  $t = 0$  is given by



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**450.** If  $x = \frac{2t}{1 + t^2}, y = \frac{1 - t^2}{1 + t^2}$ , then find  $\frac{dy}{dx}$ .



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**451.** If  $f_r(x), g_r(x), h_r(x), r = 1, 2, 3$  are polynomials such that  $f_r(a) = g_r(a) = h_r(a), r = 1, 2, 3$  and

$$F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix} \text{ then } F'(x) \text{ at } x = a \text{ is } \underline{\hspace{10cm}}$$



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452. Find  $\frac{dy}{dx}$  if  $x = a(\theta - \sin \theta)$  and  $y = a(1 - \cos \theta)$ .



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453. If  $f(x) = (\log)_x(\log x)$ , then  $f'(x)$  at  $x = e$  is equal to (a)  $\frac{1}{e}$  (b)  $e$  (c) 1 (d) zero



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454. Differentiate

$$\tan^{-1}\left(\frac{x}{1 + \sqrt{(1 - x^2)}}\right) + \left\{ 2 \tan^{-1} \sqrt{\left(\frac{1 - x}{1 + x}\right)} \right\} w. r. t. x$$



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**455.** If  $x = a \sec^3 \theta$  and  $y = a \tan^3 \theta$ , find  $\frac{dy}{dx}$  at  $\theta = \frac{\pi}{3}$ .



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**456.** Differentiate  $(\log x)^{\cos x}$  with respect to  $x$ .



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**457.** Find the differential equation of the curves given by  $y = Ae^{2x} + Be^{-2x}$  where A and B are parameters.



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**458.** If  $f(x) = x \sin x$ , then find  $f' \left( \frac{\pi}{2} \right)$



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**459.** about to only mathematics



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**460.** If  $y = \sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}$ , then find  $\frac{dy}{dx}$ .



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**461.** Find  $\frac{dy}{dx}$  of  $y = x^3$



**Watch Video Solution**

**462.** Find  $\frac{dy}{dx}$  for  $y = \sin(x^2 + 1)$ .



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**463.** Let  $f(x) = x + \frac{1}{2x + \frac{1}{2x + \frac{1}{2x}}}$  Compute the value of  $f(5)$  .



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464.  $y = \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right)$ . Find  $\frac{dy}{dx}$ .



Watch Video Solution

465. Find  $y'$  of  $y = 5x^5$



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466.  $y = \sin^{-1} \left( \frac{1}{\sqrt{1 + x^2}} \right) + \cos^{-1} \left( \frac{1}{\sqrt{1 + x^2}} \right)$ . find  $\frac{dy}{dx}$ .



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467. If  $x^2 + y^2 = R^2$  (where  $R > 0$ ) and  $k = \frac{y''}{\sqrt{(1 + y'^2)^3}}$  then find  $k$  in terms of  $R$  alone.



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468.  $y = \tan^{-1} \left( \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$ , where  $-1 < x < 1$ , find  $\frac{dy}{dx}$



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469. Find the derivative of  $\sec^{-1} \left( \frac{1}{2x^2 - 1} \right)$  w.r.t  $\sqrt{1-x^2}$  at  $x = \frac{1}{2}$



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470.  $y = \tan^{-1} \left( \frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right)$ , where  $-\frac{\pi}{2} < x < \pi$  and  $\frac{a}{b} \tan x > -1$ . Find  $\frac{dy}{dx}$



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471. If  $f(9)=9$ ,  $f'(9)=4$ ,  $\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} = ?$



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**472.** Find the sum of the series  $1 + 2x + 3x^2 + \dots + (n-1)x^{n-2}$  using differentiation.



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**473.** If  $f(x)$  satisfies the relation  
 $f\left(\frac{5x - 3y}{2}\right) = \frac{5f(x) - 3f(y)}{2} \quad \forall x, y \in R,$  and  
 $f(0) = 3$  and  $f'(0) = 2,$  then the period of  $\sin(f(x))$  is (a)  $2\pi$  (b)  $\pi$  (c)  $3\pi$  (d)  $4\pi$



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**474.**  $y = 4x^2 + e^{3x}$  find  $y'$



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**475.** Instead of the usual definition of derivative  $Df(x)$ , if we define a new kind of derivative  $D^*F(x)$  by the formula  $D^*f(x) = \lim_{h \rightarrow 0} \frac{f^2(x+h) - f^2(x)}{h}$ , where  $f^2(x)$  mean  $[f(x)]^2$  and if  $f(x) = x \log x$ , then  $D^*f(x)|_{x=e}$  has the value (A) e (B) 2e (c) 4e (d) none of these



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**476.** Simplify  $y = \tan^{-1} \left( \frac{x}{1 + \sqrt{1 - x^2}} \right)$



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**477.** If  $y = |\cos x| + |\sin x|$ , then  $\left( \frac{dy}{dx} \right)$  at  $x = \frac{2\pi}{3}$  is



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**478.**  $y = \ln x + e^{2x}$  find  $y'$



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479. If  $g$  is the inverse function of  $f$  and  $f'(x) = \sin x$ , then  $g'(x)$  is (a)  $\cos ec\{g(x)\}$  (b)  $\sin\{g(x)\}$  (c)  $-\frac{1}{\sin\{g(x)\}}$  (d) none of these



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480. Find  $\frac{dy}{dx}$  for the function:  $y = \sin x + \ln x$



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481. If  $x = \varphi(t)$ ,  $y = \psi(t)$ , then  $\frac{d^2y}{dx^2}$  is (a)  $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^2}$  (b)  $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^3}$  (c)  $\frac{\varphi''}{\psi''}$  (d)  $\frac{\psi''}{\varphi''}$



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**482.** If  $y = (1 + x)(1 + x^2)(1 + x^4)\dots(1 + x^{2^n})$ , then find  $\frac{dy}{dx}$  at  $x = 0$ .

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**483.**  $f(x) = e^x - e^{-x}$  then find  $f'(x)$

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**484.** Find  $\frac{dy}{dx}$  for the function:  $y = \sqrt{\sin \sqrt{x}}$

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**485.** If  $y = \frac{\sqrt{a+x} - \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}}$ , then  $\frac{dy}{dx}$  is equal to (a)  $\frac{ay}{x\sqrt{a^2 - x^2}}$  (b)  $\frac{ay}{\sqrt{a^2 - x^2}}$  (c)  $\frac{ay}{x\sqrt{a^2 - x^2}}$  (d) none of these

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**486.** Find  $\frac{dy}{dx}$  for the function:  $y = e^x + \cos x$



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**487.** Find  $\frac{dy}{dx}$  for the function:  $y = x^3 + e^{2x}$



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**488.** Find  $\frac{dy}{dx}$  for the function:  $y = \log \sqrt{\sin \sqrt{e^x}}$



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**489.** Let  $g(x)$  be the inverse of an invertible function  $f(x)$ , which is differentiable for all real  $x$ . Then  $g''(f(x))$  equals. (a)  $-\frac{f'''(x)}{(f'(x))^3}$  (b)  $f'(x)f''(x) - (f'(x))^3$  (c)  $\frac{f'(x)f''(x) - (f'(x))^2}{(f'(x))^2}$  (d) none of these



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**490.** Find  $\frac{dy}{dx}$  for the function:  $y = x^{\frac{1}{2}} + \sin 2x$



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**491.** Find  $\frac{dy}{dx}$  for the function:  $y = \sin 5x$



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**492.** Differentiate the function  $f(x) = x^{99}$  with respect to  $x$ .



