MATHS

BOOKS - CENGAGE

DIFFERENT PRODUCTS OF VECTORS AND THEIR GEOMETRICAL APPLICATIONS

Exercise

1. If $\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-a)^2 \end{vmatrix} = 0$ and vectors $\vec{A}, \vec{B}$ and $\vec{C}$, where $\vec{A} = a^2\hat{i} = a\hat{j} + \hat{k}$ etc. are non-coplanar, then prove that vectors $\vec{X}, \vec{Y}$ and $\vec{Z}$ where $\vec{X} = x^2\hat{i} + x\hat{j} + \hat{k}$. etc.may be coplanar.

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2. OABC is a tetrahedron where O is the origin and A,B,C have position vectors $\vec{a}, \vec{b}, \vec{c}$ respectively prove that circumcentre of tetrahedron OABC is (a^2(vecbxxvecc)+b^2(veccxxveca)+c^2(vecaxxvecb))/(2[veca vecb vecc])`

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3. Let k be the length of any edge of a regular tetrahedron (a tetrahedron whose edges are equal in length is called a regular tetrahedron). Show that the angel between any edge and a face not containing the edge is $\cos^{-1}\left(1/\sqrt{3}\right)$.

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4. In $ABC,$ a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP,$ ising vedctor method, find the are of ABC if the area of BRC is 1 unit

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5. Let O be an interior point of ΔABC such that $\overline{OA}+2\overline{OB}+3\overline{OC}=0$. Then the ratio of a ΔABC to area of ΔAOC is

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6. The lengths of two opposite edges of a tetrahedron of $aandb;$ the shortest distane between these edgesis $d,$ and the angel between them if $\dot{\theta}$ Prove using vector4s that the volume of the tetrahedron is $\frac{abdisn\theta}{6}$.

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7. Find the volume of a parallelopiped having three coterminus vectors of equal magnitude $|a|$ and equal inclination θ with each other.

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8. Let $\vec{p}$ and $\vec{q}$ any two othogonal vectors of equal magnitude 4 each. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be any three vectors of lengths $7\sqrt{15}$ and $2\sqrt{33}$, mutually perpendicular to each other. Then find the distance of the vector

$$(\vec{a}.\vec{p})\vec{p} + (\vec{a}.\vec{q})\vec{q} + (\vec{a}.(\vec{p}\times\vec{q}))(\vec{p}\times\vec{q}) + (\vec{b}.\vec{p})\vec{p} + (\vec{b}.\vec{p})\vec{q} + (\vec{b}.(\vec{b}.\vec{q}))($$

$$\left(\vec{a}.\vec{p}\right)\vec{p} + \left(\vec{a}.\vec{q}\right)\vec{q} + \left(\vec{a}.\left(\vec{p}\times\vec{q}\right)\right)\left(\vec{p}\times\vec{q}\right) + \left(\vec{b}.\vec{p}\right)\vec{p} +$$

$$+\left(\vec{c}.\left(\vec{p}\times\vec{q}\right)\right)\left(\vec{p}\times\vec{q}\right)$$

from the origin.

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9. Given that $\vec{A}, \vec{B}, \vec{C}$ form triangle such that $\vec{A} = \vec{B} + \vec{C}$. Find a,b,c,d such that area of the triangle is $5\sqrt{6}$ where $\vec{A} = a\vec{i} + b\vec{i} + c\vec{k}$. $\vec{B} = d\vec{i} + 3\vec{j} + 3\vec{k}$ and $\vec{C} = 3\vec{i} + \vec{j} - 2\vec{k}$.

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10. A line l is passing through the point $\vec{b}$ and is parallel to vector $\vec{c}$. Determine the distance of point A$(\vec{a})$ from the line l in from

$$\left|\vec{b} - \vec{a} + \frac{(\vec{a} - \vec{b})\vec{c}}{|\vec{c}|^2}\vec{c}\right| \text{ or } \frac{|(\vec{b} - \vec{a}) \times \vec{c}|}{|\vec{c}|}$$

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11. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i \cdot \vec{E}_j = 1,\ \text{if}\ i = j and \vec{e}_i \cdot \vec{E}_j = 0 and\ \text{if}\ i \neq j,$ then prove that $[\vec{e}_1 \vec{e}_2 \vec{e}_3][\vec{E}_1 \vec{E}_2 \vec{E}_3] = 1.$

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12. In a quadrilateral ABCD, it is given that $AB \mid\mid CD$ and the diagonals AC and BD are perpendicular to each other. Show that $AD.\ BC \geq AB.\ CD.$

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13. $OABC$ is regular tetrahedron in which D is the circumcentre of OAB and E is the midpoint of edge $AC^{\cdot}$ Prove that DE is equal to half the edge of tetrahedron.

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14. If $A(\vec{a}), B(\vec{b}) and C(\vec{c})$ are three non-collinear points and origin does not lie in the plane of the points $A, B and C,$ then point $P(\vec{p})$ in the plane of the ABC such that vector $\vec{O}P$ is $\perp$ to planeof ABC , show that

$\vec{O}P = \dfrac{[\vec{a}\vec{b}\vec{c}](\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{4^2}$, *where* is the area of the $ABC^{\cdot}$

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15. If $\vec{a}, \vec{b}, \vec{c}$ are three given non-coplanar vectors and any arbitrary vector $\vec{r}$ in space, where

$$\Delta_1 = \begin{vmatrix} \vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \Delta_2 = \left|\left(\vec{a}.\vec{a}, \vec{r}.\vec{a}, \vec{c}.\vec{a}\right), \left(\vec{a}.\vec{b}, \vec{r}.\vec{b}, \vec{c}.\vec{b}\right), \left(\vec{a}.\vec{c}, \vec{r}.\vec{c}\right.\right.$$

$$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{r}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix}' \Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}, \text{then prove that } \vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}$$

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16. Two vectors in space are equal only if they have equal component in a. a given direction b. two given directions c. three given directions d. in any arbitrary direction

A. a given direction

B. two given directions

C. three given direction

D. in any arbitrary direaction

Answer: c

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17. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times (\vec{a} \times \vec{b}) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: d

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18. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of equal magnitude such that the angle between each pair is $\frac{\pi}{3}$. If $\left|\vec{a} + \vec{b} + \vec{}\right| = \sqrt{6}$, then $\left|\vec{a}\right| =$

A. 2

B. -1

C. 1

D. $\sqrt{6}/3$

Answer: c

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19. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors, then the vector which is equally inclined to these vectors is (A) $\vec{a} + \vec{b} + \vec{c}$ (B) $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \vec{/}\,|\vec{c}|$ (C) $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$ (D) $|\vec{a}|\vec{a} - |\vec{b}|\vec{b} + |\vec{c}|\vec{c}$

A. $\vec{a} + \vec{b} + \vec{c}$

B. $\frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} + \frac{\vec{c}}{|\vec{c}|}$

C. $\frac{\vec{a}}{|\vec{a}|^2} + \frac{\vec{b}}{|\vec{b}|^2} + \frac{\vec{c}}{|\vec{c}|^2}$

D. $\left|\vec{a}\right|\vec{a} - \left|\vec{b}\right|\vec{b} + \left|\vec{c}\right|\vec{c}$

Answer: b

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20. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. Then the point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is (A) $(3, -1, 10$ (B) $(3, 1, -1)$ (C) $(-3, 1, 1)$ (D) $(-3, -1, -10$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $3\hat{i} - \hat{j} + \hat{k}$

C. $3\hat{i} + \hat{j} - \hat{k}$

D. $\hat{i} - \hat{j} - \hat{k}$

Answer: c

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21. If $\vec{a}$ and $\vec{b}$ are two vectors, such that $\vec{a}.\vec{b} < 0$ and $\left|\vec{a}.\vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$ then the angle between angles between the vectors $\vec{a}$ and $\vec{b}$ is

A. π

B. $7\pi/4$

C. $\pi/4$

D. $3\pi/4$

Answer: d

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22. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector and θ_1, θ_2 and θ_3 are angles between the vectors $\hat{a}, \hat{b}, \hat{b}, \hat{c}$ and $\hat{c}, \hat{a},$ respectively m then among θ_1, θ_2 and θ_3

A. all are acute angles

B. all are right angles

C. at least one is obtuse angle

D. none of these

Answer: c

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23. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = 0 = \vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \pi/3$ then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is

A. $1/2$

B. 1

C. 2

D. none of these

Answer: b

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24. P $(\vec{p})$ and $Q(\vec{q})$ are the position vectors of two fixed points and $R(\vec{r})$ is the postion vector of a variable point. If R moves such that $(\vec{r} - \vec{p}) \times (\vec{r} - \vec{q}) = \vec{0}$ then the locus of R is

A. a plane containing the origian O and parallel to two non-collinear vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

B. the surface of a sphere described on PQ as its diameter

C. a line passing through points P and Q

D. a set of lines parallel to line PQ

Answer: c

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25. Two adjacent sides of a parallelogram ABCD are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Then the value of $\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$ is

A. $20\sqrt{5}$

B. $22\sqrt{5}$

C. $24\sqrt{5}$

D. $26\sqrt{5}$

Answer: b

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26. If $\hat{a}, \hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ.

The maximum value of θ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{5}$

Answer: c

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27. Let the pair of vector $\vec{a}, \vec{b}$ and $\vec{c}, \vec{c}d$ each determine a plane. Then the planes are parallel if

A. $(\vec{a} \times \vec{c}) \times (\vec{b} \times \vec{d}) = \vec{0}$

B. $(\vec{a} \times \vec{c}).(\vec{b} \times \vec{d}) = \vec{0}$

C. $(\vec{a} \times \vec{c}) \times (\vec{c} \times \vec{d}) = \vec{0}$

D. $(\vec{a} \times \vec{c}).(\vec{c} \times \vec{d}) = \vec{0}$

Answer: c

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28. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar, then

A. $\vec{r} \perp (\vec{c} \times \vec{a})$

B. $\vec{r} \perp (\vec{a} \times \vec{b})$

C. $\vec{r} \perp \left(\vec{b} \times \vec{c}\right)$

D. $\vec{r} = \vec{0}$

Answer: d

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29. If $\vec{a}$ satisfies $\vec{a} \times \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - \hat{k}$ then $\vec{a}$ is equal to

A. $\lambda\hat{i} + (2\lambda - 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

B. $\lambda\hat{i} + (1 - 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

C. $\lambda\hat{i} + (2\lambda + 1)\hat{j} + \lambda\hat{k}, \lambda \in R$

D. $\lambda\hat{i} + (1 + 2\lambda)\hat{j} + \lambda\hat{k}, \lambda \in R$

Answer: c

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30. Vectors $3\vec{a} - 5\vec{b}$ and $2\vec{a} + \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b}$ and $\vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a} nad \vec{b}$ is

A. $\frac{19}{5\sqrt{43}}$

B. $\frac{19}{3\sqrt{43}}$

C. $\frac{19}{\sqrt{45}}$

D. $\frac{19}{6\sqrt{43}}$

Answer: a

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31. The units vectors orthogonal to the vector $-\hat{i} + 2\hat{j} + 2\hat{k}$ and making equal angles with the X and Y axes islare) :

A. $\pm\frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

B. $\frac{19}{5\sqrt{43}}$

C. $\pm \frac{1}{3}\left(\hat{i} + \hat{j} - \hat{k}\right)$

D. none of these

Answer: a

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32. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} = \hat{k} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} = x\hat{k}$, is obtuse and the angle between $\vec{b}$ and the z-axis is acute and less than $\pi/6$, are

A. $a < x < 1/2$

B. $1/2 < x < 15$

C. $x < 1/2 \text{ or } x < 0$

D. none of these

Answer: b

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33. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelograsm then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is (A) $\vec{b}+\frac{\vec{b}\times\vec{a}}{|\vec{a}|^2}$ (B) $\frac{\vec{a}.\vec{b}}{\vec{b}|^2}$ (C) $\vec{b}$ - $\frac{\vec{b}.\vec{a}}{(|\vec{a}|)^2}\Big)$ (D) $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{\vec{b}|^{20}}$

A. $\vec{b}+\frac{\vec{b}\times\vec{a}}{|\vec{a}|^2}$

B. $\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}$

C. $\vec{b}$ - $\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}$

D. $\frac{\vec{a}\times(\vec{b}\times\vec{a})}{|\vec{b}|^2}$

Answer: a

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34. A parallelogram is constructed on $3\vec{a} + \vec{b} and \vec{a} - 4\vec{b}, where \left|\vec{a}\right| = 6 and \left|\vec{b}\right| = 8, and \vec{a} and \vec{b}$ are anti-parallel. Then the length of the longer diagonal is a .40 b. 64 c. 32 d. 48

A. 40

B. 64

C. 32

D. 48

Answer: c

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35. Unit vectors $\vec{a}$ and $\vec{b}$ ar perpendicular , and unit vector $\vec{c}$ is inclined at an angle θ to both $\vec{a}$ and $\vec{b}. If \alpha\vec{a} + \beta\vec{b} + \gamma\left(\vec{a} \times \vec{b}\right)$ then.

