

India's Number 1 Education App

MATHS

BOOKS - CENGAGE

BINOMIAL THEOREM

Solved Examples And Exercises

1. that Prove

$$(2nC_0)^2 + (2nC_1)^2 + (2nC_2)^2 - + (2nC_{2n})^2 = (-1)^n 2nC_n$$



2. Find the following sum:
$$\frac{1}{n!} + \frac{1}{2!(n-2)!} + \frac{1}{4!(n-4)!} + \dots$$



- **3.** if the sum of coefficients in the expansion of $(x-2y+3z)^n$ is 128, then find the greatest coefficients in the expansion of $(1+x)^n$.
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- **4.** Find the sum of the coefficients in the expansion of $\left(1+2x+3x^2+nx^n\right)^2$.
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- **5.** In the expansion of $\left(1+x\right)^{50}$ the sum of the coefficients of the odd power of x is
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6. If the middle term in the binomial expansion of $\left(\frac{1}{x} + x \sin x\right)^{10}$ is equal to $\frac{63}{8}$, find the value of x.

7. Prove that
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

8. Find the sum $C_0-C_2+C_4-C_6+$, $where C_r=^6 C_r=^n C_r$.

9. Prove that $nC_0+^nC_3+^nC_6+\ =rac{1}{3}igg(2^n+2rac{\cos(n\pi)}{3}igg)$.







10. Given that the 4th term in the expansion of $\left[2+(3/8x)\right]^{10}$ has the maximum numerical value. Then find the range of value of x.



11. Find the sum of coefficients in the expansion of $(x-2y+3z)^n$ is 128, then find the greatest coefficients in the expansion of $(1+x)^n$.



12. Find the greatest term in the expansion of $\sqrt{3} \left(1 + \frac{1}{\sqrt{3}} \right)^{20}$.



13. In an A.P. of 99 terms, the sum of all the odd-numbered terms is 2550.

Then find the sum of all the 99 terms of the A.P.



14. Find the remainder when 27^{40} is divided by 12.



15. In the expansion of $\left(1+x\right)^n$, 7th and 8th terms are equal. Find the value of $\left(7/x+6\right)^2$.



16. Find the value of
$$\sum_{n=1}^{\infty} rac{1}{2n-1} igg(rac{1}{9^{n-1}} + rac{1}{9^{2n-1}}igg)$$



17. Find the sum
$$\sum_{r=1}^n r^2 \frac{\hat{} nC_r}{\hat{} nC_{r-1}}$$
 .



18. Show that 9^n+1 - 8n - 9 is divisible by 64, whenever n is a positive interger.



19. If the 3rd, 4th , 5th and 6th term in the expansion of $(x+\alpha)^n$ be, respectively, a,b,candd, prove that $\dfrac{b^2-ac}{c^2-bd}=\dfrac{5a}{3c}$.



20. Find the remainder when 27^{40} is divided by 12.



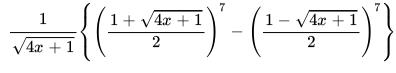
21. Show that $5^{2x}-1$ is divisible by 24 for all $n\in N$.



22. If $\left(2+\sqrt{3}\right)^n=I+f, ext{ where } I ext{ and } n ext{ are positive integers and 0}$



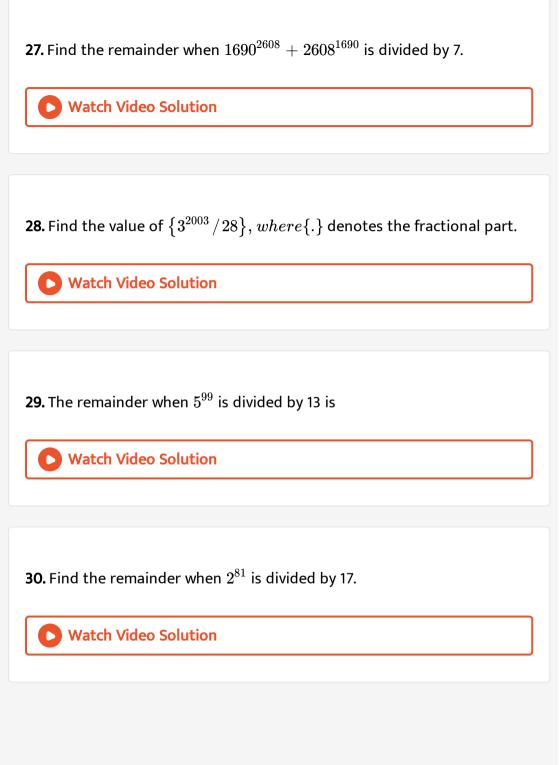
23. Find the degree of the polynomial
$$\frac{1}{\sqrt{1+\sqrt{4x+1}}} \int_{-\infty}^{7} \left(\frac{1-\sqrt{4x+1}}{\sqrt{4x+1}} \right)^{7} \right)$$



- **24.** If 9^7+7^9 is divisible b $2^n,$ then find the greatest value of $n, where n \in N$.
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- **25.** Prove that $\sqrt{10}\Big[ig(\sqrt{10}+1ig)^{100}-ig(\sqrt{10}-1ig)^{100}\Big]$ is an even integer .
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- **26.** Find the remainder when $9x^3-3x^2+x-5$ is divided by $x-\frac{2}{3}$
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31. Using Binomial theorem, prove that 6^n-5n always leaves remainder 1 when divided by 25 for all positive interger n .



32. The coefficient of the middle term in the expansion of $\left(x+2y\right)^6$ is



33. The coefficients of three consecutive terms in the expansion of $(1+a)^n$ are are in the ratio 1: 7: 42 Find n.



34. If the coefficients of r^{th} , $(r+1)^{th}$ and $(r+2)^{th}$ terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation



35.
$$(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4)......(C_{n-1} + C_n) = \frac{C_0C_1C_2.....C_{n-1}(n+1)^n}{n!}$$



36. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive terms in the expansion of $(1+x)^n$, then prove that: $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}.$



37. Find the sum of $\sum_{r=1}^{n} \frac{r^n C_r}{n C_{r-1}}$.



38. The positive integer just greater than $\left(1+.0001\right)^{10000}$ is

39. Find (i) the last digit, (ii) the last two digits, and (iii) the last three digits of
$$17^{256}$$
.



40. If $2^{x+1} = 3^{1-x}$ then find the value of x.

42. If
$$x$$
 is very large as compare to $y,$ then prove that

$$\sqrt{rac{x}{x+y}}\sqrt{rac{x}{x-y}}=1+rac{y^2}{2x^2}\,.$$

41. By mathematical induction prove that 2^{3n} -1 is divisible by 7.

43. The coefficient of
$$x^n$$
 in the expansion of $\left(1-9x+20x^2\right)^{-1}$ is



44. Find the sum
$$1-\frac{1}{8}+\frac{1}{8}\times\frac{3}{16}-\frac{1\times3\times5}{8\times16\times24}+$$

45. Show that $\sqrt{3}$ =1+(1/3)+(1/3)*(3/6)+(1/3)*((3/6)*(5/9)+......

46. Assuming
$$x$$
 to be so small that x^2 and higher power of x can be neglected, prove that
$$\frac{\left(1+\frac{3x}{4}\right)^{-4}(16-3x)^{\frac{1}{2}}}{(8+x)^{\frac{2}{3}}}=1-\left(\frac{305}{96}\right)x$$

- **47.** Find the value of `sumsum_(Olt=i
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- **48.** Find the condition for which the formula $(a+b)^m=a^m+ma^{m-1}b+rac{m(m-1)}{1 imes 2}a^{m-2}b^2+ ext{ holds}.$
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- **49.** Find the value of x, for which $1/\left(\sqrt{5+4x}\right)$ can be expanded as infinite series.
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- **50.** Find the fourth term in the expansion of $(1-2x)^{3/2}$.
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51. Prove that $\hat{\ } nC_0^{2n}C_n-^nC_1^{2n-2}C_n+^nC_2^{2n-4}C_n\equiv 2^n.$

52. Prove that $\hat{\ } nC_0^nC_0 -^{n+1} C_1^nC_1 +^{n+2} C_2^nC_2 \equiv (-1)^n$



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53. Find the sum of the coefficients of all the integral powers of x in the expansion of $\left(1+2\sqrt{x}\right)^{40}$.