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**493.** If  $f(x) = x + \tan x$  and  $g(x)$  is the inverse of  $f(x)$ , then differentiation of  $g(x)$  is (a)  $\frac{1}{1 + [g(x) - x]^2}$  (b)  $\frac{1}{2 - [g(x) + x]^2}$  (c)  $\frac{1}{2 + [g(x) - x]^2}$  (d) none of these



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**494.** Find  $\frac{dy}{dx}$  for  $y = \cos 55x$

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**495.** Find  $\frac{dy}{dx}$  for  $y = e^{6x}$

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**496.** Let  $f: R \rightarrow R$  be a one-one onto differentiable function, such that

$f(2) = 1$  and  $f'(2) = 3$ . Then, find the value of  $\left( \frac{d}{dx} (f^{-1}(x)) \right)_{x=1}$

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**497.** If  $f''(x) = -f(x)$  and  $g(x) = -f'(x)$  and  $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$

and given that  $f(5)=5$ , then  $f(10)$  is equal to

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**498.** Find  $\frac{dy}{dx}$  for the function:  $y = \sin 4x - \left(\frac{1}{x^4}\right)$



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**499.** Find  $\frac{dy}{dx}$  for the function:  $y = \sin 2x - x^4 + e^{-3x}$



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**500.** If  $x = a \cos \theta, y = b \sin \theta$ , then prove that

$$\frac{d^3y}{dx^3} = -\frac{3b}{a^3} \cos \theta c^4 \theta \cot \theta.$$



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**501.** If  $y = x \log \left\{ \frac{x}{(a + bx)} \right\}$ , then show that  $x^3 \frac{d^2y}{dx^2} = \left( x \frac{dy}{dx} - y \right)^2$ .



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**502.** A function  $f: R \rightarrow R$  satisfies the equation  $f(x + y) = f(x)f(y)$  for all  $x, y \in R$  and  $f(x) \neq 0$  for all  $x \in R$ . If  $f(x)$  is differentiable at  $x = 0$  and  $f'(0) = 2$ , then prove that  $f'(x) = 2f(x)$ .



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**503.** If  $u = x^2 + y^2$  and  $x = s + 3t, y = 2s - t$ , then  $\frac{d^2u}{ds^2}$  is (a)  $\frac{5}{2}t$  (b)  $20t^8$  (c)  $\frac{5}{16t^6}$  (d) none of these



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**504.** If  $f(x) = (1 + x)^n$ , then the value of  $f(0) + f'(0) + \frac{f''(0)}{2!} + \frac{f'''(0)}{3!} + \dots + \frac{f^n(0)}{n!}$ .



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**505.** If  $f(x) = \sin x + e^x$ , then  $f''(x)$



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506. If  $y^2 = ax^2 + bx + c$ , then  $y^3 \frac{d^2y}{dx^2}$  is



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507. If  $f(x) = \left| x \cap !2 \cos x \frac{\cos(n\pi)}{2} 4 \sin x \frac{\sin(n\pi)}{2} 8 \right|$  then find the value of  $\frac{d^n}{dx^n}([f(x)])_{x=0} n \in z$ .



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508. Let  $f\left(\frac{x+y}{2}\right) = \frac{f(x) + f(y)}{2}$  for all real  $x$  and  $y$ . If  $f'(0)$  exists and equals -1 and  $f(0) = 1$ , then find  $f(2)$ .



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**509.** If  $y^{\frac{1}{m}} = \left(x + \sqrt{1 + x^2}\right)$ , then  $(1 + x^2)y_2 + xy_1$  is (where  $y_r$  represents the  $r$ th derivative of  $y$  w.r.t.  $x$ )  
(a)  $m^2y$  (b)  $my^2$  (c)  $m^2y^2$  (d) none of these



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**510.** Suppose the function  $f(x) - f(2x)$  has the derivative 5 at  $x = 1$  and derivative 7 at  $x = 2$ . The derivative of the function  $f(x) - f(4x)$  at  $x=1$  has the value equal to  
(a) 19 (b) 9 (c) 17 (d) 14



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**511.**  $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ , for all  $x, y \in R$ . ( $xy \neq 1$ ), and  $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 2$ . Find  $f(\sqrt{3})$  and  $f'(-2)$ .



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**512.** Let  $f: R \rightarrow R$  satisfying  $|f(x)| \leq x^2$ ,  $\forall x \in R$  be differentiable at  $x = 0$ . Then find  $f'(0)$ .



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**513.** If  $f(x) = \sin^{-1} \cos x$ , then the value of  $f(10) + f'(10)$  is (a)  $11 - \frac{\pi}{2}$  (b)  $\frac{\pi}{2} - 11$  (c)  $\frac{5\pi}{2} - 11$  (d) none of these



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**514.** If  $(\sin x)(\cos y) = \frac{1}{2}$ , then  $\frac{d^2y}{dx^2}$  at  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$  is (a)  $-4$  (b)  $-2$  (c)  $-6$  (d)  $0$



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**515.** Suppose  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ . If  $|p(x)| \leq |e^{x-1} - 1| \leq 1$  then prove  $|a_1 + 2a_2 + \dots + n a_n| \leq 1$ .



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516. Let  $f(x + y) = f(x)f(y)$  for all  $x$  and  $y$ . Suppose  $f(5) = 2$  and  $f'(0) = 3$ . Find  $f'(5)$ .



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517. A function  $f$  satisfies the condition  $f(x) = f'(x) + f''(x) + f'''(x) \dots$ , where  $f(x)$  is an indefinitely differentiable function and dash denotes the order of derivatives. If  $f(0) = 1$ , then  $f(x)$  is

- (a)  $e^{\frac{x}{2}}$  (b)  $e^x$  (c)  $e^{2x}$  (d)  $e^{4x}$



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518. Let  $f(xy) = f(x)f(y) \forall x, y \in R$  and  $f$  is differentiable at  $x = 1$  such that  $f'(1) = 1$ . Also,  $f(1) \neq 0$ ,  $f(2) = 3$ . Then find  $f'(2)$



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**519.** Let  $f(x)$  be a polynomial of degree 3 such that  $f(3) = 1$ ,  $f'(3) = -1$ ,  $f''(3) = 0$ , and  $f'''(3) = 12$ . Then the value of  $f'(1)$  is

- (a) 12 (b) 23 (c) -13 (d) none of these



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**520.** Find  $\frac{dy}{dx}$  for the functions:  $y = x^3 e^x \sin x$



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**521.**  $\frac{d}{dx} \left[ \tan^{-1} \left( \frac{\sqrt{x}(3-x)}{1-3x} \right) \right]$  is

- (a)  $\frac{1}{2(1+x)\sqrt{x}}$  (b)  $\frac{3}{(1+x)\sqrt{x}}$  (c)  $\frac{2}{(1+x)\sqrt{x}}$  (d)  $\frac{3}{2(1+x)\sqrt{x}}$