A. $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

B. $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$

C. $0 \leq \theta \leq \frac{\pi}{4}$

D. $0 \leq \theta \leq \frac{3\pi}{4}$

Answer: a

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36. $\vec{a}$ and $\vec{c}$ are unit vectors and $\left|\vec{b}\right| = 4$ the angle between $\vec{a}$ and $\vec{c}$ $is \cos^{-1}(1/4)$ and $\vec{b} - 2\vec{c} = \lambda\vec{a}$ the value of λ is

A. 3,-4

B. 1/4,3/4

C. $-3, 4$

D. $-1/4, \frac{3}{4}$

Answer: a

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37. Let the position vectors of the points $PandQ$ be $4\hat{i} + \hat{j} + \lambda\hat{k}and2\hat{i} - \hat{j} + \lambda\hat{k}$, respectively. Vector $\hat{i} - \hat{j} + 6\hat{k}$ is perpendicular to the plane containing the origin and the points $PandQ$. Then λ equals $1/2$ b. $1/2$ c. 1 d. none of these

A. $-1/2$

B. $1/2$

C. 1

D. none of these

Answer: a

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38. A vector of magnitude $\sqrt{2}$ coplanar with the vectors $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, and perpendicular to the vector $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ is

A. $-\hat{j} + \hat{k}$

B. $\hat{i}$ and $\hat{k}$

C. $\hat{i} - \hat{k}$

D. hati- hatj`

Answer: a

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39. Let P be a point interior to the acute triangle ABC. If $PA + PB + PC$ is a null vector, then w.r.t traingel ABC, point P is its a. centroid b. orthocentre c. incentre d. circumcentre

A. centroid

B. orthocentre

C. incentre

D. circumcentre

Answer: a

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40. G is the centroid of triangle ABC and A_1 and B_1 are the midpoints of sides AB and AC, respectively. If Δ_1 is the area of quadrilateral GA_1AB_1 and Δ is the area of triangle ABC, then $\frac{\Delta}{\Delta_1}$ is equal to

A. $\frac{3}{2}$

B. 3

C. $\frac{1}{3}$

D. none of these

Answer: b

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41. Points $\vec{a}, \vec{b}, \vec{c}, and \vec{d}$ are coplanar and $(s \in \alpha)\vec{a} + (2\sin 2\beta)\vec{b} + (3\sin 3\gamma)\vec{c} - \vec{d} = 0$. Then the least value of $\sin^2\alpha + \sin^2 2\beta + \sin^2 3\gamma is \frac{1}{14}$ b. 14 c. 6 d. $1/\sqrt{6}$

A. $1/14$

B. 14

C. 6

D. $1/\sqrt{6}$

Answer: a

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42. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1and 2. respectively, and $\left(1 - 3\vec{a}.\vec{b}\right)^2 + \left|2\vec{a} + \vec{b} + 3\left(\vec{a} \times \vec{b}\right)\right|^2 = 47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\pi/3$

B. $\pi - \cos^{-1}(1/4)$

C. $\frac{2\pi}{3}$

D. $\cos^{-1}(1/4)$

Answer: c

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43. If $\vec{a} and \vec{b}$ are any two vectors of magnitudes 2 and 3, respectively, such that $\left|2\left(\vec{a}\times\vec{b}\right)\right| + \left|3\left(\vec{a}\dot{}\vec{b}\right)\right| = k,$ then the maximum value of k is a.$\sqrt{13}$ b. $2\sqrt{13}$ c. $6\sqrt{13}$ d. $10\sqrt{13}$

A. $\sqrt{13}$

B. $2\sqrt{13}$

C. $6\sqrt{13}$

D. $10\sqrt{13}$

Answer: c

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44. $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vecrtors such that $\left|\vec{a} + \vec{b} + 3\vec{c}\right| = 4$ Angle between $\vec{a}$ and $\vec{b} is \theta_1$, between $\vec{b}$ and $\vec{c} is \theta_2$ and between $\vec{a}$ and $\vec{b}$ varies $[\pi/6, 2\pi/3]$. Then the maximum value of $\cos\theta_1 + 3\cos\theta_2$ is

A. 3

B. 4

C. $2\sqrt{2}$

D. 6

Answer: b

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45. If the vector product of a constant vector $\vec{O}A$ with a variable vector $\vec{O}B$ in a fixed plane OAB be a constant vector, then the locus of B is a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

A. a straight line perpendicular to $\overset{\rightarrow}{OA}$

B. a circle with centre O and radius equal to $\left|\overset{\rightarrow}{OA}\right|$

C. a striaght line parallel to $\overset{\rightarrow}{OA}$

D. none of these

Answer: c

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46. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $\left|\vec{u}\right| = 1, \left|\vec{v}\right| = 2 and \left|\vec{w}\right| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $\left|\vec{u} - \vec{v} + \vec{w}\right|$ equals a. 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: c

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47. If the two adjacent sides of two rectangles are reprresented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}, \vec{q} = -\vec{a} - 2\vec{b}$ and $\vec{r} = -4\vec{a} - \vec{b}, \vec{s} = -\vec{a} + \vec{b}$, respectively, then the angle between the vectors $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s})$ and $\vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$ is

A. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. cannot of these

Answer: b

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48. If $\vec{\alpha}\ |\ |\ \left(\vec{b}\times\vec{\gamma}\right)$, $then\left(\vec{\alpha}\times\vec{\beta}\right).\left(\vec{\alpha}\times\vec{\gamma}\right)=$ (A) $\left|\vec{\alpha}\right|^{2}\left(\vec{\beta}.\vec{\gamma}\right)$ (B) $\left|\vec{\beta}\right|^{2}\left(\vec{\gamma}.\vec{\alpha}\right)$ (C) $\left|\vec{\gamma}\right|^{2}\left(\vec{\alpha}.\vec{\beta}\right)$ (D) $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

A. $\left|\vec{\alpha}\right|^{2}\left(\vec{\beta}.\vec{\gamma}\right)$

B. $\left|\vec{\beta}\right|^{2}\left(\vec{\gamma}.\vec{\alpha}\right)$

C. $\left|\vec{\gamma}\right|^{2}\left(\vec{\alpha}.\vec{\beta}\right)$

D. $\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\left|\vec{\gamma}\right|$

Answer: a

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49. The position vectors of points A,B and C are $\hat{i} + \hat{j}, \hat{i} + 5\hat{j} - \hat{k}$ and $2\hat{i} + 3\hat{j} + 5\hat{k}$, respectively the greatest angle of triangle ABC is

A. 120°

B. 90°

C. $\cos^{-1}(3/4)$

D. none of these

Answer: b

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50. Given three vectors e$\vec{a}, \vec{b}$ and $\vec{c}$ two of which are non-collinear. Futrther if $\left(\vec{a} + \vec{b}\right)$ is collinear with $\vec{c}, \left(\vec{b} + \vec{c}\right)$ is collinear with $\vec{a}, \left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = \sqrt{2}$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

A. 3

B. -3

C. 0

D. cannot of these

Answer: b

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51. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\left(\vec{a}+\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)\times\left(3\vec{a}-2\vec{b}\right)=\vec{0}$ then angle between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. π

D. indeterminate

Answer: d

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52. If in a right-angled triangle ABC, the hypotenuse $AB = p, then \vec{A}B\dot{A}C + \vec{B}C\vec{B}A + \vec{C}A\vec{C}B$ is equal to $2p^2$ b. $\frac{p^2}{2}$ c. p^2 d. none of these

A. $2p^2$

B. $\frac{p^2}{2}$

C. p^2

D. none of these

Answer: c

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53. Resolved part of vector $\vec{a}$ and along vector $\vec{b}$ is $\vec{a}1$ and that prependicular to $\vec{b}$ is $\vec{a}2$ then $\vec{a}1 \times \vec{a}2$ is equl to

A. $\dfrac{\left(\vec{a} \times \vec{b}\right).\vec{b}}{\left|\vec{b}\right|^2}$

B. $\dfrac{\left(\vec{a}.\vec{b}\right)\vec{a}}{\left|\vec{a}\right|^2}$

C. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b}\right|^2}$

D. $\dfrac{\left(\vec{a}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)}{\left|\vec{b} \times \vec{a}\right|}$

Answer: c

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54. Let $\vec{a} = 2\hat{i} = \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors . A vector in the pland of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\left(\frac{2}{3}\right)}$is (A) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (C) $-2\hat{i} - \hat{j} + 5\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $-2\hat{i} - \hat{j} + 5\hat{k}$

C. $2\hat{i} + 3\hat{j} + 3\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: b

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55. If P is any arbitrary point on the circumcirlce of the equllateral trangle of side length l units, then $\left|\vec{P}A\right|^2 + \left|\vec{P}B\right|^2 + \left|\vec{P}C\right|^2$ is always equal to $2l^2$ b. $2\sqrt{3}l^2$ c. l^2 d. $3l^2$

A. $2l^2$

B. $2\sqrt{3}l^2$

C. l^2

D. $3l^2$

Answer: a

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56. If $\vec{r} and \vec{s}$ are non-zero constant vectors and the scalar b is chosen such that $\left|\vec{r} + b\vec{s}\right|$ is minimum, then the value of $\left|b\vec{s}\right|^2 + \left|\vec{r} + b\vec{s}\right|^2$ is equal to $2\left|\vec{r}\right|^2$ b. $\left|\vec{r}\right|^2/2$ c. $3\left|\vec{r}\right|^2$ d. $|r|^2$

A. $2\left|\vec{r}\right|^2$

B. $\left|\vec{r}\right|^2/2$

C. $3\left|\vec{r}\right|^2$

D. $\left|\vec{r}\right|^2$

Answer: b

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57. $\vec{a}$ and $\vec{b}$ are two unit vectors that are mutually perpendicular. A unit vector that if equally inclined to $\vec{a}, \vec{b}$ and $\vec{a} \times \vec{b}$ is equal to

A. $\frac{1}{\sqrt{2}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

B. $\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{a}+\vec{b}\right)$

C. $\frac{1}{\sqrt{3}}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

D. $\frac{1}{3}\left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$

Answer: a

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58. Given that $\vec{a},\vec{b},\vec{p},\vec{q}$ are four vectors such that $\vec{a}+\vec{b}=\mu\vec{p}, \vec{b}.\vec{q}=0$ and $\left(\vec{b}\right)^2=1$ where μ is a sclar. Then $\left|\left(\vec{a}.\vec{q}\right)\vec{p}-\left(\vec{p}.\vec{q}\right)\vec{a}\right|$ is equal to

A. $2\left|\vec{p}\vec{q}\right|$

B. $(1/2)\left|\vec{p}.\vec{q}\right|$

C. $\left|\vec{p}\times\vec{q}\right|$

D. $\left|\vec{p}.\vec{q}\right|$

Answer: d

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59. The position vectors of the vertices A, B and C of a triangle are three unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. A vector $\vec{d}$ is such that $\vec{d}.\hat{a} = \vec{d}.\hat{b} = \vec{d}.\hat{c}$ and $\vec{d} = \lambda\left(\hat{b} + \hat{c}\right)$. Then triangle ABC is

A. acute angled

B. obtuse angled

C. right angled

D. none of these

Answer: a

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60. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4\tan A + a\tan B}\sqrt{a^2 + 4\tan c} = 6a,$ then the least vale of $\tan^2 A + \tan^2 b + \tan^2 C is$ 6 b. 10 c. 12 d. 3

A. 6

B. 10

C. 12

D. 3

Answer: d

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61. The vertex A triangle ABC is on the line $\vec{r} = \hat{i} + \hat{j} + \lambda\hat{k}$ and the vertices $BandC$ have respective position vectors $\hat{i} and \hat{j}$ Let Delta be the area of the triangle and $\text{Delta}\left[3/2, \sqrt{33}/2\right]$. Then the range of values of λ corresponding to A is $[-8, 4] \cup [4, 8]$ b. $[-4, 4]$ c. $[-2, 2]$ d. $[-4, -2] \cup [2, 4]$

A. [-8, -4]cup[4,8]`

B. $[-4, 4]$

C. [-2,2]

D. $[-4, -2] \cup [2, 4]$

Answer: c

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62. A non-zero vecto $\vec{a}$ is such tha its projections along vectors $\frac{\hat{i}+\hat{j}}{\sqrt{2}}, \frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ and $\hat{k}$ are equal , then unit vector along $\vec{a}$ us

A. $\frac{\sqrt{2}\hat{j}-\hat{k}}{\sqrt{3}}$

B. $\frac{\hat{j}-\sqrt{2}\hat{k}}{\sqrt{3}}$

C. $\frac{\sqrt{2}}{\sqrt{3}}\hat{j}+\frac{\hat{k}}{\sqrt{3}}$

D. $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

Answer: a

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63. Position vector $\hat{k}$ is rotated about the origin by angle 135^0 in such a way that the plane made by it bisects the angel between $\hat{i} and \hat{j}$. Then its new position is $\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$ b. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$ c. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$ d. none of these

A. $\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}$

B. $\pm\frac{\hat{i}}{2}\pm\frac{\hat{j}}{2}-\frac{\hat{k}}{\sqrt{2}}$

C. $\frac{\hat{i}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}$

D. none of these

Answer: d

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64. In a quadrilateral $ABCD$, $\vec{AC}$ is the bisector of the $\left(\vec{AB} \wedge \vec{AD}\right)$ which is $\frac{2\pi}{3}$, $15\left|\vec{AC}\right| = 2\left|\vec{AB}\right| = 5\left|\vec{AD}\right|$ then $\cos\left(\vec{BA} \wedge \vec{CD}\right)$ is