54. If the sum of the coefficient in the expansion of $\left(\alpha^2x^2-2\alpha x+1\right)^{51}$ vanishes, then find the value of lpha



55. Prove that $\sum_{lpha+eta+\gamma=10} rac{10!}{lpha!eta!\gamma!} = 3^{10}\cdot$



56. If $\left(1+x-2x^2\right)^{20}=a_0+a_1x+a_2x^2+a_3x^3+...+a_{40}x^{40},\,\,$ then find the value of $a_1+a_3+a_5+...+a_{39}.$



57. Find the sum of the series $~\hat{}~15C_0+^{15}C_1+^{15}C_2+~+^{15}C_7$



58. Find the sum $\sum_{k=0}^{10} \hat{}(20)C_k$



59. Find the sum of all the coefficients in the binomial expansion of $\left(x^2+x-3
ight)^{319}$.



60. If the sum of coefficient of first half terms in the expansion of $\left(x+y\right)^nis256$, then find the greatest coefficient in the expansion.



61. Find the value of $\sum_{p=1}^n \left(\sum_{m=p}^n \hat{\ } nC_m^mC_p\right)$. And hence, find the value of

$$(\lim_{n \to \infty} \frac{1}{3^n} \sum_{p=1}^n \left(\sum_{m=p}^n \hat{n} C_m^m C_p \right).$$



62. Show that the middle term in the expansion of
$$(1+x)^{2n}$$
 is
$$\frac{1.3.5....(2n-1)}{n!}2^nx^n$$
, where n is a positive integer.



- **63.** If the middle term in the expansion of $\left(x^2+1/x\right)^n$ is $924x^6,\,$ then find the value of $n\cdot$
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- **64.** The first three terms in the expansion of $(1+ax)^n (n
 eq 0)$ are
- $1,6x and 16x^2$. Then find the value of aand n.
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65. If x^4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then find the value of r.

66. Find the constant term in the expansion of $(x-1/x)^6$



67. If the coefficients of $(r-5)^t h$ and $(2r-1)^t h$ terms in the expansion of $(1+x)^{34}$ are equal, find r.



68. In $\left(2^{\frac{1}{3}} + \frac{1}{3^{\frac{1}{3}}}\right)^n$ if the ratio of 7th term from the beginning to the

7th term from the end is 1/6, then find the value of n-



69. If the coefficient of 4th term in the expansion of $\left(a+b\right)^n$ is 56, then n is



70. Find the number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$.



71. If x^p occurs in the expansion of $\left(x^2+1/x\right)^{2n}$, prove that its coefficient is $\frac{(2n)!}{\left\lceil\frac{1}{3}(4n-p)\right\rceil!\left\lceil\frac{1}{3}(2n+p)\right\rceil!}.$

- **72.** Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc+ca+ab)^6$
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73. Find the coefficient of x^7 in the expansion of $\left(1+3x-2x^3\right)^{10}$.



74. If the number of terms in the expansion of $(x+y+z)^n$ are 36, then find the value of n.



75. Find the coefficient of a^3b^4c in the expansion of $(1+a-b+c)^9$.



76. The coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^{11}$ is



77. Find the number of terms which are free from radical signs in the expansion of $\left(x^{1/5}+y^{1/10}\right)^{55}$.



78. Find the coefficient of x^5 in the expansion of $\left(1+x^2\right)^5 \left(1+x\right)^4$



79. Find the coefficient of x^{13} in the expansion of $(1-x)^5 imes (1+x+x^2+x^3)^4$.



80. Find the sum $\hat{\ }$ $10C_1+^{10}C_3+^{10}C_5+^{10}C_7+^{10}C_9$



81. Find the sum of $\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)} + \frac{1}{5!(n-5)} +$



82. If n is an even positive integer, then find the value of x if the greatest term in the expansion of $(1+x)^n$ may have the greatest coefficient also.



83. If |x|<1, then find the coefficient of x^n in the expansion of $\left(1+2x+3x^2+4x^3+
ight)^{1/2}.$



84. If (r+1)th term is the first negative term in the expansion of $(1+x)^{7/2}$, then find the value of r.



85. If |x|<1, then find the coefficient of x^n in the expansion of $\left(1+x+x^2+x^3+....\right)^2$.



86. Find the cube root of 217, correct to two decimal places.



87. Find the coefficient of $x^2 \in \left(\dfrac{a}{a+x} \right)^{1/2} + \left(\dfrac{a}{a-x} \right)^{1/2}$



88. If the third term in the expansion of $(1+x)^m is - \frac{1}{8}x^2$, then find the value of m.



89. Prove that
$$1-^n C_1 \frac{1+x}{1+nx} +^n C_2 \frac{1+2x}{\left(1+nx\right)^2} -^n C_3 \frac{1+3x}{\left(1+nx\right)^3} + .$$

$$\dots (n+1)terms = 0$$



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91. Find the coefficient of x^{50} in the expansion o $\left(1+x
ight)^{101} imes\left(1-x+x^2
ight)^{100}$.

90. Find the coefficient of x^{20} in $\left(x^2+2+rac{1}{x^2}
ight)^{-5} \left(1+x^2
ight)^{40}$.

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92. Find the term independent of x in the expansion o $\left(1+x+2x^3\right)\left[\left(3x^2/2\right)-\left(1/3x\right)\right]^9$

93. If a and b are distinct integers, prove that a-b is a factor of a^n - b^n , whenever n is a positive integer.



94. Find a,b,and n in the expansion of $\left(a+b\right)^n$ if the first three terms of the expansion are 729. 7290 and 30375, respectively.



95. Find the coefficient of x^{25} in expansion of expression

$$\sum_{r=0}^{50} \hat{}(50)C_r(2x-3)^r(2-x)^{50-r}.$$



96. If the sum of the coefficients of the first, second, and third terms of the expansion of $\left(x^2+\frac{1}{x}\right)^mis46$, then find the coefficient of the term that does not contain x.



97. If
$$p+q=1, \,$$
 then show that $\displaystyle \sum_{r=0}^n r^2 \, \hat{\,\,\,} \, n C_r p^r q^{n-r} = npq + n^2 p^2 .$

98. If $\left(18x^2+12x+4\right)^n=a_0+a_{1x}+a2x2+ +a_{2n}x^{2n},$ prove that



$$a_r = 2^n 3^r \Big(\hat{\ } (2n) C_r +^n C_1^{2n-2} C_r +^n C_2^{2n-4} C_r + \Big) \,.$$



- **99.** Prove that $\stackrel{\smallfrown}{n} C_1^n C_m -^m C_2^{2n} C_m +^m C_3^{3n} C_m \equiv (-1)^{m-1} n^m$
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100. Prove that

$$^{n}C_{0}^{2n}C_{n} - ^{n}C_{1}^{2n-1}C_{n} + ^{n}C_{2} imes ^{2n-2}C_{n} + + (-1)^{n} \hat{\ \ } nC_{n}^{n}C_{n} = 1.$$



101. Find the sum $\sum_{r=0}^{n}$ $(n+r)C_r$.



102. Find the $\sum_{0 < i < j < n} \sum_{n=1}^{\infty} 1$.



103. Find the value of $\sum \sum_{1 \le i \le n-1} (ij)^n c_i^n c_j$.



104. Find the value of 'sumsum (Olt=i



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105. Find the sum $\sum \sum_{0 < i < j \leq n} {}^n C_i {}^n C_j$



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106. Prove that $\displaystyle\sum_{r=0}^{s} \displaystyle\sum_{s=1}^{n} \; \hat{\;} \; nC_{s}^{s}C_{r} = 3^{n}-1.$



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107. Find the sum $\sum \sum_{0 \le i < j \le n} {}^n C_i$



108. The coefficient of x^4 in the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is



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109. Find the term in $\left(3\sqrt{\frac{a}{\sqrt{b}}}+\sqrt{\frac{b}{3\sqrt{a}}}\right)^{21}$ which has the same power of aa n dbdot



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110. Using the binomial theorem, evaluate $\left(102\right)^5$.



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111. Find the 6th term in expansion of $\left(2x^2-1/3x^2\right)^{10}$



112. Find a if the 17^th and 18^th terms of the expansin $\left(2+a\right)^{50}$ are equal.



113. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$.



114. Simplify:
$$x^5 + 10x^4a + 40x^3a^2 + 80x^2a^3 + 80xa^4 + 32a^5$$



115. Find the value of $18^3 + 7^3 + 3 \times 18 \times 7 \times 25$

 $3^6 + 6 imes 243 imes 2 + 15 imes 18 imes 4 + 20 imes 27 imes 8 + 15 imes 9 imes 16 + 6 imes 3 imes 3$

116. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.



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117. Find the sum $\sum_{i=0}^r .^{n_1} C_{r-i}.^{n_2} C_i$.