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**522.**

If

$$y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right)$$

upto  $n$  terms, then find the value of  $y'(0)$



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**523.** Let  $g(x)$  be the inverse of an invertible function  $f(x)$  which is differentiable at  $x = c$ . Then  $g'(f(x))$  equal. (a)  $f'(c)$  (b)  $\frac{1}{f'(c)}$  (c)  $f(c)$  (d) none of these



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**524.** If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$  then prove that  
$$\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$$



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525. Let  $f: R \rightarrow R$  be such that  $f(1) = 3$  and  $f'(1) = 6$ . Then,

$$\lim_{x \rightarrow 0} \left( \frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}} \text{ equals}$$



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526. If  $\cos y = x \cos(a+y)$ , ( $a \neq 0$ ), then show that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$



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527. Let  $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ , where  $p$  is a constant. Then

$$\frac{d^3}{dx^3}(f(x)) \text{ at } x = 0 \text{ is}$$

- (a)  $p$  (b)  $p - p^3$  (c)  $p + p^3$  (d) independent of  $p$



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528. Find  $\frac{dy}{dx}$  for  $y = \sin^{-1}(\cos x)$



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529. True or False -The derivative of an even function is always an odd function.



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530. If  $y = \sqrt{(a-x)(x-b)} - (a-b)\tan^{-1} \sqrt{\frac{a-x}{x-b}}$  then  $\frac{dy}{dx}$  is equals to



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531. Let  $F(x) = f(x)g(x)h(x)$  for all real  $x$ , where  $f(x)$ ,  $g(x)$ , and  $h(x)$  are differentiable functions. At some point  $x_0$ ,  $F'(x_0) = 21F(x_0)$ ,  $f'(x_0) = 4f(x_0)$ ,  $g'(x_0) = -7g(x_0)$ , and  $h'(x_0) = kh(x_0)$ . Then  $k$  is \_\_\_\_\_



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532. Find  $\frac{dy}{dx}$  for  $y = \tan^{-1} \left\{ \frac{1 - \cos x}{\sin x} \right\}$ .



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533. If  $x = y + \sin^2 x$  then at  $x = 0, \frac{dy}{dx} =$



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534.  $f(x) = \sin^{-1} \left( 2x \sqrt{1 - x^2} \right)$  then  $f'(x) =$



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535. If  $f(x) = |x - 2|$  and  $g(x) = f[f(x)]$ , then  $g'(x) =$  \_\_\_\_\_ for  $x > 20$



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536. Find  $\frac{dy}{dx}$  for  $y = \tan^{-1} \sqrt{\frac{\sec^2 x}{\cos ec^2 x}}$



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537. If  $y = (\sin x)^{\tan x}$ , then  $\frac{dy}{dx} =$

(a)  $(\sin x)^{\tan x}(1 + \sec^2 x \log \sin x)$   
(b)  $\tan x(\sin x)^{\tan x - 1} \cos x$   
(c)  $(\sin x)^{\tan x}$   
(d)  $\sec^2 x \log \sin x$   
 $\tan x(\sin x)^{\tan x - 1}$



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538. If  $y = \sin^{-1} \left( \sqrt{1 - x^2} \right)$  and  $0 < x < 1$ , then find  $\frac{dy}{dx}$



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539. Suppose that  $f(x)$  is a quadratic expression positive for all real  $x$ . If  $g(x) = f(x) + f'(x) + f''(x)$ , then for any real  $x$  (where  $f'(x)$  and

$f''(x)$  represent 1st and 2nd derivative, respectively).

- (a)  $g(x) < 0$  (b).  $g(x) > 0$  (c).  $g(x) = 0$  (d).  $g(x) \geq 0$



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540. If  $y = \cos^{-1} x$ , find  $\frac{d^2y}{dx^2}$ .



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541. If  $y^2 = P(x)$ , where  $P(x)$  is a polynomial of degree 3, then

$$2 \frac{d}{dx} \left( y^3 \frac{d^2y}{dx^2} \right)$$



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542. If  $g(x) = \begin{vmatrix} f(x+c) & f(x+2c) & f(x+3c) \\ f(c) & f(2c) & f(3c) \\ f'(c) & f'(2c) & f'(3c) \end{vmatrix}$ , where  $c$  is a constant,

then  $\lim_{x \rightarrow 0} \frac{g(x)}{x}$  is equal to



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**543.** If  $f(a) = 2$ ,  $f'(a) = 1$ ,  $g(a) = -1$ ,  $g'(a) = 2$ , then the value of

$$\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$$
 is (a) -5 (b)  $\frac{1}{5}$  (c) 5 (d) none of these



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**544.** If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , find  $\frac{d^2y}{dx^2}$



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**545.** Prove that  $\frac{2^{(\log_2 \frac{1}{4})x} - 3^{(\log_{27}(x^2+1))^3} - 2x}{7^{4(\log_{49}x)} - x - 1} > 0$



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**546.** ( $\lim_{h \rightarrow 0}$ )  $\frac{(e+h)^{1n(e+h)} - e}{h}$  is --



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**547.** If  $(x - a)^2 + (y - b)^2 = c^2$ , for some  $c > 0$ , prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.



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**548.** if  $y = (x^2 - 1)^m$ , then the  $(2m)th$  differential coefficient of  $y$  is



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**549.** If function  $f$  satisfies the relation  $f(x) \cdot f'(-x) = f(-x) \cdot f'(x)$  for all  $x$ , and  $f(0)=3$ , and if  $f(3)=3$ , then the value of  $f(-3)$  is \_\_\_\_\_



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550. If  $y = \frac{a + bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}$  and  $y' = 0$  at  $x = 5$ , then the value of  $\frac{a^2}{b^2}$  is \_\_\_\_\_



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551. If  $y = e^{a \cos^{-1} x}$ ,  $-1 \leq x < 1$ , show that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$



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552. Let

$f(x) = (x - 1)(x - 2)(x - 3)\dots(x - n)$ ,  $n \in N$ , and  $f'(n) = 5040$ .

Then the value of  $n$  is \_\_\_\_\_



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553. If  $y = \frac{(ax + b)}{(x^2 + c)}$ , then find  $\frac{dy}{dx}$ .



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554. If  $y = x \log \left\{ \frac{x}{(a + bx)} \right\}$ , then show that  $x^3 \frac{d^2y}{dx^2} = \left( x \frac{dy}{dx} - y \right)^2$ .