A. $\cos^{-1}\frac{\sqrt{14}}{7\sqrt{2}}$

B. $\cos^{-1}\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\cos^{-1}\frac{2}{\sqrt{7}}$

D. $\cos^{-1}\frac{2\sqrt{7}}{14}$

Answer: c

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65. In fig. 2.33 AB, DE and GF are parallel to each other and AD, BG and EF ar parallel to each other . If CD: CE = CG:CB = 2:1 then the value of area $(\triangle AEG) : area(\triangle ABD)$ is equal to

A. $7/2$

B. 3

C. 4

D. $9/2$

Answer: b

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66. Vectors $\hat{a}$ in the plane of $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ is such that it is equally inclined to $\vec{b}$ and $\vec{d}$ where $\vec{d} = \hat{j} + 2\hat{k}$ the value of $\hat{a}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

C. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

D. $\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}$

Answer: b

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67. Let $ABCD$ be a tetrahedron such that the edges $AB, ACandAD$ are mutually perpendicular. Let the area of triangles $ABC, ACDandADB$ be 3, 4 and 5sq. units, respectively. Then the area of triangle BCD is $5\sqrt{2}$ b. 5 c. $\frac{\sqrt{5}}{2}$ d. $\frac{5}{2}$

A. $5\sqrt{2}$

B. 5

C. $\frac{\sqrt{5}}{2}$

D. $\frac{5}{2}$

Answer: a

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68. Let $\overrightarrow{f}(t) = [t]\hat{i} + (t - [t])\hat{j} + [t + 1]\hat{k}$, *where*[.] denotes the greatest integer function. Then the vectors `vecf(5/4)a n df(t),0

A. parallel to each other

B. perpendicular to each other

C. inclined at $\dfrac{\cos^{-1}2}{\sqrt{7}\left(1 - t^2\right)}$

D. inclined at $\dfrac{\cos^{-1}(8 + t)}{9\sqrt{1 + t^2}}$

Answer: d

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69. If $\vec{a}$ is parallel to $\vec{b} \times \vec{c}$, *then* $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$ is equal to

A. $|\vec{a}|^2\left(\vec{b}.\vec{c}\right)$

B. $\left|\vec{b}\right|^2\left(\vec{a}.\vec{c}\right)$

C. $|\vec{c}|^2\left(\vec{a}.\vec{b}\right)$

D. none of these

Answer: a

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70. The three vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume:

A. $1/3$

B. 4

C. $\left(3\sqrt{3}\right)/4$

D. $4\sqrt{3}$

Answer: d

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71. If $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a on zero vector and $\left|\left(\vec{d}.\vec{c}\right)\left(\vec{a} \times \vec{b}\right) + \left(\vec{d}.\vec{a}\right)\left(\vec{b} \times \vec{c}\right) + \left(\vec{d}.\vec{b}\right)\left(\vec{c} \times \vec{a}\right)\right| = 0$ then (A) $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \left|\vec{d}\right|$ (B) $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$ (C) $\vec{a}, \vec{b}, \vec{c}$ are coplanar (D) $\vec{a} + \vec{c} = 2\vec{b}$

A. $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \left|\vec{d}\right|$

C. $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

D. none of these

Answer: c

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72. If $\left|\vec{a}\right| = 2$ and $\left|\vec{b}\right| = 3$ and $\vec{a}.\vec{b} = 0$, then $\left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)\right)\right)$ is equal

A. $48\hat{b}$

B. $-48\hat{b}$

C. $48\hat{a}$

D. $-48\hat{a}$

Answer: a

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73. If two diagonals of one of its faces are $6\hat{i} + 6\hat{k}$ and $4\hat{j} + 2\hat{k}$ and of the edges not containing the given diagonals is $\vec{c} = 4\hat{j} - 8\hat{k}$, then the volume of a parallelpiped is

A. 60

B. 80

C. 100

D. 120

Answer: d

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74. The volume of a tetrahedron fomed by the coterminus edges $\vec{a}, \vec{b}$ and $\vec{c} is 3$. Then the volume of the parallelepiped formed by the coterminus edges $\vec{a} + \vec{b}, \vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ is

A. 6

B. 18

C. 36

D. 9

Answer: c

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75. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually orthogonal unit vectors , then the triple product $\left[\vec{a} + \vec{b} + \vec{c} \vec{a} + \vec{b} \vec{b} + \vec{c}\right]$ equals

A. 0

B. 1 or -1

C. 1

D. 3

Answer: b

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76. Vector $\vec{c}$ is perpendicular to vectors $\vec{a} = (2, -3, 1) and \vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{c}.\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10.$ Then vector $\vec{c}$ is equal to a.$(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

A. 7,5,1

B. (-7, -5, -1)

C. 1,1,-1

D. none of these

Answer: a

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77. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j}, \vec{a} \perp \vec{b}, \vec{a}.\vec{c} = 4$ then

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2 = |\vec{a}|$

B. $\left[\vec{a}\vec{b}\vec{c}\right] = |\vec{a}|$

C. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

D. $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

Answer: d

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78. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non zero vectors such that $\vec{c}$ is a unit vector perpendicular to both

$\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

D. $\frac{3}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$

Answer: c

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79. Let $\vec{r}, \vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r}.\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right|, \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|$ then

$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] =$

A. |a||b||c|

B. - $|a||b||c|$

C. 0

D. none of these

Answer: c

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80. If $\vec{a}, \vec{b}$ and $\vec{c}$are such that $\left[\vec{a}\vec{b}\vec{c}\right] = 1, \vec{c} = \lambda\vec{a} \times \vec{b},$ angle between $\vec{a}$ and $\vec{b} is 2\pi/3, \left|\vec{a}\right| = \sqrt{2}\left|\vec{b}\right| = \sqrt{3}$ and $\left|\vec{c}\right| = \frac{1}{\sqrt{3}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: b

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81. If $4\vec{a}+5\vec{b}+9\vec{c}=0,$ then $\left(\vec{a}\times\vec{b}\right)\times\left[\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right]$ is equal to a. vector perpendicular to the plane ofa, b, c b. a scalar quantity c. $\vec{0}$ d. none of these

A. a vector perpendicular to the plane of $\vec{a}, \vec{b}$ and $\vec{c}$

B. a scalar quantity

C. $\vec{0}$

D. none of these

Answer: c

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82. Value of $\left[\vec{a}\times\vec{b}\vec{a}\times\vec{c}\vec{d}\right]$ is always equal to $\left(\vec{a}.\vec{d}\right)\left[\vec{a}\vec{b}\vec{c}\right]$ b. $\left(\vec{a}.\vec{c}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ c. $\left(\vec{a}.\vec{b}\right)\left[\vec{a}\vec{b}\vec{d}\right]$ d. none of these

A. $(\vec{a}.\vec{d})[\vec{a}\vec{b}\vec{c}]$

B. `(veca.vecc)[veca vecb vecd]

C. $(\vec{a}.\vec{b})[\vec{a}\vec{b}\vec{d}]$

D. none of these

Answer: a

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83. Let $\vec{a} and \vec{b}$ be mutually perpendicular unit vectors. Then for any arbitrary $\vec{r}$, a. $\vec{r} = (\vec{r}.\hat{a})\hat{a} + (\vec{r}.\hat{b})\hat{b} + (\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$ b. $\vec{r} = (\vec{r}.\hat{a}) - (\vec{r}.\hat{b})\hat{b} - (\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$ c. $\vec{r} = (\vec{r}.\hat{a})\hat{a} - (\vec{r}.\hat{b})\hat{b} + (\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$ d.none of these

A. $\vec{r} = (\vec{r}.\hat{a})\hat{a} + (\vec{r}.\hat{b})\hat{b} + (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

B. $\vec{r} = (\vec{r}.\hat{a}) - (\vec{r}.\hat{b})\hat{b} - (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

C. $\vec{r} = (\vec{r}.\hat{a})\hat{a} - (\vec{r}.\hat{b})\hat{b} - (\vec{r}.(\vec{a}\times\hat{b}))(\hat{a}\times\hat{b})$

D. none of these

Answer: a

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84. Let $\vec{a}$ and $\vec{b}$ be unit vectors that are perpendicular to each other l. then $\left[\vec{a} + \left(\vec{a} \times \vec{b}\right)\vec{b} + \left(\vec{a} \times \vec{b}\right)\vec{a} \times \vec{b}\right]$ will always be equal to

A. 1

B. 0

C. -1

D. none of these

Answer: a

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85. $\vec{a}$ and $\vec{b}$ are two vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a}. Vecb = 2. If\vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ then find angle between $\vec{b}$ and $\vec{c}$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{6}$

Answer: d

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86. Then for any arbitary vector $\vec{a}$, $\left(\left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{b}\times\vec{c}\right)\right)\left(\vec{b}-\vec{c}\right)$ is always equal to

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87. If $\vec{a}.\vec{b}=\beta$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}$ is

A. $\frac{\left(\beta\vec{a} - \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

B. $\frac{\left(\beta\vec{a} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

C. $\frac{\left(\beta\vec{c} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

D. $\frac{\left(\beta\vec{c} + \vec{a} \times \vec{c}\right)}{|\vec{a}|^2}$

Answer: a

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88. If $a\left(\vec{\alpha} \times \vec{\beta}\right) + b\left(\vec{\beta} \times \vec{\gamma}\right) + c\left(\vec{\gamma} \times \vec{\alpha}\right) = 0$ and at least one of $a, bandc$ is nonzero, then vectors $\vec{\alpha}, \vec{\beta} and \vec{\gamma}$ are a. parallel b. coplanar c. mutually perpendicular d. none of these

A. parallel

B. coplanar

C. mutually perpendicular

D. none of these

Answer: b

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89. If $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right) = \vec{b}, where \vec{a}, \vec{b}$ and $\vec{c}$ are non zero vectors then (A) $\vec{a}, \vec{b}$ and $\vec{c} can be coplanar (B)$veca,vecb and vecc$\mu st be coplanar (C)$ veca,vecb and vecc cannot be coplanar (D) none of these

A. $\vec{a}, \vec{b}$ and $\vec{v}$ can be coplanar

B. $\vec{a}, \vec{b}$ and $\vec{c}$ must be coplanar

C. $\vec{a}, \vec{b}$ and $\vec{c}$ cannot be coplanar

D. none of these

Answer: c

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90. If $\vec{r}.\vec{a} = \vec{r}.\vec{b} = \vec{r}.\vec{c} = \frac{1}{2}$ for some non zero vector $\vec{r}$ and $\vec{a}, \vec{b}, \vec{c}$ are non coplanar, then the area of the triangle whose vertices are $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ is

A. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\right|$

B. $|\vec{r}|$

C. $\left|\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}\right|$

D. none of these

Answer: c

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91. A vector of magnitude 10 along the normal to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at its point $P(1, 0)$ can be $6\hat{i} + 8\hat{j}$ b. $-8\hat{i} + 3\hat{j}$ c. $6\hat{i} - 8\hat{j}$ d. $8\hat{i} + 6\hat{j}$

A. $6\hat{i} + 8\hat{j}$

B. $-8\hat{i} + 3\hat{j}$

C. $6\hat{i} - 8\hat{j}$

D. $8\hat{i} + 6\hat{j}$

Answer: a

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92. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3 then \left\{\vec{a} \times \left(\vec{b} + \vec{a} \times \vec{b}\right)\right\}.\vec{b}$ is equal to

A. $\frac{-3}{4}$

B. $\frac{1}{4}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: a

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93. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r} \times \vec{a}$ is equal to

A. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{b} - \left(\vec{r}.\vec{b}\right)\left(\vec{b} \times \vec{a}\right)$

B. $\left[\vec{r}\vec{a}\vec{b}\right]\left(\vec{a} + \vec{b}\right)$

C. $\left[\vec{r}\vec{a}\vec{b}\right]\vec{a} + \left(\vec{r}.\vec{a}\right)\vec{a} \times \vec{b}$

D. none of these

Answer: a

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94. If $\vec{a} + \vec{b}, \vec{c}$ are any three non- coplanar vectors then the equation

$\left[\vec{b} \times \vec{c}\vec{c} \times \vec{a}\vec{a} \times \vec{b}\right]x^2 + \left[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}\right]x + 1 + \left[\vec{b} - \vec{c}\vec{c} - \vec{c} - \vec{a}\vec{a} - \vec{b}\right] = 0$

has roots

A. real and distinct

B. real

C. equal

D. imaginary

Answer: c

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95. Sholve the simultasneous vector equations for `vecx aedn vecy: vecx+veccxxvecy=veca and vecy+veccxxvecx=vecb, vec!=0

A. $\vec{x} = \dfrac{\vec{b} \times \vec{c} + \vec{a} + \left(\vec{c}.\vec{a}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

B. $\vec{x} = \dfrac{\vec{c} \times \vec{b} + \vec{b} + \left(\vec{c}.\vec{a}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

C. $\vec{y} = \dfrac{\vec{a} \times \vec{c} + \vec{b} + \left(\vec{c}.\vec{b}\right)\vec{c}}{1 + \vec{c}.\vec{c}}$

D. none of these

Answer: b

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96. The condition for equations $\vec{r} \times \vec{a} = \vec{b} and \vec{r} \times \vec{c} = \vec{d}$ to be consistent is

.

a.$\vec{b}. \vec{c} = \vec{a}. \vec{d}$ b. $\vec{a}. \vec{b} = \vec{c}. \vec{d}$ c. $\vec{b}. \vec{c} + \vec{a}. \vec{d} = 0$ d. $\vec{a}\vec{b} + \vec{\cdot}\vec{d} = 0$