118. Prove that $\sum_{r=0}^{2n} rig(.^{2n} \, C_rig)^2 = n^{4n} C_{2n}$.



119. Prove that $\sum_{r=0}^{n} \left(-1
ight)^{r-1} \left(1+rac{1}{2}+rac{1}{3}+rac{1}{r}
ight)^{n} C_{r} = rac{1}{n}$.



$$rac{C_1}{1} - rac{C_2}{2} + rac{C_3}{3} - rac{C_4}{4} + + rac{{{(- 1)}^{n - 1}}}{n}C_n = 1 + rac{1}{2} + rac{1}{3} + + rac{1}{n} \cdot$$



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121. Prove that $\sum_{r=0}^n \ \hat{} \ nC_r \sin rx \cos(n-r)x = 2^{n-1} \sin(nx)$.



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122. Find the last two digits of the number $(23)^{14}$.



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123. Find the last three digits of the number 27^{27} .



$$\left(1+3\sqrt{2}x\right)^{9}+\left(1-3\sqrt{2}x\right)^{9}$$



125. Find the value of $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$

prove

127. that Prove

126. Using binomial theorem (without using the formula for $\hat{\ } nC_r$) ,

 $^{n}C_{4} + ^{m}C_{2} - ^{m}C_{1}^{n}C_{2} = ^{m}C_{4} - ^{m+n}C_{1}^{m}C_{3} + ^{m+n}C_{2}^{m}C_{2} - ^{m+n}C_{3}^{m}C_{1} + ^{m+n}C_{3}^{m}C_{2} - ^{m+n}C_{3}^{m}C_{1} + ^{m+n}C_{3}^{m}C_{2} - ^{m+n}C_{3}^{m}C_{3} - ^{m+n}C_{$

that

124. Find the number of nonzero terms in the expansion of

 $(r+1)^n C_r - r^n C_r + (r-1)^n C_2 - ^n C_3 + + (-1)^r \hat{\ } n C_r = (-1)^r \hat{\ } (n - 1)^r \hat{\ } n C_r = (-1)^r \hat{\ } n C_r = (-1)^$

128. Find the sum
$$\hat{\ } nC_0+^nC_4+^nC_8+.....$$



129. Find the value of $\hat{\ }$ $4nC_0+^{4n}C_4+^{4n}C_8+ +^{4n}C_{4n}$.



$$rac{1}{\sqrt{4x+1}}igg\{igg(rac{1+\sqrt{4x+1}}{2}igg)^n-igg(rac{1-\sqrt{4x+1}}{2}igg)^nigg\}=a_0+a_1x$$

If

then find the possible value of n-



130.

 $(x + {}^{n}C_{0})(x + 3{}^{n}C_{1}) \times (x + 5{}^{n}C_{2})[x + (2n + 1){}^{n}C_{n}].$

132. If
$$(1+x)^{15}=C_0+C_1x+C_2x^2++C_{15}x^{15},\,\,$$
 then find the value of $C_2+2C_3+3C_4+\,+14C_{15}.$

 $\frac{\hat{n}C_0}{1} + \frac{\hat{n}C_2}{3} + \frac{\hat{n}C_4}{5} + \frac{\hat{n}C_6}{7} + \dots + = \frac{2^n}{n+1}$

the coefficient of x^n in the polynomial

that

131.

133.

Find

134. Find the sum $\sum \sum_{0 < i < i < z}^{n} C_i^n C_j$

135. Find the sum
$$\sum_{i\neq j}\sum_{j=1}^n C_i^nC_j$$



136. Show that the integer next above $\left(\sqrt{3}+1\right)^{2m}$ contains 2^{m+1} as a factor.

Prove that $rac{1^2}{3}{}^nC_1 + rac{1^2+2^2}{5^n}C_2 + rac{1^1+2^2+3^2}{7^n}C_3 + rac{1^2+2^2}{5^n}C_3 + rac{1^2+2^2}{5$

that



137.

138.

$$+rac{1^2+2^2++n^2}{\left(2n+1
ight)^n}C_n=rac{n(n+3)}{6}\cdot 2^{n-2}.$$



$$\frac{\hat{n}C_0}{1} + \frac{\hat{n}C_2}{3} + \frac{\hat{n}C_4}{5} + \frac{\hat{n}C_6}{7} + \dots + = \frac{2^n}{n+1}$$

Prove

$$\frac{1}{81^n} - \frac{10}{(81^n)^{2n}} C_1 + \frac{10^2}{(81^n)^{2n}} C_2 - \frac{10^3}{(81^n)^{2n}} C_3 + + \frac{10^{2n}}{81^n}$$

 $\hat{\ \ }20C_0-rac{\hat{\ \ \ }(20)C_1}{2}+rac{\hat{\ \ \ }(20)C_2}{3}-rac{\hat{\ \ \ }(20)C_3}{4}+.$

139. Find the sum $2C_0+rac{2^2}{2}C_1+rac{2^3}{2}C_2+rac{2^4}{4}C_3+\ldots\ldots+rac{2^{11}}{11}C_{10}$

the

value

value

of

of

140.

141.

Find

142. Find the sum
$$1C_0+2C_1+3C_2+\ +(n+1)C_n, where $C_r=^n C_r$$$

find the value of
$$a_1+2a_2+3a_3+\stackrel{\cdot \cdot }{+}npa_{np}.$$

143. If $\left(1+x+x^2++x^p\right)^n=a_0+a_1x+a_2x^2++a_{np}x^{np},$ then

 $C_1(a-1) - C_2 imes (a-2) + \ + (-1)^{n-1} C_n(a-n) = a, where C_r =^n C_r$

145. Find the sum $1 imes 2 imes C_1 + 2 imes 3C_2 +$

144. If
$$n>2,$$
 then prove that

 $+n(n+1)C_n$, where $C_r = {}^n C_r$.

146. If $x+y=1, \,\,$ prove that $\sum_{r=0}^{n} r \cdot^n C_r x^r y^{n-r} = nx \cdot n$



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147. Find the sum $3^nC_0 - 8^nC_1 + 13^nC_2 - 18^nC_3 + \dots$



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148. Prove that $\frac{\cdot^n C_1}{2} + \frac{\cdot^n C_3}{4} + \frac{\cdot^n C_5}{6} + \ldots = \frac{2^n - 1}{n + 1}$.



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If $(1+x)^n=\sum_{r=1}^n \hat{\ } nC_r$ 149. $C_0 + \frac{C_1}{2} + \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$.

show

that

$$n+1$$
 $n+1$



150. If $\sum_{r=0}^{2n}a_r(x-2)^r=\sum_{r=0}^{2n}b_r(x-3)^randa_k=1$ for all $k\geq n,$ then show that $b_n=^{2n+1}C_{n+1}$.



151. Statement 1: $3^{2n+2}-8n-9$ is divisible by $64,\ orall\,n\in N$. Statement 2: $\left(1+x\right)^n-nx-1$ is divisible by $x^2,\ orall\,n\in N$.



152. Statement 1: The number of distinct terms in $\left(1+x+x^2+x^3+x^4\right)^{1000}is4001$. Statement 2: The number of distinct terms in expansion $\left(a_1+a_2+{}+a_m\right)^nis^{n+m-1}C_{m-1}$



Statement1: if $n \in Nandn$ is not a multiple of 3 and

$$\left(1+x+x^2
ight)^n=\sum_{r=0}^{2n}a_rx^r,$$
 then the value of $\sum_{r=0}^n\left(\,-\,1
ight)^rar^nC_r$ is zero

Statement 2: The coefficient of x^n in the expansion of $\left(1-x^3
ight)^n$ is zero, if

$$n = 3k + 1$$
 or $n = 3k + 2$.



154. Statement 1:Three consecutive binomial coefficients are always in A.P.

Statement 2: Three consecutive binomial coefficients are not in H.P. or G.P.