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555.  $y=f(x)$ , where  $f$  satisfies the relation  
 $f(x+y) = 2f(x) + xf(y) + y\sqrt{f(x)}$ ,  $\forall x, y \in R$  and  $f'(0)=0$ . Then  $f(6)$  is equal \_\_\_\_\_

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556. If  $x^2 + y^2 = t - \frac{1}{t}$  and  $x^4 + y^4 = t^2 + \frac{1}{t^2}$ , then  $x^3 y \frac{dy}{dx} =$

- (a) 0 (b) 1 (c) -1 (d) none of these

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557. Prove that  $\frac{d^n}{dx^n} (e^{2x} + e^{-2x}) = 2^n [e^{2x} + (-1)^n e^{-2x}]$



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558. If  $e^y(x + 1) = 1$ , prove that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$



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559.  $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right)$  show that  $\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$



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560. If  $y = \cos^{-1}(\cos x)$ , then  $\frac{dy}{dx}$  at  $x=\frac{5\pi}{4}$  is equal to

- (a)1 (b)-1 (c)0 (d)4



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561. If  $y = \cos^{-1}(\cos x)$ , then  $\frac{dy}{dx}$  at  $x=\frac{5\pi}{4}$  is equal to

- (a) 1 (b) -1 (c) 0 (d) 4



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562. If  $\sin^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = \log a$ , then  $\frac{dy}{dx}$  is equal to (a)  $\frac{x}{y}$  (b)  $\frac{y}{x^2}$  (c)  $\frac{x^2 - y^2}{x^2 + y^2}$  (d)  $\frac{y}{x}$



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563. If  $y = \tan^{-1} \sqrt{\frac{x+1}{x-1}}$ , then  $\frac{dy}{dx}$  is (a)  $\frac{-1}{2|x|\sqrt{x^2-1}}$  (b)  $\frac{-1}{2x\sqrt{x^2-1}}$  (c)  $\frac{1}{2x\sqrt{x^2-1}}$  (d) none of these



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564. If  $y = \cos^{-1}\left(\frac{5\cos x - 12\sin x}{13}\right)$ , where  $x \in \left(0, \frac{\pi}{2}\right)$ , then  $\frac{dy}{dx}$  is.

- (a) 1 (b) -1 (c) 0 (d) none of these



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565. The derivative of  $y = (1 - x)(2 - x) \dots (n - x)$  at  $x = 1$  is

- (a) 0 (b)  $(-1)(n - 1)!$  (c)  $n! - 1$  (d)  $(-1)^{n-1}(n - 1)!$



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566. If  $y = \sqrt{\frac{1-x}{1+x}}$  find  $\frac{dy}{dx}$  and prove that  $(1 - x^2) \frac{dy}{dx} + y = 0$



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567.  $\frac{d^{20}}{dx^{20}}(2 \cos x \cos 3x)$  is equal to (a)  $2^{20}(\cos 2x - 2^{20} \cos 3x)$  (b)  $2^{20}(\cos 2x + 2^{20} \cos 4x)$  (c)  $2^{20}(\sin 2x + 2^{20} \sin 4x)$  (d)  $2^{20}(\sin 2x - 2^{20} \sin 4x)$



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**568.** Differentiate  $x^2 + 1$  with respect to x .



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**569.** Let  $g(x) = \begin{cases} \frac{x^2 + x \tan x - x \tan 2x}{ax + \tan x - \tan 3x}, & x \neq 0, x = 0 \text{ if } g'(0) \text{ exists} \\ b, & \text{and is equal to nonzero value } b, \text{ then } 52\frac{b}{a} \text{ is equal to } \underline{\hspace{2cm}} \end{cases}$



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**570.** Given  $y = \sin 2x + x^3$ ,  $\frac{dy}{dx}$



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**571.** Find the derivative of  $f(x) = e^{4x} + \cos 3x$



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**572.** A nonzero polynomial with real coefficient has the property that  $f(x) = f'(x)f''(x)$ . If  $a$  is the leading coefficient of  $f(x)$ , then the value of  $1/2a$  is \_\_\_\_



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**573.** Suppose  $f(x) = e^{ax} + e^{bx}$ , where  $a \neq b$ , and that  $f''(x) - 2f'(x) - 15f(x) = 0$  for all  $x$ . Then the value of  $\frac{|ab|}{3}$  is \_\_\_\_



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**574.** Let  $z = (\cos x)^5$  and  $y = \sin x$ . Then the value of  $2\frac{d^2z}{dy^2}atx = \frac{2\pi}{9}$  is \_\_\_\_.



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**575.** A function is represented parametrically by the equations

$$x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t} \quad \text{then the value of } \left| \frac{dy}{dx} - x \left( \frac{dy}{dx} \right)^3 \right|$$

is \_\_\_\_\_



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**576.** Let  $f$  be a twice differentiable function such that  $f''(x) = -f(x)$ , and  $f'(x)$

$= g(x)$ ,  $h(x) = [f(x)]^2 + [g(x)]^2$  Find  $h(10)$ , if  $h(5) = 11$



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**577.** Let  $y = e^{x \sin x^3} + (\tan x)^x$  Find  $\frac{dy}{dx}$



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**578.** If  $ax^2 + 2hxy + by^2 = 1$ , then  $\frac{d^2y}{dx^2}$  is (a)  $\frac{h^2 - ab}{(hx + by)^2}$  (b)

$\frac{ab - h^2}{(hx + by)^2} \frac{h^2 + ab}{(hx + by)^2}$  (d) none of these



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579. If  $y = \sin px$  and  $y_n$  is the  $n$ th derivative of  $y$ , then  $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$  is

- (a) 1 (b) 0 (c) -1 (d) none of these



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580. A function  $f$ , defined for all positive real numbers, satisfies the equation  $f(x^2) = x^3$  for every  $x > 0$ . Then the value of  $f'(4)$  is (a) 12 (b) 3 (c) 3/2 (d) cannot be determined



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581. If  $y = x - x^2$ , then the derivative of  $y^2$  with respect to  $x^2$  is (a)  $1 - 2x$  (b)  $2 - 4x$  (c)  $3x - 2x^2$  (d)  $1 - 3x + 2x^2$



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**582.** The first derivative of the function  $\left[ \cos^{-1} \left( \sin \sqrt{\frac{1+x}{2}} \right) + x^x \right]$

with respect to  $x$  at  $x = 1$  is (a)  $3/4$  (b)  $0$  (c)  $1/2$  (d)  $-1/2$



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**583.** The function  $f(x) = e^x + x$ , being differentiable and one-to-one, has a differentiable inverse  $f^{-1}(x)$ . The value of  $\frac{d}{dx}(f^{-1})$  at the point  $f(\log 2)$  is (a)  $\frac{1}{1n2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d) none of these



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**584.** Let  $h(x)$  be differentiable for all  $x$  and let  $f(x) = (kx + e^x)h(x)$ , where  $k$  is some constant. If  $h(0) = 5$ ,  $h'(0) = -2$ , and  $f'(0) = 18$ , then the value of  $k$  is (a) 5 (b) 4 (c) 3 (d) 2.2.



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**585.** Suppose  $y = e^{ax} + e^{bx}$ , find  $y''$

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**586.** If  $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{(\sqrt{a-x} + \sqrt{x-b})}$ , then  $\frac{dy}{dx}$  wherever it is defined is (a)  $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$  (b)  $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$  (c)  $-\frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$  (d)  $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

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**587.** If  $y = 7x^5$ , then  $\frac{dy}{dx}$

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**588.** The  $n$ th derivative of the function  $f(x) = \frac{1}{1-x^2}$  [where  $x \in (-1, 1)$ ] at the point  $x = 0$  where  $n$  is even

is (a) 0 (b)  $n!$  (c)  $n^n C_2$  (d)  $2^n C_2$



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**589.** Let  $u(x)$  and  $v(x)$  be differentiable functions such that  $\frac{u(x)}{v(x)} = 7$ .