A. $\vec{b}. \vec{c} = \vec{a}. \vec{d}$

B. $\vec{a}. \vec{b} = \vec{c}. \vec{d}$

C. $\vec{b}. \vec{c} + \vec{a}. \vec{d} = 0$

D. $\vec{a}. \vec{b} + \vec{c}. \vec{d} = 0$

Answer: c

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97. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $\left[\vec{a}\vec{b}\vec{i}\right]\hat{i} + \left[\vec{a}\vec{b}\vec{j}\right]\hat{j} + \left[\vec{a}\vec{b}\hat{k}\right]k$

is equal to

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98. If

$\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + \hat{j} + 2\hat{k}$ and $(1 + \alpha)\hat{i} + \beta(1 + \alpha)\hat{j} + \gamma(1 + \alpha)($

A. $-2, \ -4, \ -\frac{2}{3}$

B. $2, \ -4, \ \frac{2}{3}$

C. $-2, 4, \frac{2}{3}$

D. $2, 4, \ -\frac{2}{3}$

Answer: a

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99. Let $\left(\vec{a}(x) = (\sin x)\hat{i} + (\cos x)\hat{j}\right.$ and $\vec{b}(x) = (\cos 2x)\hat{i} + (\sin 2x)\hat{j}$ be two variable vectors $(x \in R)$. Then $\vec{a}(x)$ and $\vec{b}(x)$ are

A. collinear for unique value of x

B. perpendicular for infinte values of x.

C. zero vectors for unique value of x

D. none of these

Answer: b

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100. For any vectors $\vec{a}$ and $\vec{b}$, $\left(\vec{a}\times\hat{i}\right)+\left(\vec{b}\times\hat{i}\right)+\left(\vec{a}\times\hat{j}\right).\left(\vec{b}\times\hat{j}\right)+\left(\vec{a}\times\hat{k}\right).\left(\vec{b}\times\hat{k}\right)$ is always equal to

A. $\vec{a}.\vec{b}$

B. $2\vec{a}. Vecb$

C. zero

D. none of these

Answer: b

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101. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors and $\vec{r}$ is any vector in space, then $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{r}\times\vec{c}\right)+\left(\vec{b}\times\vec{c}\right)\times\left(\vec{r}\times\vec{a}\right)+\left(\vec{c}\times\vec{a}\right)\times\left(\vec{r}\times\vec{b}\right)=$

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

B. $2\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

C. $3\left[\vec{a}\vec{b}\vec{c}\right]\vec{r}$

D. none of these

Answer: b

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102. If $\vec{P}=\dfrac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}. \vec{q}=\dfrac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$ and $\vec{r}=\dfrac{\vec{a}\times\vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are three non- coplanar vectors then the value of the expression

$\left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{q}+\vec{q}+\vec{r}\right)$ is

A. 3

B. 2

C. 1

D. 0

Answer: a

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103. $A\left(\vec{a}\right), B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ are the vertices of triangle ABC and $R\left(\vec{r}\right)$ is any point in the plane of triangle ABC, then $\vec{r}.\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$ is always equal to

A. zero

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. - $\left[\vec{a}\vec{b}\vec{c}\right]$

D. none of these

Answer: b

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104. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar vectors and $\vec{a} \times \vec{c}$ is perpendicular to $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$, then the value of $\left[\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right] \times \vec{c}$ is equal to

A. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{c}$

B. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{b}$

C. $\vec{0}$

D. $\left[\vec{a}\vec{b}\vec{c}\right]\vec{a}$

Answer: c

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105. If V be the volume of a tetrahedron and V' be the volume of another tetrahedran formed by the centroids of faces of the previous tetrahedron and $V = KV^{'}$, $thenK$ is equal to 9 b. 12 c. 27 d. 81

A. 9

B. 12

C. 27

D. 81

Answer: c

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106. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\left(\vec{b} \times \vec{c}\right) \times \left(\vec{c} \times \vec{a}\right)\left(\vec{c} \times \vec{a}\right) \times \left(\vec{a} \times \vec{b}\right)\right]$ is equal to (where $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero non- colanar vectors).

A. $\left[\vec{a}\vec{b}\vec{c}\right]^2$

B. $\left[\vec{a}\vec{b}\vec{c}\right]^3$

C. $\left[\vec{a}\vec{b}\vec{c}\right]^4$

D. $\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: c

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107. If

$\vec{r} = x_1\left(\vec{a} \times \vec{b}\right) + x_2\left(\vec{b} \times \vec{a}\right) + x_3\left(\vec{c} \times \vec{d}\right)$ and $4\left[\vec{a}\vec{b}\vec{c}\right] = 1$ then $x_1 + x_2 + x_3$ is equal to

A. $\frac{1}{2}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\frac{1}{4}\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

C. $2\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

D. $4\vec{r}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

Answer: d

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108. If the vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other then a vector $\vec{v}$ in terms of $\vec{a}$ and $\vec{b}$ satisfying the equations $\vec{v}.\vec{a} = 0, \vec{v}.\vec{b} = 1$ and $\left[\vec{v} \quad \vec{a} \quad \vec{b}\right] = 1$ is

A. $\dfrac{\vec{b}}{\left|\vec{b}\right|^2} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$

B. $\dfrac{\vec{b}}{\left|\vec{b}\right|} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|^2}$

C. $\dfrac{\vec{b}}{\left|\vec{b}\right|} + \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$

D. none of these

Answer: a

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109. If $\vec{a}, ' = \hat{i} + \hat{j}, \vec{b}' = \hat{i} - \hat{j} + 2\hat{k}nad\vec{c}' = 2\hat{i} = \hat{j} - \hat{k}$ then the altitude of the parallelepiped formed by the vectors, $\vec{a}, \vec{b}$ and $\vec{c}$ having baswe formed by

$\vec{b}$ and $\vec{c}$ is (where $\vec{a}'$ is recipocal vector $\vec{a}$, , etc.

A. 1

B. $3\sqrt{2}/2$

C. $1/\sqrt{6}$

D. $1/\sqrt{2}$

Answer: d

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110. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$ then in the reciprocal system of vectors $\vec{a}, \vec{b}, \vec{c}$ reciprocal $\vec{a}$ of vector $\vec{a}$ is

A. $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2}$

B. $\dfrac{\hat{i} - \hat{j} + \hat{k}}{2}$

C. $\dfrac{-\hat{i} - \hat{j} + \hat{k}}{2}$

D. $\dfrac{\hat{i} + \hat{j} - \hat{k}}{2}$

Answer: d

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111. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined of an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$ then θ in the interval

A. $[0, \pi/6)$

B. $(5\pi/6, \pi]$

C. $[\pi/6, \pi/2]$

D. $(\pi/2, 5\pi/6]$

Answer: a,b

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112. $\vec{b}$ and $\vec{c}$ are non- collinear if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a}.\vec{b}\right)\vec{b} = (4 - 2x - \sin y)\vec{b} + \left(x^2 - 1\right)\vec{c}$ and $d\left(\vec{.}\,\vec{c}\right)\vec{a} = \vec{a}$ then

A. x =1

B. x = -1

C. $y = (4n+1)\frac{\pi}{2}, n \in I$

D. $y(2n+1)\frac{\pi}{2}, n \in I$

Answer: a,c

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113. Let $\vec{a}.\vec{b} = 0$ where $\vec{a}$ and $\vec{b}$ are unit vectors and the vector $\vec{c}$ is inclined an anlge θ to both $\vec{a}$ and $\vec{b}$. $If \vec{c} = m\vec{a} + n\vec{b} + p\left(\vec{a} \times \vec{b}\right), (m, n, p \in R)$ then

A. $\alpha = \beta$

B. $\gamma^2 = 1 - 2\alpha^2$

C. $\gamma^2 = -\cos 2\theta$

D. $\beta^2 = \frac{1+\cos 2\theta}{2}$

Answer: a,b,c,d

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114. $\vec{a}$ and $\vec{b}$ are two given vectors. On these vectors as adjacent sides a parallelogram is constructed. The vector which is the altitude of the parallelogam and which is perpendicular to $\vec{a}$ is not equal to

A. $\frac{(\vec{a}.\vec{b})}{|\vec{a}|^2}\vec{a} - \vec{b}$

B. $\frac{1}{|\vec{a}|^2}\left\{|\vec{a}|^2\vec{b} - (\vec{a}.\vec{b})\vec{a}\right\}$

C. $\frac{\vec{a} \times (\vec{a} \times \vec{b})}{|\vec{a}|^2}$

D. $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{|\vec{b}|^2}$

Answer: a,b,c

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115. If $\vec{a} \times (\vec{b} \times \vec{c})$ is perpendicular to $(\vec{a} \times \vec{b}) \times \vec{c}$, we may have a. $(\vec{a}.\vec{c})|\vec{b}|^2 = (\vec{a}.\vec{b})(\vec{b}.\vec{c})$ b. $\vec{a}.\vec{b} = 0$ c. $\vec{a}.\vec{c} = 0$ d. $\vec{b}.\vec{c} = 0$

A. $(\vec{a}.\vec{b})|\vec{b}|^2 = (\vec{a}.\vec{b})(\vec{b}.\vec{c})$

B. $\vec{a}.\vec{b} = 0$

C. $\vec{a}.\vec{c} = 0$

D. $\vec{b}.\vec{c} = 0$

Answer: a,c

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116. If $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a} \ \vec{b} \ \vec{b}]}$ where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then the value of the expression $(\vec{a} + \vec{b} + \vec{c}).(\vec{p} + \vec{q} + \vec{r})$ is

A. $x[\vec{a}\vec{b}\vec{c}] + \frac{[\vec{p}\vec{q}\vec{r}]}{x}$ has least value 2

B. $x^2\left[\vec{a}\vec{b}\vec{c}\right]^2 + \frac{\left[\vec{p}\vec{q}\vec{r}\right]}{x^2}$ has least value $\left(3/2^{2/3}\right)$

C. $\left[\vec{p}\vec{q}\vec{r}\right] > 0$

D. none of these

Answer: a,c

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117. $a_1, a_2, a_3 \in R - \{0\}$ and $+a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ " for all " x in R\`

then

A. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ are parallel to each each other

C. if vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then on of the ordered trippplet $\left(a_1, a_2, a_3\right) = (1, -1, -2)$

D. if $2a_1 + 3a_2 + 6a_3 + 6a_3 = 26$, then $\left|\vec{a}\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

Answer: a,b,c,d

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118. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ , then

A. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

B. $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2$, if $\theta = \pi/4$

C. $\vec{a} \times \vec{b} = \left(\vec{a}. Vecb\right)\hat{n}$ (where $\hat{n}$ is a normal unit vector) if $\theta f = \pi/4$

D. $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} + \vec{b}\right) = 0$

Answer: a,b,c,d

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119. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

A. $\vec{b} - \dfrac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$

B. $2\vec{b} - \dfrac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$

C. $|\vec{a}|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$

D. $|\vec{b}|\vec{b} - \dfrac{\vec{a} \times \vec{b}}{|\vec{b}|^2}$

Answer: a,b,cd,

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120. If vector $\vec{b} = \left(tan\alpha, -12\sqrt{\sin\alpha/2}\right) and \vec{c} = \left(tan\alpha, tan\alpha - \dfrac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vector $\vec{a} = (13, \sin 2\alpha)$ makes an obtuse angle with the z-axis, then the value of α is $\alpha = (4n+1)\pi + \tan^{-1}2$ b. $\alpha = (4n+1)\pi - \tan^{-1}2$

c. $\alpha = (4n+2)\pi + \tan^{-1}2$ d. $\alpha = (4n+2)\pi - \tan^{-1}2$

A. $\alpha = (4n+1)\pi + \tan^{-1}2$

B. $\alpha = (4n+1)\pi - \tan^{-1}2$

C. $\alpha = (4n+2)\pi + \tan^{-1}2$

D. $\alpha = (4n+2)\pi - \tan^{-1}2$

Answer: b,d

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121. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \sqrt{2}$

A. $\vec{r} = \frac{2}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

B. $\vec{r} = \frac{1}{3}\left(\vec{a} + \vec{a} \times \vec{b}\right)$

C. $\vec{r} = \frac{2}{3}\left(\vec{a} - \vec{a} \times \vec{b}\right)$

D. $\vec{r} = \frac{1}{3}\left(-\vec{a} + \vec{a} \times \vec{b}\right)$

Answer: b,d

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122. If $\vec{a}$ and $\vec{b}$ are unequal unit vectors such that $\left(\vec{a} - \vec{b}\right) \times \left[\left(\vec{b} + \vec{a}\right) \times \left(2\vec{a} + \vec{b}\right)\right] = \vec{a} + \vec{b}$ then angle θ between $\vec{a}$ and $\vec{b}$ is

A. 0

B. $\pi/2$

C. $\pi/4$

D. π

Answer: b,d

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123. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpenicualar to each other and $\vec{c} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\left(\vec{a} \times \vec{b}\right)$, then which of the following is (are) true ?