155. The value of

$$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} + \dots + \binom{30}{20}\binom{30}{30} =$$

 $60C20 \text{ b.} \ \hat{\ } 30C10 \text{ c.} \ \hat{\ } 60C30 \text{ d.} \ \hat{\ } 40C30$



If $f(x) = x^n, f(1) + \frac{f^1(1)}{1} + \frac{f^2(1)}{2!} + \frac{f^n(1)}{n!}, where f^r(x)$ 156. denotes the rth order derivative of f(x) with respect to x, is

A. a.nB. b. 2^n

C. c. 2^{n-1}

D. d. none of these

Answer: null



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157. The fractional part of $rac{2^{4n}}{15}$ is $(n\in N)$ (A) $rac{1}{15}$ (B) $rac{2}{15}$ (C) $rac{4}{15}$ (D) none of these



158. The value of $\hat{}$ $15C02 - {}^{15}C12 + {}^{15}C22 - {}^{15}C152$ is 15 b. - 15 c. 0 d.

51



159. If the sum of the coefficients in the expansion of $\left(1-3x+10x^2\right)^n$ is a and the sum of the coefficients in the expansion of $\left(1-x^2
ight)^n$ is b, then



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160. If $(1+x-2x^2)^6=1+a_1x+a_2x^2+a_2x^3+...$, then the value of $a_2 + a_4 + a_6 + \dots + a_{12}$ will be

A. (a) 32

B. (b) 31

C. (c) 64

D. (d) 1024

Answer: null



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161. Maximum sum of coefficient in the expansion of $\left(1-x\sin\theta+x^2\right)^n$ is 1 b. 2^n c. 3^n d. 0



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162. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is



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number of distinct terms in the expansion of $\left(x+rac{1}{x}+x^2+rac{1}{x^2}
ight)^{15}$ is/are (with respect to different power of x) 255b. 61 c. 127 d. none of these

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164. The sum of the coefficients of even power of x in the expansion of $\left(1+x+x^2+x^3\right)^5$ is a .256 b. 128 c. 512 d. 64



165. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation



166. If the coefficients of the (2r+4)th, (r-2)th term in the expansion of $\left(1+x\right)^{18}$ are equal, then the value of r is.



167. If the coefficients of the rth, (r+1)th, (r+2)th terms is the expansion of $(1+x)^{14}$ are in A.P, then the largest value of r is.



168. If the three consecutive coefficients in the expansion of $(1+x)^n$ are 28, 56, and 70, then the value of n is.



169. Degree of the polynomial
$$\left[\sqrt{x^2+1}+\sqrt{x^2-1}
ight]^8+\left[rac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}}
ight]^8$$
 is.



- **170.** Least positive integer just greater than $\left(1+0.00002\right)^{50000}$ is.
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171. If
$$U_n=\left(\sqrt{3}+1\right)^{2n}+\left(\sqrt{3}-1\right)^{2n}$$
 , then prove that $U_{n+1}=8U_n-4U_{n-1}$



172. Prove that the coefficient of
$$x^n$$
 in the expansion of
$$\frac{1}{(1-x)(1-2x)(1-3x)}is\frac{1}{2}\big(3^{n+2}-2^{n+3}+1\big).$$

 $(30,0)(30,10) - (30,1)(30,11) + (30,2)(30,11) - \dots + (30,20)(30,11)$

, where $(n,r) = nC_r$ is a. (30,10) b. (30,15) c. (60,30) d. (31,10)

value

of



The

173.

174.

174. If
$$n=12m(m\in N),$$
 prove that $\hat{C}_0 - \frac{\hat{C}_0}{\left(2+\sqrt{3}\right)^2} + \frac{\hat{C}_0}{\left(2+\sqrt{3}\right)^4} - \frac{\hat{C}_0}{\left(2+\sqrt{3}\right)^6} + = \left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^n.$

175. In the expansion of $(1+x)^n(1+y)^n(1+z)^n$, the sum of the co-

 $rac{.^{101} \ C_1}{.^{101} \ C_2} + rac{2. \, .^{101} \ C_2}{.^{101} \ C_3} + rac{3. \, .^{101} \ C_3}{.^{101} \ C_2} + \ldots + rac{101. \, .^{101} \ C_{101}}{.^{101} \ C_{102}} \, ext{is} \, \ldots.$

177. Prove that $\sum_{r=1}^{m-1} rac{2r^2 - r(m-2) + 1}{(m-r)^m C_r} = m - rac{1}{m}$

that

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efficients of the terms of degree 'r' is

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176. The sum of the series

$$\hat{ }$$
 n

178. Find the coefficient of
$$x^{50}$$
 in the expansion of $(1+x)^{101} imes \left(1-x+x^2\right)^{100}$.



179. If b_1,b_2b_n are the nth roots of unity, then prove that ${}^{\hat{}} nC_1\dot{b}_1+{}^nC_2\dot{b}_2+ +{}^nC_n\dot{b}_n=rac{b_1}{b_2}ig\{(1+b_2)^n-1\}$



180. If
$$\ \hat{}\ n+1C_{r+1}$$
: $\ C_r$: $\ C_r$: $\ C_{r-1}=11$: $\ 6$: $\ 3$, then $\ nr=1$

A. a.20

u.2(

B. b. 30

C. c. 40

Answer: null



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- **181.** If the last tem in the binomial expansion of $\left(2^{\frac{1}{3}}-\frac{1}{\sqrt{2}}\right)^n is\left(\frac{1}{3^{\frac{5}{3}}}\right)^{\log_3 8} \text{ , then 5th term from the beginning is } 210 \text{ b.}$
- $420\ \mathrm{c.}\ 105\ \mathrm{d.}$ none of these



- **182.** Find the last two digits of the number $(23)^{14}$.
 - A. 01
 - B. 03
 - C. 09
 - D. None of these

Answer: null



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183. The value of x for which the sixth term in the expansion of

$$\left[2^{\log} - 2^{\sqrt{9^{(x-1)}+7}} + rac{1}{2^{rac{1}{5}}(\log)_2\left(3^{(x-1)}+1
ight)}
ight]^{\gamma}$$
 is 84 is

A. a.4

B. b. 1 or 2

C. c. 0 or 1

D. d. 3

Answer: null



184. If the 6th term in the expansion of $\left(\frac{1}{x^{\frac{8}{3}}} + x^2(\log)_{10}x\right)^8$ is 5600,

then x equals

A. a.1

B. b. $(\log)_e 10$

C. c. 10

D. d. x

Answer: null



185. The total number of terms which are dependent on the value of x in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$ is equal to

A. a.2n+1

B. b. 2n

 $\mathsf{C}.\,\mathsf{c}.\,n$

 $\mathsf{D.\,d.}\,n+1$

Answer: null



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186. In the expansion of $\left(3^{-x/4}+3^{5x/4}\right)^n$ the sum of binomial coefficient is 64 and term with the greatest binomial coefficient exceeds the third by (n-1) , the value of x must be

 $\mathsf{A.\ a.}0$

B. b. 1

 $\mathsf{C.}\;\mathsf{c.}\;2$

D. d. 3

Answer: null



187. If n is an integer between 0 and 21, then the minimum value of n!(21-n)! is attained for n=1 b. 10 c. 12 d. 20



188. If R is remainder when $6^{83}+8^{83}$ is divided by 49, then the value of R/5 is.



189. Let a and b be the coefficients of x^3 in $\left(1+x+2x^2+3x^3\right)^4 and \left(1+x+2x^2+3x^3+4x^4\right)^4,$ then respectively. Then the value of 4a/b is.



190. Let $1+\sum_{r=0}^{10}\left(3^{r}\dot{10}C_{r}+r\dot{10}C_{r}\right)=2^{10}ig(lpha.\,4^{5}+etaig)where lpha,\,eta\in N$ and $f(x) = x^2 - 2x - k^2 + 1$. If α, β lies between the roots of f(x) = 0, then find the smallest positive integral value of k.



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191. Let $a=3^{\frac{1}{223}}+1$ and for all t>3, let

$$f(n) = ^n C_0 \dot{a}^{n-1} - ^n C_1 \dot{a}^{n-2} + ^n C_2 \dot{a}^{n-3} - + (-1)^{n-1} \stackrel{\cdot}{\frown} n C_{n-1} \dot{a}^0 \;\; .$$

If the value of $f(2007) + f(2008) = 3^k where k \in N$, then the value of k is.



binomial expansion 192. the constant term in the of $\left(x^2-rac{1}{x}
ight)^n, n\in N$ is 15, then the value of n is equal to.



193. The largest value of x for which the fourth tem in the expansion

$$\left(5^{\left(\frac{2}{5}\right)\,(\log)_{\,5}\sqrt{4^x+44}}+\frac{1}{5^{\log_5}\!\left(2^{\,(\,x\,-\,1\,)\,+\,7}\right)^{\frac{1}{3}}}\right)^8 \text{ is 336 is.}$$



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194. The number of values in set of values of r for which

$$\hat{\ }(23)C_r + 2.^{23}\ C_{r+1} + ^{23}\ C_{r+2} \geq^{25}\ C_{15}$$
 is



195. If the second term of the expansion $\left[a^{rac{1}{13}}+rac{a}{\sqrt{a^{-1}}}
ight]^n$ is $14a^{5/2}$, then the value of $\frac{\hat{} nC_3}{\hat{} nC_2}$ is.