If  $\frac{u'(x)}{v'(x)} = p$  and  $\left(\frac{u(x)}{v(x)}\right)' = q$ , then  $\frac{p+q}{p-q}$  has the value of (a) 1 (b) 0  
(c) 7 (d) -7



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**590.** Statement 1: Let  $f: R \rightarrow R$  be a real-valued function  $\forall x, y \in R$

such that  $|f(x) - f(y)| \leq |x - y|^3$ . Then  $f(x)$  is a constant function.

Statement 2: If the derivative of the function w.r.t.  $x$  is zero, then function is constant.



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**591.** Statement 1: For  $f(x) = \sin x$ ,  $f'(\pi) = f'(3\pi)$

Statement 2: For  $f(x) = \sin x$ ,  $f(\pi) = f(3\pi)$ .

- a. Statement 1 and Statement 2, both are correct and Statement 2 is the correct explanation for Statement 1
- b. Statement 1 and Statement 2, both are correct and Statement 2 is not the correct explanation for Statement 1
- c. Statement 1 is correct but Statement 2 is wrong.
- d. Statement 2 is correct but Statement 1 is wrong.



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**592.**  $f: R^+ \rightarrow R$  is a continuous function satisfying

$$f\left(\frac{x}{y}\right) = f(x) - f(y) \quad \forall x, y \in R^+. \text{ If } f'(1)=1, \text{ then}$$

(a)  $f$  is unbounded (b)  $\lim_{x \rightarrow 0} f\left(\frac{1}{x}\right) = 0$  (c)  $\lim_{x \rightarrow 0} \frac{f(1+x)}{x} = 1$  (d)  $\lim_{x \rightarrow 0} x \cdot f(x) = 0$



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**593.**

$f_n(x) = e^{f_{n-1}(x)}$  for all  $n \in N$  and  $f_0(x) = x$ , then  $\frac{d}{dx}\{f_n(x)\}$  is



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**594.** Suppose  $f$  and  $g$  are functions having second derivative  $f''$  and  $g''$

everywhere. If  $f(x)g(x) = 1$  for all  $x$  and  $f'$  and  $g'$  are never zero, then

$\frac{f''(x)}{f'(x)} - \frac{g''(x)}{g'(x)}$  is equal (a)  $\frac{-2f'(x)}{f}$  (b)  $\frac{-2g'(x)}{g(x)}$  (c)  $\frac{-f'(x)}{f(x)}$  (d)  
 $\frac{2f'(x)}{f(x)}$



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**595.** If  $y = e^{-x} \cos x$  and  $y_n + k_n y = 0$ , where  $y_n = \frac{d^ny}{dx^n}$  and  $k_n$  are

constants  $\forall n \in N$ , then  $k_4 = 4$  (b)  $k_8 = -16$   $k_{12} = 20$  (d)

$k_{16} = -24$



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**596.** If a function is represented parametrically by the equations  
 $x = \frac{1 + (\log)_e t}{t^2}; y = \frac{3 + 2(\log)_e t}{t}$ , then which of the following statements are true?



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**597.** Prove  $(a + b + c)(ab + bc + ca) > 9abc$



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**598.** Statement 1:  $f(x) = x + \cos x$  is increasing  $\forall x \in R$ . Statement 2: If  $f(x)$  is increasing, then  $f'(x)$  may vanish at some finite number of points.



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**599.** If  $y = ae^{mx} + be^{-mx}$ , then  $\frac{d^2y}{dx^2}$  is equals to





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600. If  $y = \cot^{-1} \left[ \frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}} \right]$ ,  $\left(0 < x < \frac{\pi}{2}\right)$ , then  
 $\frac{dy}{dx} =$



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601. If  $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \infty}}}$ , then  $\frac{dy}{dx}$  is (a)  $\frac{x}{2y - 1}$  (b)  
 $\frac{x}{2y + 1}$  (c)  $\frac{1}{x(2y - 1)}$  (d)  $\frac{1}{x(1 - 2y)}$



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602.  $\frac{d^n}{dx^n} (\log x) = ?$  (a)  $\frac{(n-1)!}{x^n}$  (b)  $\frac{n!}{x^n}$  (c)  $\frac{(n-2)!}{x^n}$  (d)  
 $(-1)^{n-1} \frac{(n-1)!}{x^n}$



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**603.** If  $y = \sec(\tan^{-1} x)$ , then  $\frac{dy}{dx} \text{at } x = 1$  is

(a)  $\cos\left(\frac{\pi}{4}\right)$

(b)  $\sin\left(\frac{\pi}{2}\right)$

(c)  $\sin\left(\frac{\pi}{6}\right)$

(d)  $\cos\left(\frac{\pi}{3}\right)$



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**604.** The differential coefficient of  $f((\log)_e x)$  with respect to  $x$ , where

$f(x) = (\log)_e x$ , is

(a)  $\frac{x}{(\log)_e x}$  (b)  $\frac{1}{x}(\log)_e x$  (c)  $\frac{1}{x(\log)_e x}$  (d) none of these



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**605.**

If

$u = f(x^3)$ ,  $v = g(x^2)$ ,  $f'(x) = \cos x$ , and  $g'(x) = \sin x$ , then  $\frac{du}{dv}$  is



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**606.** If  $f'(x) = \sqrt{2x^2 - 1}$  and  $y = f(x^2)$ , then  $\frac{dy}{dx}$  at  $x = 1$  is (a) 2 (b) 1

(c) -2 (d) none of these



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**607.** if  $f(x) = \sqrt{1 - \sin 2x}$ , then  $f'(x)$  is equal to



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**608.** If  $x = t \cos t$ ,  $y = t + \sin t$ . Then  $\frac{d^2x}{dy^2}$  at  $t = \frac{\pi}{2}$  is

(a)  $\frac{\pi + 4}{2}$  (b)  $-\frac{\pi + 4}{2}$  (c) -2 (d) none of these



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**609.** If  $x^3 + 3x^2 - 9x = c$  is of the form  $(x - \alpha)^2(x - \beta)$ , then  $c$  is equal to 27 b. -27 c. 5 d. -5



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**610.** If  $y = f(x)$  is an odd differentiable function defined on  $(-\infty, \infty)$  such that  $f'(3) = -2$ , then  $|f'(-3)|$  equals \_\_\_\_\_.