A. $\lambda_1 = \vec{a}.\vec{c}$

B. $\lambda_2 = \left|\vec{b} \times \vec{c}\right|$

C. $\lambda_3 = \ |\ \left(\vec{a} \times \vec{b}\right| \times \vec{c}|$

D. $\lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\left(\vec{a} \times \vec{b}\right)$

Answer: a,d

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124. If vectors $\vec{a}$ and $\vec{b}$ are non collinear then $\frac{\vec{a}}{\left|\vec{a}\right|} + \frac{\vec{b}}{\left|\vec{b}\right|}$ is (A) a unit vector $\in thepla \neq of$veca and vecb$(B) \in thepla \neq of$veca and vecb $(C)equally \in cl \in edot\vec{a}s$ and $\vec{b}$ (D) $perpendic\underline{a}t \rightarrow$ veca xx vecb`

A. a unit vector

B. in the plane of $\vec{a}$ and $\vec{b}$

C. equally inclined to $\vec{a}$ and $\vec{b}$

D. perpendicular to $\vec{a} \times \vec{b}$

Answer: b,c,d

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125. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - 2\vec{b}\right|$ then

A. $2\vec{a}.\vec{b} = \left|\vec{b}\right|^2$

B. $\vec{a}.\vec{b} = \left|\vec{b}\right|^2$

C. least value of $\vec{a}. Vecb + \dfrac{1}{\left|\vec{b}\right|^2 + 2}$ is $\sqrt{2}$

D. least value of $\vec{a}.\vec{b} + \dfrac{1}{\left|\vec{b}\right|^2 + 2}$ is $\sqrt{2} - 1$

Answer: a,d

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126. Let $\vec{a}\vec{b}$ and $\vec{c}$ be non- zero vectors aned $\vec{V}_1 = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and $\vec{V}_2 = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$.vectors $\vec{V}_1$ and $\vec{V}_2$ are equal . Then

A. $\vec{a}$ and $\vec{b}$ ar orthogonal

B. $\vec{a}$ and $\vec{c}$ are collinear

C. $\vec{b}$ and $\vec{c}$ ar orthogonal

D. $\vec{b} = \lambda\left(\vec{a} \times \vec{c}\right)$ when λ is a scalar

Answer: b,d

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127. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$ then prove that $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

A. $\vec{A} = \dfrac{\left(\vec{a} \times \vec{b}\right) - \vec{a}}{a^2}$

B. $\vec{B} = \dfrac{\left(\vec{b} \times \vec{a}\right) + \vec{a}\left(a^2 - 1\right)}{a^2}$

C. $\vec{A} = \dfrac{\left(\vec{a} \times \vec{b}\right) + \vec{a}}{a^2}$

D. $\vec{B} = \dfrac{\left(\vec{b} \times \vec{a}\right) - \vec{a}\left(a^2 - 1\right)}{a^2}$

Answer: b,c,

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128. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;}\ \vec{c}, \vec{a},$ respectively. Then

A. $\vec{x}.\vec{d} = -1$

B. $\vec{y}.\vec{d} = 1$

C. vecz.vecd=0`

D. vecr.vecd=0, " where " vecr=lambda vecx + mu vecy +deltavecz`

Answer: c.d

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129. Vectors perpendicular to $\hat{i} - \hat{j} - \hat{k}$ and in the plane of $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ are (A) $\hat{i} + \hat{k}$ (B) $2\hat{i} + \hat{j} + \hat{k}$ (C) $3\hat{i} + 2\hat{j} + \hat{k}$ (D) $-4\hat{i} - 2\hat{j} - 2\hat{k}$

A. $\hat{i} + \hat{k}$

B. $2\hat{i} + \hat{j} + \hat{k}$

C. $3\hat{i} + 2\hat{j} + \hat{k}$

D. $-4\hat{i} - 2\hat{j} - 2\hat{k}$

Answer: b,d

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130. If the sides $\overrightarrow{AB}$ of an equilateral triangle ABC lying in the xy-plane is $3\hat{i}$ then the side $\overrightarrow{CB}$ can be (A) $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\right)$ (B) $\frac{3}{2}\left(\hat{i}-\sqrt{3}\right)$ (C) $-\frac{3}{2}\left(\hat{i}+\sqrt{3}\right)$ (D) $\frac{3}{2}\left(\hat{i}+\sqrt{3}\right)$

A. $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

B. $-\frac{3}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

C. $-\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

D. $\frac{3}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

Answer: b,d

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131. Let $\hat{a}$ be a unit vector and $\hat{b}$ a non zero vector non parallel to $\vec{a}$. Find the angles of the triangle tow sides of which are represented by the vectors. $\sqrt{3}\left(\hat{\ } \times \vec{b}\right)$ and $\vec{b}-\left(\hat{a}.\vec{b}\right)\hat{a}$

A. $\tan^{-1}\left(\sqrt{3}\right)$

B. $\tan^{-1}\left(1/\sqrt{3}\right)$

C. $\cot^{-1}(0)$

D. tant^(-1)(1)`

Answer: a,b,c

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132. $\vec{a}, \vec{b}$ and $\vec{c}$ are unimdular and coplanar. A unit vector $\vec{d}$ is perpendicualt to them , $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \frac{1}{6}\hat{i} - \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$, and the angle between $\vec{a}$ and $\vec{b} is 30^\circ$ then $\vec{c}$ is

A. $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)/3$

B. $\left(- \hat{i} + 2\hat{j} - 2\hat{k}\right)/3$

C. $\left(- \hat{i} + 2\hat{j} - \hat{k}\right)/3$

D. $\left(- 2\hat{i} - 2\hat{j} + \hat{k}\right)/3$

Answer: a,b

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133. If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$

A. $2\left(\vec{a} \times \vec{b}\right)$

B. $6\left(\vec{b} \times \vec{c}\right)$

C. $3\left(\vec{c} \times \vec{a}\right)$

D. $\vec{0}$

Answer: c,d

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134. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a}$ - $\left(\vec{a}.\vec{b}\right)\vec{b}$ and $\overset{\rightarrow}{=} \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $\left|\vec{u}\right|$

B. $\left|\vec{u}\right| + \left|\vec{u}.\vec{b}\right|$

C. $\left|\vec{u}\right| + \left|\vec{u}.\vec{a}\right|$

D. none of these

Answer: b,d

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135. if $\vec{a} \times \vec{b} = \vec{c}. \vec{b} \times \vec{c} = \vec{a}, \text{ where } \vec{c} \neq \vec{0}$ then

A. $\left|\vec{a}\right| = \left|\vec{c}\right|$

B. $\left|\vec{a}\right| = \left|\vec{b}\right|$

C. $\left|\vec{b}\right| = 1$

D. $\left|\vec{a}\right| = \vec{b}| = |\vec{c}\ | = 1$

Answer: a,c

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136. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non- coplanar vectors and $\vec{d}$ be a non -zero , which is perpendicular to $\left(\vec{a} + \vec{b} + \vec{c}\right)$. $Now \vec{d} = \left(\vec{a} \times \vec{b}\right) \sin x + \left(\vec{b} \times \vec{c}\right) \cos y + 2\left(\vec{c} \times \vec{a}\right)$. Then

A. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = 2$

B. $\dfrac{\vec{d}.\left(\vec{a} + \vec{c}\right)}{\left[\vec{a}\vec{b}\vec{c}\right]} = -2$

C. minimum value of $x^2 + y^2 is \pi^2/4$

D. minimum value of $x^2 + y^2 is 5\pi^2/4$

Answer: b,d

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137. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b},$ then $\left(\vec{b}\right.$ and $\vec{c}$ being non parallel)

A. angle between $\vec{a}$ and $\vec{b} is \pi/3$

B. angle between $\vec{a}$ and $\vec{c} is \pi/3$

C. angle between $\vec{a}$ and $\vec{b} is \pi/2$

D. angle between $\vec{a}$ and $\vec{c} is \pi/2$

Answer: b,c

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138. If in triangle ABC, $\overrightarrow{AB} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$, where $|\vec{u}| \neq |\vec{v}|$, then

A. $1 + \cos 2A + \cos 2B + \cos 2C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: a,b,c

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139. $\left[\vec{a}\times\vec{b}\quad\vec{c}\times\vec{d}\quad\vec{e}\times\vec{f}\right]$ is equal to

A. $\left[\vec{a}\vec{b}\vec{d}\right]\left[\vec{c}\vec{e}\vec{f}\right]$ - $\left[\vec{a}\vec{b}\vec{c}\right]\left[\vec{d}\vec{e}\vec{f}\right]$

B. $\left[\vec{a}\vec{b}\vec{e}\right]\left[\vec{f}\vec{c}\vec{d}\right]$ - $\left[\vec{a}\vec{b}\vec{f}\right]\left[\vec{e}\vec{c}\vec{d}\right]$

C. $\left[\vec{c}\vec{d}\vec{a}\right]\left[\vec{b}\vec{e}\vec{f}\right]$ - $\left[\vec{a}\vec{d}\vec{b}\right]\left[\vec{a}\vec{e}\vec{f}\right]$

D. $\left[\vec{a}\vec{c}\vec{e}\right]\left[\vec{b}\vec{d}\vec{f}\right]$

Answer: a,b,c

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140. The scalars l and m such that $l\vec{a}+m\vec{b}=\vec{c}$, where $\vec{a},\vec{b}$ and $\vec{c}$ are given vectors, are equal to

A. $l = \frac{(\vec{c} \times \vec{b}).(\vec{a} \times \vec{b})}{(\vec{a} \times \vec{b})^2}$

B. $l = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

C. $m = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})^2}$

D. $m = \frac{(\vec{c} \times \vec{a}).(\vec{b} \times \vec{a})}{(\vec{b} \times \vec{a})}$

Answer: a,c

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141. If $(\vec{a} \times v\vec{b}) \times (\vec{c} \times \vec{d}).(\vec{a} \times \vec{d}) = 0$ then which of the following may be true ?

A. $\vec{a}, \vec{b}$ and $\vec{d}$ are nenessarily coplanar

B. $\vec{a}$ lies iin the plane of $\vec{c}$ and $\vec{d}$

C. $\vec{v}b$ lies in the plane of $\vec{a}$ and $\vec{d}$

D. $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{d}$

Answer: b,c,d

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142. A,B C and dD are four points such that $\overrightarrow{AB} = m\left(2\hat{i} - 6\hat{j} + 2\hat{k}\right)\overrightarrow{BC} = \left(ahti - 2\hat{j}\right)$ and $\overrightarrow{CD} = n\left(-6\hat{i} + 15\hat{j} - 3\hat{k}\right)$. If CD intersects AB at some points E, then

A. $m \geq 1/2$

B. $n \geq 1/3$

C. m= n

D. $m < n$

Answer: a,b

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143. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$\left[l\vec{a} + m\vec{b} + n\vec{c} \quad l\vec{b} + m\vec{c} + n\vec{a} \quad l\vec{c} + m\vec{a} + n\vec{b}\right] = 0$ then

A. $l + m + n = 0$

B. roots of the equation $lx^2 + mx + n = 0$ are equal

C. $l^2 + m^2 + n^2 = 0$

D. $l^3 + m^2 + n^3 = 3lmn$

Answer: a,b,d

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144. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{v}$ is perpendicular to

A. $\vec{\alpha}$

B. $\vec{\beta}$

C. $\vec{\gamma}$

D. none of these

Answer: a,b,c

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145. If vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = \hat{i} + \hat{j} + 5\hat{k}$ and $\vec{C}$ form a left handed system then $\vec{C}$ is (A) $11\hat{i} - 6\hat{j} - \hat{k}$ (B) $-11\hat{i} + 6\hat{j} + \hat{k}$ (C) $-11\hat{i} + 6\hat{j} - \hat{k}$ (D) $-11\hat{i} + 6\hat{j} - \hat{k}$

A. $11\hat{i} - 6\hat{j} - \hat{k}$

B. $-11\hat{i} - 6\hat{j} - \hat{k}$

C. $-11\hat{i} - 6\hat{j} + \hat{k}$

D. $-11\hat{i} + 6\hat{j} - \hat{k}$

Answer: b,d

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146. If

$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k}$ and $\vec{c} = z\hat{i} + x\hat{j} + y\hat{k},$, then $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

Answer: a,b,c,d

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147. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) \times \vec{c}$ for non coplanar $\vec{a}, \vec{b}, \vec{c}$ then......

A. $\left(\vec{c} \times \vec{a}\right) \times \vec{b} = \vec{0}$

B. $\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \vec{0}$

C. $\vec{b} \times \left(\vec{c} \times \vec{a}\right) = \vec{0}$

D. $\vec{c} \times \vec{a} \times \vec{b} = \vec{b} \times (\vec{c} \times \vec{a}) = \vec{0}$

Answer: a,c,d

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148. A vector $\vec{d}$ is equally inclined to three vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{j} - 2\hat{k}$. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{;} \vec{c}, \vec{a},$ respectively. Then

A. $\vec{z}. \vec{d} = 0$

B. $\vec{x}. \vec{d} = 1$

C. $\vec{y}. \vec{d} = 32$

D. $\vec{r}. \vec{d} = 0, \text{ where } \vec{r} = \lambda\vec{x} + \mu\vec{y} + \gamma\vec{z}$

Answer: a,d

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149. A parallelogram is constructed on the vectors $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}. If \left|\vec{\alpha}\right| = \left|\vec{\beta}\right| = 2$ and angle between $\vec{\alpha}$ and $\vec{\beta} is \frac{\pi}{3}$ then the length of a diagonal of the parallelogram is (A) $4\sqrt{5}$ (B) $4\sqrt{3}$ (C) 4sqrt(7)` (D) none of these

A. $4\sqrt{5}$

B. $4\sqrt{3}$

C. $4\sqrt{7}$

D. none of these

Answer: b,c

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150. Statement 1: Vector $\vec{c} = 5\hat{i} + 7\hat{j} + 2\hat{k}$ is along the bisector of angle between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -8\hat{i} + \hat{j} - 4\hat{k}$.