196. Given $\left(1-2x+5x^2-10x^3\right)\left(1+x\right)^n=1+a_1x+a_2x^2+$ and that $a12=2a_2$ then the value of n is.



197. Sum of last three digits of the number $N=7^{100}-3^{100}$ is.



198. Let n be a positive integer and $\left(1+x+x^2\right)^n=a_0+a_1x++a^{2n}x^{2n}$. Show that $a_0^2-a_1^2+a_2^2++{'a_2n'x^2}=a_n$.



199. Prove that $\sum_{r=1}^k \left(-3\right)^{r-13n} C_{2r-1} = 0,$ where k = 3n/2 and n is an even integer.

200. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals



201. If in the expansion of $(1+x)^n, a, b, c$ are three consecutive coefficients, then $n=\frac{ac+ab+bc}{b^2+ac}$ b. $\frac{2ac+ab+bc}{b^2-ac}$ c. $\frac{ab+ac}{b^2-ac}$ d. none of these



202. Prove that
$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = 1 - \frac{1}{n+1}$$
.



203. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all n = 1, 2,



204. The coefficient of 1/x in the expansion of $(1+x)^n(1+1/x)^n$ is $\frac{n!}{(n-1)!(n+1)!}$ b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(2n-1)!(2n+1)!}$ d. none of these



205. The coefficient x^5 in the expansion of $\left(1+x\right)^{21}+\left(1+x\right)^{22}+{}+\left(1+x\right)^{30}$ is a. $^{51}C_5$ b. 9C_5 c. $^{31}C_6-^{21}C_6$ d.



 $^{30}C_{5}+^{20}C_{5}$

206. If x^m occurs in the expansion $\left(x+1/x^2\right)^2n$ then the coefficient of

206. If
$$x^m$$
 occurs in the expansion $(x+1/x^2)$ n then the coefficient of x^m is $\frac{(2n)!}{(m)!(2n-m)!}$ b. $\frac{(2n)!3!3!}{(2n-m)!}$ c. $\frac{(2n)!}{\left(\frac{2n-m}{3}\right)!\left(\frac{4n+m}{3}\right)!}$ d. none of

these



207. If the coefficients of 5th, 6th, and 7th terms in the expansion of $(1+x)^n$ are in A.P., then n=1 a. 7 only b. 14 only c. 7 or 14 d. none of these



208. If
$$\left(1+2x+x^2
ight)^n=\sum_{r=0}^{2n}a_rx^r, thena=\left(\stackrel{\cdot}{n}C_2
ight)^2$$
 b.

^
$$nC_r$$
 $\stackrel{ extstyle }{\frown}$ nC_{r+1} c. ^ $2nC_r$ d. ^ $2nC_{r+1}$



209. In the expansion of $\left(x^3-rac{1}{x^2}
ight)^n, n\in N$ if sum of the coefficients of x^5 and x^{10} is 0 then n is



210. If the coefficients of rth and (r+1)th terms in the expansion of $(3+7x)^{29}$ are equal, then r equals a. 15 b. 21 c. 14 d. none of these



211. In the expansion of $\left(1+3x+2x^2\right)^6$, the coefficient of x^{11} is a. 144 b. 288 c. 216 d. 576

212. If
$$n-1C_r=\left(k^2-3\right)^nC_{r+1},$$
 then (a) $(\,-\infty,\,-2]$ (b) $[2,\infty)$ (c)

$$\left[-\sqrt{3},\sqrt{3}
ight]$$
 (d) $\left(\sqrt{3},2
ight]$

213. Prove that
$$\dfrac{3!}{2(n+3)}=\sum_{r=0}^n \left(-1
ight)^r \left(\dfrac{\hat{} nC_r}{\hat{} (r+3)C_r}
ight)$$



214. If
$$s_n=\sum_{r=0}^n rac{1}{{}^nC_r}$$
 and $t_n=\sum_{r=0}^n rac{r}{{}^nC_r}$,then $rac{t_n}{s_n}$ is equal to



215. The expression
$$\left(x+\frac{\left(x^{3}-1\right)^{\frac{1}{2}}}{2}\right)^{5}+\left(x-\frac{\left(x^{3}-1\right)^{\frac{1}{2}}}{2}\right)^{5}$$
 is a polynomial of degree a. 5 b. 6 c. 7 d. 8



216. The roots of the equation are | x C r x+1 C r x+2 C r n-1 C r n C r n+1 Cr n-1 Cr-1 n Cr-1 n+1 Cr-1 | =0 are a. x=n, b. x=n+1, c. x=n-1, d. x=n-2.



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217. In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of 5^{th} and 6^{th} term $\frac{a}{h}$ equals



218. Coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$ is 1051 b. 1106 c. 1113 d. 1120



219. r and n are positive integers r>1, n>2 and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n



equals

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220. The coefficient of x^4 in $\left(x/2 - 3/x^2 \right)^{10}$ is $\frac{405}{256}$ b. $\frac{504}{259}$ c. $\frac{450}{263}$ d. none of these



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221. If C_r stands for nC_r , then the sum of the series

$$rac{2\left(rac{n}{2}
ight)!\left(rac{n}{2}
ight)!}{n!}ig[C_0^2-2C_1^2+3C_2^2-......+(-1)^n(n+1)C_n^2ig]$$
 ,where

n is an even positive integer, is



222. The sum $\sum_{i=0}^{m} \binom{10}{i} \binom{20}{m-1}$, where $\binom{p}{q} = 0$ if p < q, is maximum when m is equal to (A) 5 (B) 10 (C) 15 (D) 20



223. The coefficient of
$$X^{24}$$
in the expansion of $\left(1+X^2\right)^{12}\left(1+X^{12}\right)\left(1+X^{24}\right)$



224. The term independent of
$$a$$
 in the expansion of
$$\left(1+\sqrt{a}+\frac{1}{\sqrt{a}-1}\right)^{-30}$$
 is (a) $30C_{20}$ (b) 0 (c) $30C_{10}$ (d) non of these



225. The coefficient of x^{53} in the expansion

$$\sum_{m=0}^{100} \ \hat{}\ 100 C_m (x-3)^{100-m} 2^m$$
 is (a) $100 C_{47}$ (b.) $100 C_{53}$ (c.) $-100 C_{53}$ (d.)



none of these

226. The coefficient of the term independent of \boldsymbol{x} in the exampansion of

$$\left(rac{x+1}{x^{2/3}-x^{1/3}+1}-rac{x-1}{x-x^{1/2}}
ight)^{10}$$
 is 210 b. 105 c. 70 d. 112



227. In the expansion of $\left(1+x+x^3+x^4\right)^{10}$, the coefficient of x^4 is

 $\hat{\ }$ $40C_4$ b. $\hat{\ }$ $10C_4$ c. 210 d. 310



228. If coefficient of $a^2b^3c^4\in (a+b+c)^m$ (where $n\in N$) is L(L
eq 0),then in same expansion coefficient of $a^4b^4c^1$ will be (A) L (B) $\frac{L}{2}$ (C) $\frac{mL}{4}$



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229. The last two digits of the number 3^{400} are (A) 81 (B) 43 (C) 29 (D) 01



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230. The expression

$$\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}
ight)^6+\left(rac{2}{\left(\sqrt{2x^2+1}+\sqrt{2x^2-1}
ight)^{\square}}
ight)^6$$
 i

polynomial of degree 6 b. 8 c. 10 d. 12



231. The coefficient of $x^r[0 \le r \le (n-1)]$ in Ithe expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2++(x+2)^{n-1}$ is $\hat{\ } nC_r(3^r-2^n)$ b. $\hat{\ } nC_r(3^{n-r}-2^{n-r})$ c. $\hat{\ } nC_r(3^r+2^{n-r})$ d. none

232. If $(1+2x+3x^2)^{10}=a_0+a_1x+a_2x^2+a_{20}x^{20}$, then a_1 equals

233. In the expansion of $\left(5^{1/2}+7^{1/8}\right)^{1024}$, the number of integral



of these



terms is 128 b. 129 c. 130 d. 131



234. For which of the following values of x,5th term is the numerically greatest term in the expansion of $\left(1+x/3\right)^{10}$, a. -2 b. 1.8 c. 2 d. -1.9



235. For natural numbers
$$m,n, \quad ext{if} \quad (1-y)^m(1+y)^n=1+a_1y+a_2y^2+, and a_1=a_2=10, then$$
 `m nc. m+n=80 d . m-n=20`



sum of possible real values of x is.