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**611.**  $f'(x) = \phi(x)$  and  $\phi'(x) = f(x)$  for all  $x$ . Also,  $f(3) = 5$  and  $f'(3) = 4$ . Then the value of  $[f(10)]^2 - [\phi(10)]^2$  is \_\_\_\_\_



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**612.** Match the value of  $x$  in column II where derivative of the function in column I is negative. Column I Column II  $y = |x^2 - 2|$  p. (1, 2)  $y = |(\log)_e|x||$  q. (-3, -2)  $y = x\left[\frac{x}{2}\right]$ , where [.] represent r. (-1, 0)  $y = |\sin x|$  s. (0, 1)



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**613.** If  $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ , then  $\frac{dy}{dx}$  is  $\frac{-2}{1+x^2}$  for all  $x$  (b)  $\frac{-2}{1+x^2}$  for all  $|x| < 1$   $\frac{2}{1+x^2}$  for  $|x| > 1$  (d) none of these



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**614.** Each question contains statements given in two columns which have to be matched. Statements a,b,c,d in column I have to be matched with statements p,q,r,s in column II. If the correct matches are a-p, q-s, b-q, r-c-p, q and d-s, then the correctly bubbled  $4 \times 4$  matrix should be as follows:  
Figure Column I, Column II: Differential equation order 1, p.of all parabolas whose axis is the x-axis order 2, q.of family of curves

$y = a(x + a)^2$ , where  $a$  is an arbitrary constant degree 1, r.

$$\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = \frac{4d^3y}{dx^3} \text{ degree 3, s. of family of curve } y^2 = 2c(x + \sqrt{c}),$$

where  $c > 0$



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**615.** Suppose the function  $f(x)$  satisfies the relation

$$f(x + y^3) = f(x) + f(y^3) \quad \forall x, y \in R \text{ and is differentiable for all } x.$$

Statement 1: If  $f'(2) = a$ , then  $f'(-2) = a$  Statement 2:  $f(x)$  is an odd function.



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**616.** If for some differentiable function  $f(\alpha) = 0$  and  $f'(\alpha) = 0$ ,

Statement 1: Then sign of  $f(x)$  does not change in the neighbourhood of  $x = \alpha$  Statement 2:  $\alpha$  is repeated root of  $f(x) = 0$



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**617.** Statement 1: If differentiable function  $f(x)$  satisfies the relation

$$f(x) + f(x - 2) = 0 \forall x \in R, \quad \text{and} \quad \text{if}$$

$$\left( \frac{d}{dx} f(x) \right)_{x=a} = b, \text{ then } \left( \frac{d}{dx} f(x) \right)_{x=a+4000} = b.$$

Statement 2:  $f(x)$  is a periodic function with period 4.

- (a) Statement 1 and Statement 2, both are correct. Statement 2 is the correct explanation for Statement 1
- (b) Statement 1 and Statement 2, both are correct. Statement 2 is not the correct explanation for Statement 1
- (c) Statement 1 is correct but Statement 2 is not correct.
- (d) Both Statement 1 and Statement 2 are not correct.



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**618.** Let  $\alpha$  be a repeated root of a quadratic equation

$f(x) = 0$  and  $A(x), B(x), C(x)$  be polynomials of degrees 3, 4, and 5, respectively, then show that

$|A(x)B(x)C(x)A(\alpha)B(\alpha)C(\alpha)A'(\alpha)B'(\alpha)C'(\alpha)|$  is divisible by  $f(x)$ , where prime (') denotes the derivatives.



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619. If  $y = \{(\log)_{\cos x} \sin x\} \{(\log)_{\sin x} \cos x\}^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ , find  
 $\frac{dy}{dx} \text{ at } x = \frac{\pi}{4}$



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620. find  $\frac{dy}{dx}$ ,  $\left(\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}\right)$  is equal to,  $0 < x < \frac{\pi}{2}$



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621. If  $y = \left(x + \sqrt{x^2 + a^2}\right)^n$ , then  $\frac{dy}{dx}$  is (a)  $\frac{ny}{\sqrt{x^2 + a^2}}$  (b)  $-\frac{ny}{\sqrt{x^2 + a^2}}$   
(c)  $\frac{nx}{\sqrt{x^2 + a^2}}$  (d)  $-\frac{nx}{\sqrt{x^2 + a^2}}$



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**622.** If  $f(x) = \sqrt{1 + \cos^2(x^2)}$ , then  $f'\left(\frac{\sqrt{\pi}}{2}\right)$  is (a)  $\frac{\sqrt{\pi}}{6}$  (b)  $-\sqrt{\pi/6}$  (c)  $1/\sqrt{6}$  (d)  $\pi/\sqrt{6}$



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**623.**  $\frac{d}{dx}(\cos^{-1}\sqrt{\cos x})$  is equal to (a)  $\frac{1}{2}\sqrt{1+\sec x}$  (b)  $\sqrt{1+\sec x}$  (c)  $-\frac{1}{2}\sqrt{1+\sec x}$  (d)  $-\sqrt{1+\sec x}$



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**624.** If  $f(0) = 0$ ,  $f'(0) = 2$ , then the derivative of  $y = f(f(f(x)))$  at  $x = 0$  is 2 (b) 8 (c) 16 (d) 4



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**625.** If  $y = ax^{n+1} + bx^{-n}$ , then  $x^2 \frac{d^2y}{dx^2}$  is equal to (a)  $n(n-1)y$  (b)  $n(n+1)y$  (c)  $ny$  (d)  $n^2y$



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626. If  $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$ , then  $\frac{dy}{dx}$  is equal to (a)  $y$  (b)  $y + \frac{x^n}{n!}$  (c)  $y - \frac{x^n}{n!}$  (d)  $y - 1 - \frac{x^n}{n!}$



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627. If  $y = a \sin x + b \cos x$ , then  $y^2 + \left(\frac{dy}{dx}\right)^2$  is a (a) function of  $x$  (b) function of  $y$  (c) function of  $x$  and  $y$  (d) constant



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628. If  $y = (\log)_{\sin x}(\tan x)$ , then  $\left(\left(\frac{dy}{dx}\right)\right)_{\frac{\pi}{4}}$  is equal to (a)  $\frac{4}{\log 2}$  (b)  $-4 \log 2$  (c)  $\frac{-4}{\log 2}$  (d) none of these



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**629.** If  $y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$ , then  $(1 - x^2) \frac{dy}{dx}$  is equal to (a)  $x + y$  (b)  $1 + xy$  (c)  $1 - xy$  (d)  $xy - 2$

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**630.** If  $(\lim)_{x \rightarrow 0} \frac{ae^x + b \cos x + ce^{-x}}{e^{2x} - 2e^x + 1} = 4$ , then a.  $a = 2$  b.  $b = -4$  c.  $c = 2$  d.  $a + b + c = -8$

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**631.** If  $\lim_{x \rightarrow \infty} x \log_e \begin{pmatrix} \alpha/x & 1 & \gamma \\ 0 & 1/x & \beta \\ 1 & 0 & 1/x \end{pmatrix} = -5$ . where  $\alpha, \beta, \gamma$  are finite real numbers, then

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**632.** Evaluate:  $(\lim)_{x \rightarrow \frac{3\pi}{4}} \frac{1 + (\tan x)^{\frac{1}{3}}}{1 - 2 \cos^2 x}$



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633. Let  $f: R \rightarrow R$  be a differentiable function at  $x = 0$  satisfying  $f(0) = 0$

and  $f'(0) = 1$ , then the value of  $\lim_{x \rightarrow 0} \frac{1}{x} \cdot \sum_{n=1}^{\infty} (-1)^n \cdot f\left(\frac{x}{n}\right)$ , is



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634. ( $\lim_{x \rightarrow 0^+}$ )  $\frac{1}{x\sqrt{x}} \left( \text{atan}^{-1} \frac{\sqrt{x}}{a} - b \frac{\tan^{-1}(\sqrt{x})}{b} \right)$  has the value equal to  $\frac{a-b}{3}$  b. 0 c.  $\frac{(a^2 - b^2)}{6a^2b^2}$  d.  $\frac{a^2 - b^2}{3a^2b^2}$