Statement 2 : $\vec{c}$ is equally inclined to $\vec{a}$ and $\vec{b}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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151. Statement1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular to the direction of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$

Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: c

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152. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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153. Prove that $\left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]=0$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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154. Statement 1: If $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b}_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are three mutually perpendicular unit vectors then $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}, a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$ may be mutually perpendicular unit vectors.

Statement 2 : value of determinant and its transpose are the same.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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155. Statement 1: $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$ then $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = 243$ Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right). \vec{C}\right| = \left|\vec{A}\right|^2 \left|\left[\vec{A}\vec{B}\vec{C}\right]\right|$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: d

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156. Statement 1: $\vec{a}, \vec{b}$ and $\vec{c}$ arwe three mutually perpendicular unit vectors and $\vec{d}$ is a vector such that $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are non- coplanar. If $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] = 1$, then $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

Statement 2: $\left[\vec{d}\vec{b}\vec{c}\right] = \left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right] \Rightarrow \vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: b

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157. Consider three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

Statement 1: $\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right).\vec{b}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right).\vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a}\right).\vec{b}\Big)\hat{k}$

Statement 2: $\vec{c} = \left(\hat{i}.\vec{c}\right)\hat{i} + \left(\hat{j}.\vec{c}\right)\hat{j} + \left(\hat{k}.\vec{c}\right)\hat{k}$

A. Both the statements are true and statement 2 is the correct explanation for statement 1.

B. Both statements are true but statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true and Statement 2 is false

D. Statement 1 is false and Statement 2 is true.

Answer: a

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158. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times \left(\vec{v} \times \vec{w}\right) = \vec{b}, \left(\vec{u} \times \vec{v}\right) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and

Vector $\vec{u}$ is

A. $\vec{a} - \frac{2}{3}\vec{b} + \vec{c}$

B. $\vec{a} + \frac{4}{3}\vec{b} + \frac{8}{3}\vec{c}$

C. $2\vec{a} - \vec{b} + \frac{1}{3}\vec{c}$

D. $\frac{4}{3}\vec{a} - \vec{b} + \frac{2}{3}\vec{c}$

Answer: b

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159. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and

Vector $\vec{u}$ is

A. $2\vec{a} - 3\vec{c}$

B. $3\vec{b} - 4c$

C. $-4\vec{c}$

D. $\vec{a} + \vec{b} + 2\vec{c}$

Answer: c

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160. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be three unit vectors such that $\vec{u} + \vec{v} + \vec{w} = \vec{a}, \vec{u} \times (\vec{v} \times \vec{w}) = \vec{b}, (\vec{u} \times \vec{v}) \times \vec{w} = \vec{c}, \vec{a}.\vec{u} = 3/2, \vec{a}.\vec{v} = 7/4$ and

Vector $\vec{u}$ is

A. $\frac{2}{3}(2\vec{c} - \vec{b})$

B. $\frac{1}{3}(\vec{a} - \vec{b} - \vec{c})$

C. $\frac{1}{3}\vec{a} - \frac{2}{3}\vec{b} - 2\vec{c}$

D. $\frac{4}{3}(\vec{c} - \vec{b})$

Answer: d

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161. vertors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of 60° with each other . $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$

Vector $\vec{x}$ is

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \left(\vec{a} + \vec{b}\right)\right]$

B. $\frac{1}{2}\left[\left(\vec{a} + \vec{b}\right) \times \vec{c} + \left(\vec{a} - \vec{b}\right)\right]$

C. $\frac{1}{2}\left[-\left(\vec{a} + \vec{b}\right) \times \vec{c} + \left(\vec{a} + \vec{b}\right)\right]$

D. $\frac{1}{2}\left[\left(\vec{a} + \vec{b}\right) \times \vec{c} - \left(\vec{a} + \vec{b}\right)\right]$

Answer: d

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162. vertors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of 60° with each other . $\vec{x} \times (\vec{y} \times \vec{z}) = \vec{a}, \vec{y} \times (\vec{z} \times \vec{x}) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$

Vector $\vec{x}$ is

A. $\frac{1}{2}\left[\left(\vec{a} + \vec{c}\right) \times \vec{b} - \vec{b} - \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{b} + \vec{b} + \vec{a}\right]$

C. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} + \vec{a}\right]$

D. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{a} + \vec{b} - \vec{a}\right]$

Answer: c

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163. vertors $\vec{x}, \vec{y}$ and $\vec{z}$ each of magnitude $\sqrt{2}$, make an angle of 60° with each other . $\vec{x} \times \left(\vec{y} \times \vec{z}\right) = \vec{a}, \vec{y} \times \left(\vec{z} \times \vec{x}\right) = \vec{b}$ and $\vec{x} \times \vec{y} = \vec{c}$

Vector $\vec{x}$ is

A. $\frac{1}{2}\left[\left(\vec{a} - \vec{c}\right) \times \vec{c} - \vec{b} + \vec{a}\right]$

B. $\frac{1}{2}\left[\left(\vec{a} - \vec{b}\right) \times \vec{c} + \vec{b} - \vec{a}\right]$

C. $\frac{1}{2}\left[\vec{c} \times \left(\vec{a} - \vec{b}\right) + \vec{b} + \vec{a}\right]$

D. none of these

Answer: b

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164. If $\vec{x} \times \vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$ then find x,y,z in terms of $\vec{a}, \vec{b}$ and γ.

A. $\frac{1}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\frac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} \times \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: b

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165. If $\vec{\times} x\vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$ then find x,y,z in terms of `veca,vecb and gamma.

A. $\dfrac{\vec{a} \times \vec{b}}{\gamma}$

B. $\vec{a} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

C. $\vec{a} + \vec{b} + \dfrac{\vec{a} \times \vec{b}}{\gamma}$

D. none of these

Answer: a

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166. If $\vec{\times} x\vec{y} = \vec{a}, \vec{y} \times \vec{z} = \vec{b}, \vec{x}.\vec{b} = \gamma, \vec{x}.\vec{y} = 1$ and $\vec{y}.\vec{z} = 1$ then find x,y,z in terms of `veca,vecb and gamma.

A. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} \times \left(\vec{a} \times \vec{b}\right)\right]$

B. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} - \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

C. $\dfrac{\gamma}{\left|\vec{a} \times \vec{b}\right|^2}\left[\vec{a} + \vec{b} + \vec{a} \times \left(\vec{a} \times \vec{b}\right)\right]$

D. none of these

Answer: c

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167. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A}$ - $\vec{P}$. then $\left(\vec{P} \times \vec{B}\right) \times \vec{B}$ is equal to

A. $\vec{P}$

B. -$\vec{P}$

C. $2\vec{B}$

D. $\vec{A}$

Answer: b

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168. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then

$\left(\vec{P} \times \vec{B}\right) \times \vec{B}$ is equal to

A. $\frac{\vec{A}}{2} + \frac{\vec{A} \times \vec{B}}{2}$

B. $\frac{\vec{A}}{2} + \frac{\vec{B} \times \vec{A}}{2}$

C. $\frac{\vec{A} \times \vec{B}}{2} - \frac{\vec{A}}{2}$

D. $\vec{A} \times \vec{B}$

Answer: b

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169. Given two orthogonal vectors $\vec{A}$ and $VecB$ each of length unity. Let $\vec{P}$ be the vector satisfying the equation $\vec{P} \times \vec{B} = \vec{A} - \vec{P}$. then

which of the following statements is false ?

A. vectors $\vec{P}, \vec{A}$ and $\vec{P} \times \vec{B}$ ar linearly dependent.

B. vectors $\vec{P}, \vec{B}$ and $\vec{P} \times \vec{B}$ ar linearly independent

C. $\vec{P}$ is orthogonal to $\vec{B}$ and has length $\frac{1}{\sqrt{2}}$.

D. none of these

Answer: d

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170. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a} on \vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1 on \vec{c}$. Then $\vec{a}_2$ is equal to

A. $\frac{943}{49}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

B. $\frac{943}{49^2}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

C. $\frac{943}{49}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

D. $\frac{943}{49^2}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Answer: b

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171. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a} on \vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1 on \vec{c}$. Then $\vec{a}_1 . \vec{b}$ is equal to

A. -41

B. $-41/7$

C. 41

D. 287

Answer: a

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172. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a} on \vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a} on \vec{c}$. Then which of the following is true ?

A. $\vec{a}$ and $vcea_2$ are collinear

B. $\vec{a}_1$ and $\vec{c}$ are collinear

C. $\vec{a} m \vec{a}_1$ and $\vec{b}$ are coplanar

D. $\vec{a}, \vec{a}_1$ and a_2 are coplanar

Answer: c

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173. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the vector $\overline{AG}$ is

A. $\sqrt{17}$

B. $\sqrt{51}/3$

C. $3/\sqrt{6}$

D. $\sqrt{59}/4$

Answer: b

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174. Consider a triangular pyramid $ABCD$ the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the perpendicular from the vertex D on the opposite face

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. none of these

Answer: c

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175. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the vector $\bar{AG}$ is

A. $14/\sqrt{6}$

B. $2/\sqrt{6}$

C. $3/\sqrt{6}$

D. none of these

Answer: a

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176. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

The distance between the parallel lines AB and CD is

A. $\sqrt{6}$

B. $3\sqrt{6/5}$

C. $2\sqrt{2}$

D. 3

Answer: c

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177. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

A. $\frac{4\sqrt{6}}{9}$

B. $\frac{32\sqrt{6}}{9}$

C. $\frac{16\sqrt{6}}{9}$

D. none

Answer: b

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178. Vertices of a parallelogram taken in order are A, (2,-1,4) , B (1,0,-1) , C (1,2,3) and D.

Distance of the point P (8, 2,-12) from the plane of the parallelogram is

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179. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j}-8\hat{i}-\vec{r}\right)=40$ and $p_1=\max\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2=\min\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. A tangent line is drawn to the curve $y = \frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B

A. 9

B. $2\sqrt{2} - 1$

C. $6\sqrt{6} + 3$

D. $9 - 4\sqrt{2}$

Answer: d

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180. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j} - 8\hat{i} - \vec{r}\right) = 40$ and $p_1 = \max\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}, p_2 = \min\left\{\left|\vec{r} + 2\hat{i} - 3\hat{j}\right|^2\right\}$. A tangent line is drawn to the curve $y = \frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B

A. 2

B. 10

C. 18

D. 5

Answer: c

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181. Let $\vec{r}$ is a positive vector of a variable pont in cartesian OXY plane such that $\vec{r}.\left(10\hat{j}-8\hat{i}-\vec{r}\right)=40$ and $p_1=\max\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}, p_2=\min\left\{\left|\vec{r}+2\hat{i}-3\hat{j}\right|^2\right\}$. A tangent line is drawn to the curve $y=\frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B

Then $\overrightarrow{AB}.\overrightarrow{OB}$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer: c

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182. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB} \times \overrightarrow{AC}$ and $\overrightarrow{AD} \times \overrightarrow{AB} = \vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times (\vec{b} - \vec{c})}{|\vec{a}|^2} + \frac{3(\vec{b} \times \vec{a})}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}-\frac{3\left(\vec{b}\times\vec{a}\right)}{|\vec{a}|^2}$

D. none of these

Answer: a

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183. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB}\times\overrightarrow{AC}$ and $\overrightarrow{AD}\times\overrightarrow{AB}=\vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AC}$ is

A. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}+\frac{3\left(\vec{b}\times\vec{a}\right)}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}-\frac{3\left(\vec{b}\times\vec{a}\right)}{|\vec{a}|^2}$

D. none of these

Answer: b

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184. Ab, AC and AD are three adjacent edges of a parallelpiped. The diagonal of the praallelepiped passing through A and direqcted away from it is vector $\vec{a}$. The vector of the faces containing vertices A, B , C and A, B, D are $\vec{b}$ and $\vec{c}$, respectively , i.e. $\overrightarrow{AB}\times\overrightarrow{AC}$ and $\overrightarrow{AD}\times\overrightarrow{AB}=\vec{c}$ the projection of each edge AB and AC on diagonal vector $\vec{a} is \frac{|\vec{a}|}{3}$ vector $\overrightarrow{AD}$ is

A. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}$

B. $\frac{1}{3}\vec{a}+\frac{\vec{a}\times\left(\vec{b}-\vec{c}\right)}{|\vec{a}|^2}+\frac{3\left(\vec{b}\times\vec{a}\right)}{|\vec{a}|^2}$

C. $\frac{1}{3}\vec{a} + \frac{\vec{a} \times \left(\vec{b} - \vec{c}\right)}{|\vec{a}|^2} - \frac{3\left(\vec{b} \times \vec{a}\right)}{|\vec{a}|^2}$

D. none of these

Answer: c

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185.

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186.

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187.

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188. Given two vectors $\vec{a} = -\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{j} - \hat{k}$

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189. Given two vectors $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 2\hat{k}$

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190.

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191. Valume of parallelpiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq. units.