236. If the middle term in the expansion of $\left(\frac{x}{2}+2\right)^8$ is 1120, then the



237. If

$$(1+x)^n = C_0 + C_1 x + C 2 x 2 + + C_n x^n, then' C_0 - (C_0 + C_1 +) + (C_0 + C_1 +) + C_0$$
 is

A. a. even integer

B. b. a positive value

C. c.a negative value divisible by 2^{n-1}

D. d.divisible by 2^n

Answer: null



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238. In the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n, n\in N$, (a)number of terms is 2n+1 (b)coefficient of constant terms is 2^{n-1} (c)coefficient of $x^{2n-1}isn$ (d)coefficient of x^2 in n



239. The value of $\hat{\ } nC_1 +^{n+1} C_2 +^{n+2} C_3 + \ +^{n+m-1} C_m$ is equal to (a) $\hat{\ } m + nC_{n-1}$ (b) $\hat{\ } m + nC_{n-1}$ (c)

a)
$$m+n$$
C $_{n-1}$ (b) $m+n$ C $_{n-1}$ (c) $^{\hat{}}mC_1+^{m+1}C_2+^{m+2}C_3+^{m+n-1}$ (d) $^{\hat{}}m+1$ C $_{m-1}$

If





240.

 $(1+x)^n=C_0+C_1x+C2x2+ +C_nx^n, n\in N, then C_0-C_1+C_2- +$

is equal to (m < n)



(b)a rational number (c)a positive integer (d)a negative integer

241. The 10th term of $\left(3-\sqrt{rac{17}{4}+3\sqrt{2}}
ight)^{20}$ is (a)a irrational number

242. For the expansion $\left(x\sin p + x^{-1}\cos p\right)^{10}, (p\in R),$ The greatest value of the term independent of x is $(a)10!/2^5(5!)^2$ (b)the least value of sum of coefficient is zero (c)the greatest value of sum of coefficient is 32 (d)the least value of the term independent of x occurs when $p=(2n+1)rac{\pi}{4}, n\in Z$



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243. Let $\left(1+x^2\right)^2(1+x)^n=\sum_{k=0}^{n+4}a_kx^k\dot{I}fa_1,a_2anda_3$ are in arithmetic progression, then the possible value/values of n is/are a. 5 b. 4 c. 3 d. 2



244. The middle term in the expansion of $(x/2+2)^8$ is 1120, then $x\in R$ is equal to a. -2 b. 3 c. -3 d. 2



245. If
$$\left(1-x^2\right)^n=\sum_{r=0}^n a_r x^r (1-x)^{2n-r},$$
 then a_r is equal to a.) $\hat{\ }nC_r$ b.) $\hat{\ }nC_r 3^r$ c.) $\hat{\ }2nC_r$ d.) $\hat{\ }nC_r 2^r$



$$\Big[(\ \hat{\ }nC_0+^nC_3+)1/2(\ \hat{\ }nC_1+^nC_2+^nC_4+^nC_5]^2+3/4(\ \hat{\ }nC_1-^nC_2+^nC_4+^nC_5)^2\Big]$$
3 b. 4 c. 2 d. 1

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 $f(x) = a_0 + a_1 x + a_2 x^2 + a_n x^n + a_n$ 248. and

247. If $\sum_{r=0}^{10} \left(rac{r+2}{r+1}
ight)$. n $C_{r} = rac{2^{8}-1}{6}$, then n is (A) 8 (B) 4 (C) 6 (D) 5

 $rac{f(x)}{1-x}=b_0+b_1x+b_2x^2+{}+b_nx^n+$, then $b_n+b_{n-1}=a_n$ b.

 $b_n-b_{n-1}=a_n$ c. $b_n/b_{n-1}=a_n$ d. none of these



249. $\sum_{r=0}^{300}a_rx^r=\left(1+x+x^2+x^3
ight)^{100}$. If $a=\sum_{r=0}^{300}a_r,then\sum_{r=0}^{300}ra_r$ is equal to 300a b. 100a c. 150a d. 75a



250. The value of $\sum_{r=1}^{n+1} \left(\sum_{k=1}^n {}^kC_{r-1}\right)$ (where $r,k,n\in N$) is equal to a. $2^{n+1}-2$ b. $2^{n+1}-1$ c. 2^{n+1} d. none of these



251. If $\dfrac{x^2+x+1}{1-x}=a_0+a_1x+a_2x^2+$, $then\sum_{i=1}^{50}a_r$ is equal to 148 b.

 $146\ \mathsf{c}.\ 149\ \mathsf{d}.$ none of these



252. p is a prime number and `n



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The coefficient of x^9 in the expansion 253. of $(1+x)(1+x^2)(1+x^3)....(1+x^{100})$ is



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254. The coefficients of three consecutive terms of $\left(1+x\right)^{n+5}$ are in the ratio 5:10:14. Then n=



255.

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 $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + a_rx^r + thena_0 + a_1 + a_2 + a_r$

If

257. The coefficient of x^{10} in the expansion of $\left(1+x^2-x^3\right)^8$ is 476 b.

is equal to (a) $\frac{n(n+1)(n+2)(n+r)}{r!}$ (b) $\frac{(n+1)(n+2)(n+r)}{r!}$ (c)

 $\frac{n(n+1)(n+2)(n+r-1)}{r!}$ (d)none of these

256. The value of $\sum_{r=0}^{20} r(20-r) \left(.^{20} C_r\right)^2$ is equal to

a. $400^{39}C_{20}$ b. $400^{40}C_{19}$ c. $400^{39}C_{19}$ d. $400^{38}C_{20}$

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258. If the term independent of
$$x$$
 in the $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then k equals $2,-2$ b. $3,-3$ c. $4,-4$ d. $1,-1$

259. The coefficient of
$$x^2y^3$$
 in the expansion of $(1-x+y)^{20}$ is $\frac{20!}{213!}$ b.

 $-rac{20!}{213!}$ c. $rac{20!}{15!2!3!}$ d. none of these

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- **260.** The coefficient of x^5 in the expansion of $\left(x^2-x-2\right)^5$ is -83 b.
 - -82 c. -86 d. -81
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261. The coefficient of $a^8b^4c^9d^9$ in $(abc+abd+acd+bcd)^{10}$ is 10! b.

 $\frac{10.}{8!4!9!9!}$ c. 2520 d. none of these

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262. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{hx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left\lceil ax - \left(rac{1}{hx^2}
ight)
ight
ceil^{11}$, then a and b satisfy the relation



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263. If $(1+x)^5 = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$, then the value of $\left(a_0-a_2+a_4
ight)^2+\left(a_1-a_3+a_5
ight)^2$ is equal to 243 b. 32 c. 1 d. 2^{10}



264. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is



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265. The coefficient of x^{28} in the expansion of $\left(1+x^3-x^6\right)^{30}$ is 1 b. 0 c. $30C_6$ d. $^{\circ} 30C_3$

266. The coefficient of x^n in $(1+x)^{101} \big(1-x+x^2\big)^{100}$ is nonzero, then n cannot be of the form 3r+1 b. 3r c. 3r+2 d. none of these



267.

$$\sum_{r=0}^{n} \left(\, -1
ight)^{r} \, \, \hat{} \, \, n C_{r} igg[rac{1}{2^{r}} + rac{3}{2^{2r}} + rac{7}{2^{3r}} + rac{15}{2^{4r}} + up
ightarrow mterms igg] = rac{2^{mn}}{2^{mn} (2^{n})} \, \, .$$

Prove

that



A. (a)there are exactly 730 rational term

268. In the expansion of $\left(7^{1/3} + 11^{1/9}\right)^{6561}$,

B. (b)there are exactly 5831 irrational terms

C. (c)the term which involves greatest binomial coefficients is

irrational

D. (d)the term which involves greatest binomial coefficients is rational

Answer: null



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269. If for
$$z$$
 as real or complex, $\left(1+z^2+z^4\right)^8=C_0+C1z2+C2z4++C_{16}z^{32}then$ (a)

(a)

$$C_0-C_1+C_2-C_3+ \ + C_{16}=1$$
 (b)

$$C_0 - C_1 + C_2 - C_3 + + C_{16} = 1$$
 (b)
$$C_0 + C_3 + C_6 + C_0 + C_{12} + C_{15} = 3^7$$
 (c)

$$C_2 + C_5 + C_6 + C_{11} + C_{14} = 3^6$$
 (d)

$$C_1 + C_4 + C_7 + C_{10} + C_{13} + C_{16} = 3^7$$



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270. The sum of coefficient in the expansion of $\left(1+ax-2x^2\right)^n$ is (a)positive, when $a<1 and n=2k, k\in N$ (b)negative, when

 $a < 1 and n = 2k+1, k \in N$ (c)positive, when $a < 1 and n \in N$ (d)zero, when a = 1



271. If the 4th term in the expansion of $\left(ax+1/x\right)^n$ is 5/2, then (a) $a=rac{1}{2}$ b. n=8 c. $a=rac{2}{3}$ d. n=6



272. Find the value (s) of r satisfying the equation

$$\hat{\ }$$
 69 C_{3r-1} $-^{69}$ C_{r^2} $=^{69}$ C_{r^2-1} $-^{69}$ C_{3r}



273. If $\left(4+\sqrt{15}\right)^n=I+f, whren$ is an odd natural number, I is an integer and '0



274. In the expansion of $(x+a)^n$ if the sum of odd terms is P and the sum of even terms is Q, then (a) $P^2-Q^2=\left(x^2-a^2\right)^n$ (b)

$$4PQ = (x+a)^{2n} - (x-a)^{2n}$$
 (c)

$$2ig(P^2+Q^2ig)=(x+a)^{2n}+(x-a)^{2n}$$
 (d)all of these



275. If the coefficients of the rth, (r+1)th, (r-2)th terms is the expansion of $(1+x)^{14}$ are in A.P, then the largest value of r is.