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635. If  $f'(a) = \frac{1}{4}$ , then ( $\lim_{h \rightarrow 0}$ )  $\frac{f(a + 2h^2) - f(a - 2h^2)}{f(a + h^3 - h^2) - f(a - h^3 + h^2)}$  = 0 b. 1 c. -2 d. none of these



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636. The value of  $\lim_{x \rightarrow 0} \frac{e^{x^2} - e^x + x}{1 - \cos 2x}$  is



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637. Evaluate  $\lim_{x \rightarrow 1} \frac{1 + \log x - x}{1 - 2x + x^2}$



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638. The value of  $\lim_{x \rightarrow 1} \frac{x^{\frac{1}{13}} - x^{\frac{1}{7}}}{\left(x^{\frac{1}{5}} - x^{\frac{1}{3}}\right)}$  is a.  $\frac{44}{91}$  b.  $\frac{45}{91}$  c.  $\frac{45}{89}$  d.  $\frac{40}{93}$



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639. If  $f(x) = \begin{cases} \frac{x}{s \in x}, & x > 0 \\ 2 - x, & x \leq 0 \end{cases}$  and  
 $g(x) = \begin{cases} x + 3, & x < 1 \\ x^2 - 2x - 2, & 1 \leq x < 2 \\ x - 5, & x \geq 2 \end{cases}$  Then the value of  $(\lim_{x \rightarrow 0} g(f(x)))$  a. is -2 b. is -3 c. is 1 d. does not exist



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**640.** If  $k \in I$  such that  $(\lim)_{x \rightarrow \infty} \left( \cos \frac{k\pi}{4} \right)^{2n} - \left( \cos - \frac{k\pi}{6} \right)^{2n} = 0$ ,

then (a)  $k$  must not be divisible by 24 (b)  $k$  is divisible by 24 or  $k$  is divisible neither by 4 nor by 6 (c)  $k$  must be divisible by 12 but not necessarily by 24 (d) none of these



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**641.**  $\lim_{x \rightarrow \infty} \frac{\sum_{r=1}^{10} (x+r)^{2010}}{(x^{1006}+1)(2x^{1004}+1)} =$



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**642.** If  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = a$  and  $\lim_{x \rightarrow 0} \frac{f(1-\cos x)}{g(x)\sin^2 x} = b$  (where  $b \neq 0$ ),

then  $\lim_{x \rightarrow 0} \frac{g(1-\cos 2x)}{x^4}$  is



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**643.** The value of  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$  is 8 b. -4 c. -8 d. -2



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**644.** (  $\lim$  ) $_{x \rightarrow \frac{\pi}{2}}$   $\frac{(1 - \sin x)(8x^3 - \pi^3)\cos x}{(\pi - 2x)^4}$

- a.  $\frac{\pi^2}{6}$  b.  $\frac{3\pi^2}{16}$  c.  $\frac{\pi^2}{16}$  d.  $-\frac{3\pi^2}{16}$



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**645.** The value of  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$  is



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**646.** The value of (  $\lim$  ) $_{x \rightarrow \infty}$   $\left(e^{\sqrt{x^4+1}} - e^{x^2+1}\right)$  is (a) 0 b. e c.  $1/e$  d.  $-\infty$



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**647.** If  $a_n$  and  $b_n$  are positive integers and  $a_n + \sqrt{2}b_n = (2 + \sqrt{2})^n$ , then  $(\lim)_{n \rightarrow \infty} \left( \frac{a_n}{b_n} \right) =$

a. 2 b.  $\sqrt{2}$  c.  $e^{\sqrt{2}}$  d.  $e^2$



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**648.** If  $f(x) = \begin{cases} x + \frac{1}{2}, & x < 0 \\ 2x + \frac{3}{4}, & x \geq 0 \end{cases}$ , then  $[(\lim)_{x \rightarrow 0} f(x)]$  = (where  $[.]$  denotes the greatest integer function)



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**649.**  $(\lim)_{X \rightarrow (-7)} \frac{[x]^2 + 15[x] + 56}{\sin(x+7)\sin(x+8)} =$  (where  $[.]$  denotes the greatest integer function)

a. is 0 b. is 1 c. is -1 d. does not exist



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**650.** Let  $L_1 = (\lim)_{x \rightarrow 4} (x - 6)^x$  and  $L_2 = (\lim)_{x \rightarrow 4} (x - 6)^4$ . Which of the following is true? Both  $L_1$  and  $L_2$  exists Neither  $L_1$  and  $L_2$  exists  $L_1$  exists but  $L_2$  does not exist  $L_2$  exists but  $L_1$  does not exist



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**651.** If  $f: R \rightarrow R$  is defined by  $f(x) = [x - 3] + |x - 4|$  for  $x \in R$ , then  $\lim_{x \rightarrow 3} f(x)$  is equal to (where  $[.]$  represents the greatest integer function)



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**652.** If  $[.]$  denotes the greatest integer function, then  $(\lim)_{x \rightarrow 0} \frac{x}{a} \left[ \frac{b}{x} \right]$  a.  $\frac{b}{a}$  b. 0 c.  $\frac{a}{b}$  d. does not exist



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653.  $\lim_{x \rightarrow \frac{-1}{3}} \frac{1}{x} \left[ \frac{-1}{x} \right] =$  (where  $[.]$  denotes the greatest integer function)



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654. ( $\lim_{x \rightarrow \infty}$ )  $x^2 \sin\left((\log)_e \sqrt{\frac{\cos \pi}{x}}\right)$  a. 0 b.  $\frac{\pi^2}{2}$  c.  $\frac{\pi^2}{4}$  d.  $\frac{\pi^2}{8}$



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655.  $\lim_{x \rightarrow 0} \frac{1}{x^2} \begin{vmatrix} 1 - \cos 3x & \log_e(1 + 4x) \\ \sin^{-1}(x^x) & \tan^{-1}(2x) \end{vmatrix}$  is equal to



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656. If graph of the function  $y = f(x)$  is continuous and passes through point  $(3, 1)$  then ( $\lim_{x \rightarrow 3}$ )  $\frac{(\log)_e(3f(x) - 2)}{2(1 - f(x))}$  is equal a.  $\frac{3}{2}$  b.  $\frac{1}{2}$  c.  $-\frac{3}{2}$  d.  $-\frac{1}{2}$



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657.  $\lim_{x \rightarrow \infty} \left[ x - \log_e \left( \frac{e^x + e^{-x}}{2} \right) \right] =$

- a)  $(\log)_e 4$  b. 0 c.  $\infty$  d.  $(\log)_e 2$



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658. The value of  $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1 - x)}{x^3}$  is

(a)  $\frac{1}{2}$

(b)  $-\frac{1}{2}$

(c) 0

(d) none of these



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**659.** If  $\lim_{x \rightarrow \infty} \left( \frac{x+c}{x-c} \right)^x = 4$  then the value of  $e^c$  is