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192. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\dfrac{3\left|\vec{a} + \vec{b}\right|}{2} + 2\left|\vec{a} - \vec{b}\right|$

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193. Let $\vec{u}$ be a vector on rectangular coodinate system with sloping angle 60° suppose that $\left|\vec{u} - \hat{i}\right|$ is geomtric mean of $\left|\vec{u}\right|$ and $\left|\vec{u} - 2\hat{i}\right|$, where $\hat{i}$ is the unit vector along the x-axis . Then find the value of $\dfrac{\sqrt{2} - 1}{\left|\vec{u}\right|}$

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194. Find the absolute value of parameter t for which the area of the triangle whose vertices the $A(-1, 1, 2); B(1, 2, 3) and C(5, 1, 1)$ is minimum.

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195. If

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ and $\left[3\vec{a} + \vec{b}3\vec{b} + \vec{c}\right.$

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196. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value of 6α. Such that $\left\{\left(\vec{a} \times \vec{b}\right) \times \left(\vec{b} \times \vec{c}\right)\right\} \times \left(\vec{c} \times \vec{a}\right) = 0$

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197. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying $\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}+\left[(a-2)\gamma^2+(b-3)\gamma+c\right.$

are three distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)$.

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198. Let $\vec{u} and \vec{v}$ be unit vectors such that $\vec{u}\times\vec{v}+\vec{u}=\vec{w}$ and $\vec{w}\times\vec{u}=\vec{v}$.

Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$.

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199. The volume of the tetrahedron whose vertices are the points with positon vectors $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+7\hat{k}$, $5\hat{i}-\hat{j}+\hat{k}$ and $7\hat{i}-4\hat{j}+7\hat{k}$ is 11 cubic units if the value of λ is

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200. Given that

$\vec{u} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{v} = 2\hat{i} + \hat{k} + 4\hat{k}, \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k}$ and $\left(\vec{u}.\vec{R} - 15\right)\hat{i} + \left(\vec{c}.\vec{R} - 30\right)\hat{j}$

. Then find the greatest integer less than or equal to $\left|\vec{R}\right|$.

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201. Let a three- dimensional vector $\vec{V}$ satissgy the condition , $2\vec{V} + \vec{V} \times \left(\hat{i} + 2\hat{j}\right) = 2\hat{i} + \hat{k}. If 3\left|\vec{V}\right| = \sqrt{m}$. Then find the value of m.

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202. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}. Vecb = 0 = \vec{a}. \vec{c}$ and the angle between $\vec{b}$ and $\vec{c} is \frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$

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203. Let $\vec{O}A - \vec{a}, \vec{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq$, then find$k$.

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204. Find the work done by the force $F = 3\hat{i} - \hat{j} - 2\hat{k}$ acting on a particle such that the particle is displaced from point $A(-3, -4, 1)$ and $B(-1, -1, -2)$.

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205. From a point O inside a triangle ABC, perpendiculars OD, OE and OF are drawn to the sides BC, CA and AB, respectively. Prove that the perpendiculars from A, B and C to the sides EF, FD and DE are concurrent

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206. $A_1, A_2, \dots A_n$ are the vertices of a regular plane polygon with n sides and O ars its centre. Show that $\sum_{i=1}^{n-1}\left(\overrightarrow{OA_i}\times\overrightarrow{OA_{i+1}}\right)=(1-n)\left(\overrightarrow{OA_2}\times\overrightarrow{OA_1}\right)$

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207. If $\vec{c}$ is a given non-zero scalar, and $\vec{A} and \vec{B}$ are given non-zero vector such that $\vec{A}\perp B$, then find vector $\vec{X}$ which satisfies the equation $\vec{A}.\vec{X}=c$ and $\vec{A}\times\vec{X}=\vec{B}^{\cdot}$

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208. $A, B, CandD$ are any four points in the space, then prove that $\left|\vec{AB}\times\vec{CD}+\vec{BC}\times\vec{AD}+\vec{CA}\times\vec{BD}\right|=4$ (area of ABC .)

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209. If the vectors $\vec{a}, \vec{b},$ and $\vec{c}$ are coplanar show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = 0$$

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210. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ Determine a vector $\vec{R}$ satisfying

$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R}\vec{A} = 0.$

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211. Determine the value of c so that for all real x , vectors

$cx\hat{i} - 6\hat{j} - 3\hat{k} and x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

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212. If vectors, $\vec{b}, vcec$ and $\vec{d}$ are not coplanar, the pove that vector $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)+\left(\vec{a}\times\vec{c}\right)\times\left(\vec{d}\times\vec{b}\right)+\left(\vec{a}\times\vec{d}\right)\times\left(\vec{b}\times\vec{c}\right)$ is parallel to $\vec{a}$.

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213. The position vectors of the vertices A, B and C of a tetrahedron ABCD are $\hat{i}+\hat{j}+\hat{k}$, $\hat{k}$, $\hat{i}$ and $\hat{3}i$,respectively. The altitude from vertex D to the opposite face ABC meets the median line through Aof triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is2/2/3, find the position vectors of the point E for all its possible positfons

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214. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\pi/3then\left\{\vec{a}\times\left(\vec{b}+\vec{a}\times\vec{b}\right)\right\}.\vec{b}$ is equal to

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215. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $|\vec{b}| = |\vec{c}|$ then $\left\{\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} + \vec{c}\right)\right\} \times \left(\vec{b} \times \vec{c}\right). \left(\vec{b} + \vec{c}\right) =$

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216. for any two vectors $\vec{u}$ and $\vec{v}$, prove that

$a.\ \left(\vec{u}.\vec{v}\right)^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2$ and

b.$\left(\vec{1} + |\vec{u}|^2\right)\left(\vec{1} + |\vec{v}|^2\right| = \left(1 = \vec{u}..\vec{v}\right)^2 + \left|\vec{u} + \vec{v} + \left(\vec{u} \times \vec{v}\right)\right|^2$

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217. Let $\vec{u}$ and $\vec{v}$ be unit vectors. If $\vec{w}$ is a vector such that $\vec{w} + \vec{w} \times \vec{u} = \vec{v}$, then prove that $\left|\left(\vec{u} \times \vec{v}\right). \vec{w}\right| \leq \frac{1}{2}$ and that the equality holds if and only if $\vec{u}$ is perpendicular to $\vec{v}$.

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218. Prove that if the vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{a} \times \vec{b}$

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219. Let V be the volume of the parallelepied formed by the vectors, $\vec{a} = a_1\hat{i} = a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. if $a_r b_r nadc_r$ are non- negative real numbers and $\sum_{r=1}^{3}\left(a_r + b_r + c_r\right) = 3L$ show that $V \leq L^3$

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220. $\vec{u}, \vec{v}$ and $\vec{w}$ are three nono-coplanar unit vectors and α, β and γ are the angles between $\vec{u}$ and $\vec{u}, \vec{v}$ and $\vec{w}$ and $\vec{w}$ and $\vec{u}$, respectively and $\vec{x}, \vec{y}$ and $\vec{z}$ are unit vectors along the bisectors of the angles α, β and γ. respectively, prove that $\left[\vec{x} \times \vec{y}\vec{y} \times \vec{z}\vec{z} \times \vec{x}\right) = \frac{1}{16}\left[\vec{u}\vec{v}\vec{w}\right]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}$.

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221. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct vectors such that $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$. Prove that $\left(\vec{a} - \vec{d}\right).\left(\vec{c} - \vec{b}\right) \neq 0, i.e., \vec{a}.\vec{b} + \vec{d}.\vec{c} \neq \vec{d}.\vec{b} + \vec{a}.\vec{c}.$

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222. $P_1 nd P_2$ are planes passing through origin $L_1 and L_2$ are two lines on $P_1 and P_2$, respectively, such that their intersection is the origin. Show that there exist points $A, B and C$, whose permutation $A', B' and C'$, respectively, can be chosen such that A is on $L_1, B on P_1$ but not on $L_1 and C$ not on P_1; A' is on $L_2, B' on P_2$ but not on $L_2 and C'$ not on P_2

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223. about to only mathematics

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224. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be vectors of legth , 3,4and 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}, \vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to$\vec{A}+\vec{B}$ then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is ________.

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225. The unit vector perendicular to the plane determined by P (1,-1,2) ,C(3,-1,2) is _______.

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226. The area of the triangle whose vertices are $A(1, -1, 2), B(2, 1-1)C(3, -1, 2)$ is ....

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227. If $\vec{A}, \vec{B}, \vec{C}$ are non-coplanar vectors then $\frac{\vec{A}. \vec{B} \times \vec{C}}{\vec{C} \times \vec{A}. \vec{B}} + \frac{\vec{B}. \vec{A} \times \vec{C}}{\vec{C}. \vec{A} \times \vec{B}} =$

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228. If $\vec{A} = (1, 1, 1)$ and $\vec{C} = (0, 1, -1)$ are given vectors then find a vector $\vec{B}$ satisfying equations $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{A}. \vec{B} = 3$

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229. Let $\vec{b} = 4\hat{i} + 3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. Find all vetors in te same plane having projection 1 and 2 along $\vec{b}$ and $\vec{c}$ respectively.

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230. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are ______ and ________, respectively.

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231. A unit vector coplanar with $\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 2\vec{j} + \vec{k}$ and perpendicular to $\vec{i} + \vec{j} + \vec{k}$ is _____

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232. A non vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and thepane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$ then angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ is = (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$

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233. If $\vec{b}$ and $\vec{c}$ are any two mutually perpendicular unit vectors and $\vec{a}$ is any vector, then $\left(\vec{a}.\vec{b}\right)\vec{b} + \left(\vec{a}.\vec{c}\right)\vec{c} + \frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|^2}\left(\vec{b}\times\vec{c}\right) =$ (A) 0 (B) $\vec{a}(C)$ veca/2(D)2veca`

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234. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1,1 and 2 resectively. If $\vec{a}\times\left(\vec{a}\times\vec{c}\right) + \vec{b} = \vec{0}$ then the acute angel between $\vec{a}$ and $\vec{c}$ is

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235. A, B C and D are four points in a plane with position vectors, $\vec{a}, \vec{b}\vec{c}$ and $\vec{d}$ respectively, such that $\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right) = \left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right) = 0$ then point D is the _____ of triangle ABC.

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236. If $\vec{A} = \lambda(\vec{u} \times \vec{v}) + \mu(\vec{v} \times \vec{w}) + v(\vec{w} \times \vec{u})$ and $[\vec{u}\vec{v}\vec{w}] = \frac{1}{5} then \lambda + \mu + v =$ (A) 5 (B) 10 (C) 15 (D) none of these

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237. If $\vec{a} = \hat{j} + \sqrt{3}\hat{k} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle , then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is ______

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238. If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$. Prove that $\vec{a} = \pm 2(\vec{b} \times \vec{c})$

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239. If $\vec{x}.\vec{a} = 0$, $\vec{x}.\vec{b} = 0$, $\vec{x}.\vec{c} = 0$ and $\vec{x} \neq \vec{0}$ then show yhat $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar .

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240. for any three vectors, $\vec{a}, \vec{b}$ and $\vec{c}$, $\left(\vec{a} - \vec{b}\right). \left(\vec{b} - \vec{c}\right) \times \left(\vec{c} - \vec{a}\right) = 2\vec{a}. \vec{b} \times \vec{c}$.

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241.

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242.

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243.

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244. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ then find the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$

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245. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = i + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{r}.\vec{a} = 0$ then find the value of $\vec{r}.\vec{b}$.

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246. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$ then $\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$ is

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247. Let $\vec{a}, \vec{b},$ and $\vec{c}$ be three non coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{x}=p\vec{a}+q\vec{b}+r\vec{c}$ where p,q,r are scalars then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

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Exercise 2.1

1. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$.

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2. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

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3. If the vertices A,B, C of a triangle ABC are (1,2,3),(-1, 0,0), (0, 1,2), respectively, then find $\angle ABC$.

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4. If $|a| = 3, |b| = 4 and$ the angle between $a and b$ is 120^0 , then find the value of $|4a + 3b|$

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5. If vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other, then find the locus of th point (x,y).

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6. Let $\vec{a}\vec{b}$ and $\vec{c}$ be pairwise mutually perpendicular vectors, such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2, \left|\vec{c}\right| = 2$, the find the length of $\vec{a} + \vec{b} + \vec{c}$.

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7. If $\vec{a} + \vec{b} + \vec{c} = 0, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

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8. If the angle between unit vectors $\vec{a}$ and $\vec{b}$ is 60° . Then find the value of $\left|\vec{a} - \vec{b}\right|$.

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9. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n} = 0$ and $\vec{v}.\hat{n} = 0$ then find the value of $|\vec{w}.\hat{n}|$

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10. A, B, C, D are any four points, prove that $\vec{AB}\cdot\vec{CD} + \vec{BC}\cdot\vec{AD} + \vec{CA}\cdot\vec{BD} = 0$.

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11. $P(1, 0, -1), Q(2, 0, -3), R(-1, 2, 0) and S(, -2, -1),$ then find the projection length of $\vec{P}Qon\vec{R}S$

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12. If the vectors $3\vec{P} + \vec{q}, 5\vec{P} - 3\vec{q}$ and $2\vec{p} + \vec{q}, 4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors, the find the angle between vectors $\vec{p}$ and $\vec{q}$.