276. The value of x in the expression $\left(x+x^{(\log)_{10}x}\right)^5$ if third term in the expansion is 10,00,000 is/are a. 10 b. 100 c. $10^{-5/2}$ d. $10^{-3/2}$



277. Let $R=\left(5\sqrt{5}+11\right)^{2n+1} and f=R-[R]where[]$ denotes the greatest integer function, prove that $Rf=4^{2n+1}$



278. If |x|<1, then find the coefficient of x^n in the expansion of $\left(1+x+x^2+x^3+....\right)^2$.



279. The coefficient of $x^5 \in \left(1+2x+3x^2+....
ight)^{-3/2} is(|x|<1)$ 21 b.

25 c. 26 d. none of these

280. If x is so small that x^3 and higher power of x may neglected, then

$$rac{(1+x)^{rac{3}{2}}-\left(1+rac{1}{2}x
ight)^3}{(1-x)^{rac{1}{2}}}$$
 may be approximated as



281. If x is positive, the first negative term in the expansion of $(1+x)^{rac{27}{5}}$ is



282. Value of $\sum_{k=1}^{\infty} \sum_{r=0}^{k} \frac{1}{3^k} (\hat{k}C_r)$ is $\frac{2}{3}$ b. $\frac{4}{3}$ c. 2 d. 1



283. If the expansion in powers of x of the function 1/[(1-ax)(1-bx)]

is a $a_0+a_1x+a_2x^2+a_3x^3+$,then coefficient of x^n is $\dfrac{b^n-a^n}{b-a}$ b.

$$rac{a^n-b^n}{b-a}$$
 c. $rac{b^{n+1}-a^{n+1}}{b-a}$ d. $rac{a^{n+1}-b^{n+1}}{b-a}$



284. If $f(x)=1-x+x^2-x^3+...+x^{15}+x^{16}-x^{17}$, then the coefficient of $x^2\in f(x-1)$ is 826 b. 816 c. 822 d. none of these



285. The sum of rational term in $\left(\sqrt{2} + \sqrt[3]{3} + \sqrt[6]{5}\right)^{10}$ is equal to 12632 b. 1260 c. 126 d. none of these



286. The value of $\sum_{r=0}^{10} (r)^{20} C_r$ is equal to: a. $20 \left(2^{18} + ^{19} C_{10} \right)$ b. $10 \left(2^{18} + ^{19} C_{10} \right)$ c. $20 \left(2^{18} + ^{19} C_{11} \right)$ d. $10 \left(2^{18} + ^{19} C_{11} \right)$



287. If $p=\left(8+3\sqrt{7}\right)^n and f=p-[p], where [.]$ denotes the greatest integer function, then the value of p(1-f) is equal to a.1 b. 2 c. 2^n d. 2^{2n}



288. Statement 1: Greatest term in the expansion of $(1+x)^{12}, when x=11/10$ is 7th Statement 2: 7th term in the expansion of $(1+x)^{12}$ has the factor $\hat{\ }12C_6$ which is greatest value of $\hat{\ }12C_r$.



289. Statement 1: Remainder $when 3456^{2222}$ is divided by 7 is 4. Statement

2: Remainder when 5^{2222} is divided by 7 is 4 option1: BOTH the statement are TRUE and STATEMENT 2 is the correct explaination option2: BOTH the statement are TRUE and STATEMENT 2 is NOT the correct explaination option3: STATEMENT 1 is TRUE and STATEMENT 2 is FALSE option4:

STATEMENT 1 is FALSE and STATEMENT 2 is TRUE



290. The value of \boldsymbol{x} for which the sixth term in the expansion of

$$\left[2^{\log} - 2^{\sqrt{9^{(x-1)}+7}} + rac{1}{2^{rac{1}{5}}(\log)_2ig(3^{(x-1)}+1ig)}
ight]^{7}$$
 is 84 is



- **291.** The number $51^{49} + 51^{48} + 51^{47} + \dots + 51 + 1$ is divisible by a. 10 b.
- 20 c. 25 d. 50



292. If $\sum_{r=0}^n \frac{r}{{}^{\hat{}} n C_r} = \sum_{r=0}^n \frac{n^2-3n+3}{2.^n \, C_r}$, then an=1 b. n=2 c. n=3



d. none of these

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293. If $\left(1+x\right)^n=C_0+C_1x+C_2x^2+......+C_nx^n$, then show that the sum of the products of the coefficients taken two at a time, represented by $\sum \sum_{0 \leq i < j \leq n} {}^n c_i {}^n c_j$ is equal to $2^{2n-1} - \frac{(2n)!}{2(n!)^2}$



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294. For any positive integer (m,n) (with $n \geq m$), Let $\binom{n}{m} = n^n C_m$ that Prove

$$\binom{n}{m}+2\binom{n-1}{m}+3\binom{n-2}{m}+....+(n-m+1)\binom{m}{m}$$



$$b_n - (-1)^n a_n = 0.$$

295. If $\sum_{r=0}^{n}\left\{a_{r}(x-lpha+2)^{r}-b_{r}(lpha-x-1)^{r}
ight\}=0$, then prove that



296. Let
$$a=\left(2^{1/401}-1
ight)$$
 and for each

 $n\geq 2, letb_n=^n C_1+^n C_2\dot{a}+^n C_3a^2+......+^n C_n\cdot a^{n-1}$. Find the

Prove

 $\sum_{i=0}^{n} \left[\left[i^{r} + i^{2r} + i^{3r} + i^{4r}
ight] = 2^{n} + 2^{rac{n}{2}+1} \cos(n\pi/4), wherei = 0$

that

297.

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value of $(b_{2006} - b_{2005})$.

298. Find the coefficient of
$$x^n$$
 in $\left(1+rac{x}{1!}+rac{x^2}{2!}+\ +rac{x^n}{n!}
ight)^2$.

$$1-2n+rac{2n(2n-1)}{2!}-rac{2n(2n-1)(2n-2)}{3!}+ + (-1)^{n-1}rac{2n(2n-1)(2n-2)}{(n-1)!}$$

prove that $^{n+1}C_1 + ^{n+1}C_2s_1 + ^{n+1}C_3s_2 + \dots + ^{n+1}C_{n+1}s_n = 2^nS_n$.

301. Show that $x^n=1+n\Big(1-rac{1}{x}\Big)+rac{n(n+1)}{1.2}\Big(1-rac{1}{x}\Big)^2+...$

Given,

299. If n is a positive integer, prove that



$$s_n = 1 + q + q^2 + + q^n, S_n = 1 + rac{q+1}{2} + \left(rac{q+1}{2}
ight)^2 + ... + \left(rac{q+1}{2}
ight)^2$$

300.