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**660.**

If

$$f(x) = \lim_{n \rightarrow \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}} \text{ and } \lim_{x \rightarrow \pm 1} f(x)$$

exists, then

The value of  $b$  is

- A. -1
- B. 1
- C. 0
- D. 2

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**661.** If  $\lim_{x \rightarrow 0} \left[ 1 + x + \frac{f(x)}{x} \right]^{\frac{1}{x}} = e^3$ , then find the value of  $\ln \left( \lim_{x \rightarrow 0} \left[ 1 + \frac{f(x)}{x} \right]^{\frac{1}{x}} \right)$  is --



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**662.** Let  $f(x)$  be the fourth degree polynomial such that  $f'(0) = 6$ ,  $f(0) = 2$  and  $(\lim_{x \rightarrow 1} \frac{f(x)}{(x - 1)^2}) = 1$ . The value of  $f(2)$  is 3 b.  
1 c. 0 d. 2



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**663.** Let  $f(x)$  be the fourth degree polynomial such that  $f'(0) = 6$ ,  $f(0) = 2$  and  $(\lim_{x \rightarrow 1} \frac{f(x)}{(x - 1)^2}) = 1$ . The value of  $f(2)$  is 3 b.  
1 c. 0 d. 2



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**664.**  $(\lim)_{x \rightarrow 0} \left( \frac{\sqrt{1 + x \sin x} - \sqrt{\cos 2x}}{\tan^2(x/2)} \right)$  is equal to **a.  $\frac{1}{6}$**  b. 6 c. 3 d. 2



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**665.** The value of  $\lim_{x \rightarrow \infty} x^2 \left( 1 - \cos \frac{1}{x} \right)$  is



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**666.**  $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$  is equal to



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**667.**  $\lim_{x \rightarrow (2^+)} \{x\} \frac{\sin(x-2)}{(x-2)^2} =$  (where  $\{.\}$  denotes the fractional part function) a. 0 b. 2 c. 1 d. does not exist



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**668.** The value of  $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos x^2}}{1 - \cos x}$  is a.  $\frac{1}{2}$  b. 2 c.  $\sqrt{2}$  d. none of these



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**669.** If

$$f(x) = \lim_{n \rightarrow \infty} \frac{(x^2 + ax + 1) + x^{2n}(2x^2 + x + b)}{1 + x^{2n}} \text{ and } \lim_{x \rightarrow \pm 1} f(x)$$

exists, then

The value of b is



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**670.**  $\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{3x}$



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**671.**  $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)} (1 - \sin x) \tan x =$



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672. The value of  $\lim_{x \rightarrow 3} \frac{(x^3 + 27)(\log_e(x - 2))}{x^2 - 9}$  is a. 9 b. 18 c. 27 d. 1/3



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673. If  $(\lim)_{x \rightarrow 0} \frac{e^{ax} - e^x - x}{x^2} = b$  (finite), then a.  $a = 2, b = 0$  b.  $a = 0, b = \frac{3}{2}$  c.  $a = 2, b = \frac{3}{2}$  d.  $a = 0, b = 2$



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674. If  $\lim_{x \rightarrow 0} \frac{x^3}{\sqrt{a+x}(bx - \sin x)} = 1, a > 0$ , then  $a + b$  is equal to



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675.  $\lim_{x \rightarrow 0} \left[ \frac{\sin^{-1} x}{\tan^{-1} x} \right] =$  (where  $[.]$  denotes the greatest integer function)



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676. The value of  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1 - \sqrt{\sin 2x}}}{\pi - 4x}$  is



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677.  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)$



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678. If  $f(x) = x \left( \frac{e^{|x| + [x]} - 2}{|x| + [x]} \right)$  then (where [.] represents the greatest integer function) (a)  $\lim_{x \rightarrow 0^+} f(x) = -1$  b.  $\lim_{x \rightarrow 0^-} f(x) = 0$  c.  $\lim_{x \rightarrow 0^\square} f(x) = -1$  d.  $\lim_{x \rightarrow 0^\square} f(x) = 0$



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679. If  $A = \lim_{x \rightarrow 0} \frac{\sin^{-1}(\sin x)}{\cos^{-1}(\cos x)}$  and  $B = \lim_{x \rightarrow 0} \frac{[|x|]}{x}$ , then



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680. Let  $f(x) = \lim_{m \rightarrow \infty} \left\{ \lim_{n \rightarrow \infty} \cos^{2m}(n! \pi x) \right\}$ , where  $x \in R$ . Then prove that  $f(x) = \{1, \text{if } x \text{ is rational and } 0, \text{if } x \text{ is irrational}$



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681.  $\lim_{x \rightarrow \infty} \left\{ (e^x + \pi^x)^{\frac{1}{x}} \right\} =$  where  $\{x\}$  denotes the fractional part of  $x$  is equal to



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682. Let  $f(x) = (\lim)_{x \rightarrow \infty} \frac{\tan^{-1}(\tan x)}{1 + ((\log_e x))^n}, x \neq (2n+1)\frac{\pi}{2}$  then 'AA1 e ,f(x)' is a constant function



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**683.** Assume that  $\lim_{\theta \rightarrow -1} f(\theta)$  exists and  
 $\frac{\theta^2 + \theta - 2}{\theta + 3} \leq \frac{f(\theta)}{\theta^2} \leq \frac{\theta^2 + 2\theta - 1}{\theta + 3}$  holds for certain interval containing the point  $\theta = -1$  then  $\lim_{\theta \rightarrow -1} f(\theta)$



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**684.**  $\lim_{n \rightarrow \infty} \left( \frac{\sqrt{n^2 + n} - 1}{n} \right)^{2\sqrt{n^2 + n} - 1}$



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**685.** Let  $f: \overrightarrow{RR}$  be such that  $f(a) = 1, f'(a) = 2$ . Then  
 $(\lim)_{x \rightarrow 0} \left( \frac{f^2(a+x)}{f(a)} \right)^{1/x}$  is a.  $e^2$  b.  $e^4$  c.  $e^{-4}$  d.  $1/e$

A.  $e^2$

B.  $e^4$

C.  $e^{-4}$



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**686.** If  $f(n) = \lim_{x \rightarrow 0} \left\{ \left(1 + \sin \frac{x}{2}\right) \left(1 + \sin \frac{x}{2^2}\right) \dots \left(1 + \sin \frac{x}{2^n}\right) \right\}^{\frac{1}{x}}$

then find  $\lim_{n \rightarrow \infty} f(n)$ .



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**687.** If  $a > 0, b > 0$  then  $(\lim)_{n \rightarrow \infty} \left( \frac{a - 1 + b^{\frac{1}{n}}}{a} \right)^n =$  b<sup>1/a</sup> c. a<sup>1/b</sup> d. a<sup>b</sup>



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**688.**  $(\lim)_{x \rightarrow 0} \frac{\log(e^x + 2 + 2\sqrt{x})}{\tan \sqrt{x}}$  is equal to 0 b. 1 c. 2 d. e<sup>2</sup>



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**689.** If  $f(x) = (\lim)_{n \rightarrow \infty} \left( \cos\left(\frac{x}{\sqrt{n}}\right) \right)^n$ , then the value of  $(\lim)_{x \rightarrow 0} \frac{f(x) - 1}{x}$  is

A. 0

B. 1

C. 2

D. 3/2



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