302.
$$\sum_{k=1}^{\infty} k \left(1 - rac{1}{n}
ight)^{k-1} = ext{ a.} n(n-1) ext{ b. } n(n+1) ext{ c. } n^2 ext{ d. } (n+1)^2$$



303. The coefficient of x^4 in the expansion of $\left\{\sqrt{1+x^2}-x\right\}^{-1}$ in ascending powers of x, when |x|<1, is 0 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. $-\frac{1}{8}$



304.
$$1+rac{1}{3}x+rac{1 imes4}{3 imes6}x^2+rac{1 imes4 imes7}{3 imes6 imes9}x^3+$$
 is equal to

A. a. x

B. b. $(1+x)^{1/3}$

C. c. $(1-x)^{1/3}$

D. d. $(1-x)^{-1/3}$

Answer: null

305. The value of
$$\sum_{r=1}^{15} \frac{r2^r}{(r+2)!}$$
 is $\frac{(17)!-2^{16}}{(17)!}$ b. $\frac{(18)!-2^{17}}{(18)!}$ c. $\frac{(16)!-2^{15}}{(16)!}$ d. $\frac{(15)!-2^{14}}{(15)!}$



306.
$$(n+2)nC_0ig(2^{n+1}ig)-(n+1)nC_1(2^n)+(n)nC_2ig(2^{n-1}ig)-....$$
 is equal to



307. The value of
$$\sum_{r=0}^{50}{(-1)^r}\frac{(50)C_r}{r+2}$$
 is equal to $a.$ $\frac{1}{50\times51}$ b. $\frac{1}{52\times50}$ c. $\frac{1}{52\times51}$ d. none of these



4n b. 4n-3 c. 4n+1 d. none of these



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309. Statement 1: The sum of coefficient in the expansion of $\left(3^{-x/4}+3^{5x/4}
ight)^n is 2^n$. Statement 2: The sum of coefficient in the expansion of $(x+y)^n is 2^n$

308. In the expansion of $\left[\left(1+x\right)/\left(1-x\right)\right]^2$, the coefficient of x^n will be



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310. Let n be a positive integer and k be a whole number, k < 2n.

Statement 1: The maximum value of $2nC_kis^{2n}C_n$. Statement 2:

$$rac{\hat{C}_k(2n)C_{k+1}}{\hat{C}_k(2n)C_k}igg\langle 1,f ext{ or } k=0,1,2,,n-1 and rac{\hat{C}_k(2n)C_k}{\hat{C}_k(2n)C_{k-1}} 1,f ext{ or } k=n+1$$

311. Prove that $\sum_{r=0}^{2n} rig(.^{2n}\,C_rig)^2 = n^{4n}C_{2n}$.



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312. Statement 1: '^m C_r+^m C_(r-1)^n C_1+^mC_(r-2)^n C_2++^n C r=0,ifm+n



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313.
$$1 + \left(\frac{1}{4}\right) + \left(\frac{1 \cdot 3}{4 \cdot 8}\right) + \left(\frac{1 \cdot 4 \cdot 7}{4 \cdot 8 \cdot 12}\right) + \dots =$$



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314. If
$$|x|<1, then 1+n\Big(rac{2x}{1+x}\Big)+rac{n(n+1)}{2!}\Big(rac{2x}{1+x}\Big)^2+.....$$
 is equal to

A. a.
$$\left(\frac{2x}{1+x}\right)^n$$

B. b.
$$\left(\frac{1+x}{2x}\right)^n$$
C. c. $\left(\frac{1-x}{1+x}\right)^n$
D. d. $\left(\frac{1+x}{1-x}\right)^n$

Answer: null



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 $\left[\left(2+\sqrt{5}
ight)^p
ight]-2^{p+1}$ is always divisible by p(where[.]] denotes the greatest integer function). Statement 2: if n prime, then

Statement 1: If p is a prime number $(p \neq 2)$,

 \hat{D} nC_1 , nC_2 , nC_2 , nC_{n-1} must be divisible by nC_2



316. Statement 1: The total number of dissimilar terms in the expansion of m(n+1)(n+2)

 $(x_1+x_2+{}+x_n)^3 is rac{n(n+1)(n+2)}{6}$. Statement 2: The total number

 $(x_1+x_2+x_3)^n is rac{n(n+1)(n+2)}{\epsilon}.$

317. Statement 1: In the expansion of $(1+x)^{41} (1-x+x^2)^{40}$, the coefficient of x^{85} is zero. Statement 2: In the expansion of $(1+x)^{41} and (1-x+x^2)^{40}$, x^{85} term does not occur.

dissimilar terms in the expansion



Statement

of

318.

$$\left(1+x+rac{x^2}{2!}+rac{x^3}{3!}++rac{x^n}{n!}
ight)^3$$
 is $rac{3^n}{n!}$ Statement 2: The coefficient of $x^n\in e^{3x}israc{3^n}{n!}$

1: The coefficient of x^n

is



319. The value of $\sum_{r=0}^{10} r^{10} C_r, 3^r. \ (-2)^{10-r}$ is -

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320. The value of
$$\sum_{r=1}^n \left(-1\right)^{r+1} \frac{{}^n C r}{r+1}$$
 is equal to a. $-\frac{1}{n+1}$ b. $\frac{1}{n}$ c.

 $rac{1}{n+1}$ d. $rac{n}{n+1}$

321. If
$$\hat{C}_0, C^{C}_1, C^{C}_2, C^{C}_n$$
 are the binomial coefficient, then $2 \times C_1 + 2^3 \times C^3 + 2^5 \times C_5 + \text{ equals } \frac{3^n + (-1)^n}{2}$ b. $\frac{3^n - (-1)^n}{2}$

c.
$$\frac{3^n+1}{2}$$
 d. $\frac{3^n-1}{2}$

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322. The value of
$$\frac{{}^{n}C_{0}}{n} + \frac{{}^{n}C_{1}}{n+1} + \frac{{}^{n}C_{2}}{n+2} + \dots + \frac{{}^{n}C_{n}}{2n}$$
 is equal to

A. a.
$$\int_0^1 x^{n-1} (1-x)^n dx$$

B. b.
$$\int_{1}^{2} x^{n} (x-1)^{n-1} dx$$

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Answer: null

C. c. $\int_{\cdot}^{2}x^{n-1}(1+x)^{n}dx$

D. d. $\int_{0}^{1} (1-x)^{n-1} dx$

323. The value of
$$^{20}C_0+^{20}C_1+^{20}C_2+^{20}C_3+^{20}C_4+^{20}C_{12}+^{20}C_{13}+^{20}C_{14}+^{20}C_{15}$$
 is

a.
$$2^{19}-rac{\left(\hat{\ }(20)\mathrm{C}_{10}+\ ^{20}C_{9}
ight)}{2}$$
 b. $2^{19}-rac{\left(\hat{\ }(20)C10+2 imes^{20}C9
ight)}{2}$ c. $2^{19}-rac{\hat{\ }(20)C10}{2}$ d. none of these

324. If $\left(3+x^{2008}+x^{2009}\right)^{2010}=a_0+a_1x+a_2x^2+ +a_nx^n$, then the

value of $a_0-rac{1}{2}a_1-rac{1}{2}a_2+a_3-rac{1}{2}a_4-rac{1}{2}a_5+a_6$ is 3^{2010} b. 1 c.

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 2^{2010} d. none of these

325. The sum of series $\hat{\ }(20)C_0 = {}^{20}C1 + {}^{20}C2 = {}^{20}C3 + {} + {}^{20}C10$ is

 $(1+x)^n = C_0 + C_1 x + C_2 x^2 + + C_n x^n, then C_0 C_2 + C_1 C_3 + C_2 C_4 + +$

If

A. a.1/2*
$$^{\smallfrown}$$
 $(20)C_{10}$

B. b. 0

C. c.
$$\hat{}$$
 (20) C 10

D. d. $- \wedge (20)C10$

Answer: null

326.



$$\frac{(2n)!}{(n!)^2}$$
 b. $\frac{(2n)!}{(n-1)!(n+1)!}$ c. $\frac{(2n)!}{(n-2)!(n+2)!}$ d. none of these



327. The value of $\lim_{n o\infty}\ \sum_{r=0}^n\left(\sum_{t=0}^{r-1}rac{1}{5^n}\cdot {}^nC_r\cdot {}^rC_t.\left(3^t
ight)
ight)$ is equal to



 $C_0 - 2^2 C_1 + 3^2 C_2 - 4^2 C_3 + ... + {(-1)}^n {(n+1)}^2 imes C_n = 0 where C_r =^n C_1$

Given

 $C_{1}+2C_{2}x+3C_{3}x^{2}+\ +2nC_{2n}x^{2n-1}=2n{(1+x)}^{2n-1},where C_{r}=(2n)$

that

then prove that
$$C12-2C22+3C32--2nC2n2=(-1)^{ ext{\cap}}C_n\cdot$$

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329.

331. The largest real value of
$$x$$
 such that $\sum_{k=0}^4 \left(\frac{3^{4-k}}{(4-k)!}\right) \left(\frac{x^k}{k!}\right) = \frac{32}{3}$ is.



332. In the binomial expansion of $(a-b)^n$, $n\geq 5$, the sum of 5^{th} and 6^{th} term $\frac{a}{b}$ equals