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13. Let $\vec{A}$ and $\vec{B}$ be two non-parallel unit vectors in a plane. If $\left(\alpha\vec{A} + \vec{B}\right)$ bisets the internal angle between $\vec{A}$ and $\vec{B}$ then find the value of α.

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14. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$ and $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 4, \left|\vec{c}\right| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

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15. If $\vec{a}$ and $\vec{b}$ are unit vectors, then find the greatest value of $\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$.

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16. Constant forces $P_1 = \hat{i} + \hat{j} + \hat{k}, P_2 = \hat{i} + 2\hat{j} - \hat{k} and P_3 = \hat{j} - \hat{k}$ act on a particle at a point A. Determine the work done when particle is displaced from position $A\left(4\hat{i} - 3\hat{j} - 2\hat{k}\right) \rightarrow B\left(6\hat{i} + \hat{j} - 3\hat{k}\right)$.

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17. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8$ and $\left|\vec{a} + \vec{b}\right| = 10$ then find $\left|\vec{b}\right|$

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18. If A, B, C, D are four distinct point in space such that AB is not perpendicular to CD and satisfies $\vec{AB}\vec{CD} = k\left(\left|\vec{AD}\right|^2 + \left|\vec{BC}\right|^2 - \left|\vec{AC}\right|^2 = \left|\vec{BD}\right|^2\right)$, then find the value of k.

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Exercise 2.2

1. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$ then find (m,n)

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2. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$ then find the value of $\vec{a}.\vec{b}$

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3. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq 0$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, then for some scalar k prove that $\vec{a} + \vec{c} = k\vec{b}$.

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4. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then find the value of $\left(\vec{a} \times \vec{b}\right).\left(\vec{a} \times \vec{c}\right)$

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5. if the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ from a right -handed system, then find $\vec{c}$.

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6. Given that $\vec{a}.\vec{b} = \vec{a}.\vec{c}, \vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a}$ is not a zero vector. Show that $\vec{b} = \vec{c}$.

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7. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$

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8. If $\vec{x}$ and $\vec{y}$ are unit vectors and $\left|\vec{z}\right| = \frac{2}{\sqrt{7}}$ such that $\vec{z} + \vec{z} \times \vec{x} = \vec{y}$ then find the angle θ between $\vec{x}$ and $\vec{z}$

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9. prove that $\left(\vec{a}.\hat{i}\right)\left(\vec{a}\times\hat{i}\right)+\left(\vec{a}.\hat{j}\right)\left(\vec{a}\times\hat{j}\right)+\left(\vec{a}.\hat{k}\right)\left(\vec{a}\times\hat{k}\right)=\vec{0}$

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10. Let $\vec{a}\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\lambda\vec{b}\times\vec{a}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}+\vec{a}=\vec{0}$ then find the value of λ.

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11. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)^{.}$ Find the velocity of the particle at point $P(3, 6, 4)^{.}$

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12. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm \frac{2}{\sqrt{3}}\left(\vec{b} \times \vec{c}\right)$.

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13. If $\left|\vec{a} \times \vec{b}\right|^2 + \left(\vec{a}.\vec{b}\right)^2 = 144$ and $\left|\vec{a}\right| = 4$, then $\left|\vec{b}\right|$ is equal to ....

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14. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$. If $\vec{c}$ be a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$, then $\vec{c}.\vec{b}$ is equal to

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15. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

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Exercise 2.3

1. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-coplanar unit vectors such that $\vec{d}$ makes equal angles with all the three vectors $\vec{a}, \vec{b}, \vec{c}$ then prove that $\left[\vec{d}\vec{a}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{b}\right] = \left[\vec{d}\vec{c}\vec{a}\right]$

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2. Prove that $\left[\vec{l}\vec{m}\vec{n}\right]\left[\vec{a}\vec{b}\vec{c}\right] = \begin{vmatrix} \vec{l}.\vec{a} & \vec{l}.\vec{b} & \vec{l}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$

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3. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of

α if $(\alpha > 0)$

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4. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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5. If $\vec{x}.\vec{a} = 0 \vec{x}.\vec{b} = 0$ and $\vec{x}.\vec{c} = 0$ for some non zero vector $\vec{x}$ then show that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$

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6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ then find the vector $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = \vec{b}$.

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7. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b}$ then prove that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

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8. If $\vec{a} = \vec{P} + \vec{q}, \vec{P} \times \vec{b} = \vec{0}$ and $\vec{q}.\vec{b} = 0$ then prove that $\dfrac{\vec{b} \times (\vec{a} \times \vec{b})}{\vec{b}.\vec{b}} = \vec{q}$

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9. prove that $\left(\vec{a}.\left(\vec{b} \times \hat{i}\right)\right)\hat{i} + \left(\vec{a}.\left(\vec{b} \times \hat{j}\right)\right)\hat{j} + \left(\vec{a}.\left(\vec{b} \times \hat{k}\right)\right)\hat{k} = \vec{a} \times \vec{b}$

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10. for any four vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ prove that $\vec{d}.\left(\vec{a} \times \left(\vec{b} \times \left(\vec{c} \times \vec{d}\right)\right)\right) = \left(\vec{b}.\vec{d}\right)\left[\vec{a}\vec{c}\vec{d}\right]$

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11. If $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors such that $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\frac{1}{2}\vec{b}$ then find the angle between $\vec{a}$ and $\vec{b}$.

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12. show that $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ if and only if $\vec{a}$ and $\vec{c}$ are collinear or $\left(\vec{a}\times\vec{c}\right)\times\vec{b}\times\vec{0}$

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13. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that no two are collinear and $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$ if θ is the acute angle between vectors $\vec{b}$ and $\vec{c}$ then find value of $\sin\theta$.

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14. If $\vec{p}, \vec{q}, \vec{r}$ denote vector $\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$, respectively, show that $\vec{a}$ is parallel to $\vec{q} \times \vec{r}, \vec{b}$ is parallel $\vec{r} \times \vec{p}, \vec{c}$ is parallel to $\vec{p} \times \vec{q}$.

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15. Let $\vec{a}, \vec{b}, \vec{c}$ be non -coplanar vectors and let equations $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vector $\vec{a}, \vec{b}, \vec{c}$ then prove that $\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}'$ is a null vector.

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16. Given unit vectors $\hat{m}\hat{n}$ and $\hat{p}$ such that angle between $\hat{m}$ and $\hat{n} is \alpha$ and angle between $\hat{p}$ and $\hat{m} X \hat{n} is \alpha$ find alpha

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17. $\vec{a}, \vec{b}, and\vec{c}$ are three unit vectors and every two are inclined to each other at an angel $\cos^{-1}(3/5)^{.}$ If $\vec{a} \times \vec{b} = p\vec{a} + q\vec{b} + r\vec{c}, wherep, q, r$ are scalars, then find the value of $q^{.}$

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18. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ gve three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b} is\frac{\pi}{6}$, then prove that

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} p = \frac{1}{4}\left(a_1^2 + a_2^2 + a_3^2\right)\left(b_1^2 + b_2^2 + b_3^2\right)$$

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JEE Previous Year (Single Question)

1. The scalar $\vec{A}.\left(\vec{B}+\vec{C}\right)\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals (A) 0 (B) $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$ (C) $\left[\vec{A}\vec{B}\vec{C}\right]$ (D) none of these

A. 0

B. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$

C. $\left[\vec{A}\vec{B}\vec{C}\right]$

D. none of these

Answer: a

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2. For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $\left|\left(\vec{a}\times\vec{b}\right).\vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$ holds if and only if

A. $\vec{a}.\vec{b}=0, \vec{b}.\vec{c}=0$

B. $\vec{b}.\vec{c}=0, \vec{c},\vec{a}=0$

C. $\vec{c}.\vec{a}=0, \vec{a},\vec{b}=0$

D. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Answer: d

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3. The volume of he parallelepiped whose sides are given by $\vec{O}A = 2i - 2, j, \vec{O}B = i + j - kand\vec{O}C = 3i - k$ is $4/13$ b. 4 c. $2/7$ d. 2

A. $4/13$

B. 4

C. $2/7$

D. 2

Answer: d

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4. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplanar vector and $\vec{p}$, $\vec{q}$ $\vec{r}$ are defind by the relations $\vec{p} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, $\vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\vec{b}\vec{c}\right]}$, then $\vec{p}.\left(\vec{a} + \vec{b}\right) + \vec{q}.\left(\vec{b} + \vec{c}\right) + \vec{r}.\left(\vec{c} + \vec{a}\right)$ =............

A. 0

B. 1

C. 2

D. 3

Answer: d

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5. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\vec{a}.\hat{d} = 0 = \left[\vec{b}\vec{c}\vec{d}\right]$ then $\hat{d}$ equals

A. $\pm \dfrac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$

B. $\pm \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

C. $\pm \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

D. $\pm \hat{k}$

Answer: a

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6. If $\vec{a}, \vec{b}, \vec{c}$ are three non - coplanar vector such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is ....

A. $3\pi/4$

B. $\pi/4$

C. $\pi/2$

D. π

Answer: a

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7. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vector such that $\vec{u} + \vec{v} + \vec{w} = \vec{0}$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{w}| = 5$ then $\vec{u}.\vec{v} + \vec{v}.\vec{w} + \vec{w}.\vec{u}$ is ....

A. 47

B. -25

C. 0

D. 25

Answer: b

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8. If $\vec{a}, \vec{b}$ and $\vec{c}1$ are three non-coplanar vectors, then $(\vec{a} + \vec{b} + \vec{c}).\left[(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})\right]$ equals

A. 0

B. $\left[\vec{a}\vec{b}\vec{c}\right]$

C. $2\left[\vec{a}\vec{b}\vec{c}\right]$

D. $-\left[\vec{a}\vec{b}\vec{c}\right]$

Answer: d

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9. Let $\vec{p}, \vec{q}, \vec{r}$ be three mutually perpendicular vectors of the same magnitude. If a vector $\vec{x}$ satisfies the equation

$$\vec{p} \times \{\vec{x} - \vec{q}) \times \vec{p}\} + \vec{q} \times \{\vec{x} - \vec{r}) \times \vec{q}\} + \vec{r} \times \{\vec{x} - \vec{p}) \times \vec{r}\} = \vec{0},$$

then $\vec{x}$ is given by

A. $\frac{1}{2}\left(\vec{p} + \vec{q} - 2\vec{r}\right)$

B. $\frac{1}{2}\left(\vec{p} + \vec{q} + \vec{r}\right)$

C. $\frac{1}{3}\left(\vec{p} + \vec{q} + \vec{r}\right)$

D. $\frac{1}{3}\left(2\vec{p} + \vec{q} - \vec{r}\right)$

Answer: b

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10. Let $\vec{a} = 2\vec{j} + \vec{j} - 2\vec{k}, \vec{b} = \vec{i} + \vec{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is $30°$. Find the value of $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$

A. $2/3$

B. $3/2$

C. 2

D. 3

Answer: b

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11. Let $\vec{a} = 2i + j + k, \vec{b} = i + 2j - k$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is pependicular to $\vec{a}$. Then $\vec{c}$ is

A. $\frac{1}{\sqrt{2}}(-j + k)$

B. $\frac{1}{\sqrt{3}}(i - j - k)$

C. $\frac{1}{\sqrt{5}}(i - 2j)$

D. $\frac{1}{\sqrt{3}}(i - j - k)$

Answer: a

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12. If the vectors $\vec{a}, \vec{b}, \vec{c}$ form the sides BC,CA and AB respectively of a triangle ABC then (A) $\vec{a}.\left(\vec{b} \times \vec{c}\right) = \vec{0}$ (B) $\vec{a} \times \left(\vec{b} x \vec{c}\right) = \vec{0}$ (C) $\vec{a}.\vec{b} = \vec{c} = \vec{c} = \vec{a}.a \neq 0$ (D) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} \vec{0}$

A. $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = 0$

B. $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

C. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$

D. $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

Answer: b

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13. Consider the vectors, $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) = \vec{0}$ Let P_1 and P_2 be the planes determined by the pairs of vectors, $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

Answer: a

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14. If $\vec{a}, \vec{b} and \vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a} - \vec{b} 2\vec{b} - \vec{c} 2\vec{c} - \vec{a}\right]$ is a.0 b. 1 c. $-\sqrt{3}$ d. $\sqrt{3}$

A. 0

B. 1

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: a

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15. If $\hat{a}, \hat{b}, and \hat{c}$ are unit vectors, then $\left|\hat{a} - \hat{b}\right|^2 + \left|\hat{b} - \hat{c}\right|^2 + \left|\hat{c} - \hat{a}\right|^2$ does not exceed 45^0 b. 60^0 c. $\cos^{-1}(1/3)$ d. $\cos^{-1}(2/7)$

A. 4

B. 9

C. 8

D. 6

Answer: b

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16. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is (A) 45^0 (B) 60^0 (C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{2}{7}\right)$

A. $45°$

B. $60°$

C. $\cos^{-1}(1/3)$

D. $\cos^{-1}(2/7)$

Answer: b

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17. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}^{\cdot}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is a.-1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

A. -1

B. $\sqrt{10} + \sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: c

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18. Find the value of a so that the volume of the parallelepiped formed by vectors $\hat{i} + a\hat{j} + k, \hat{j} + a\hat{k} and a\hat{i} + \hat{k}$ becomes minimum.

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: c

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19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{k}$, then $\vec{b}.\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right) =$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{i} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: c

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20. The unit vector which is orthogonal to the vector $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$is

A. $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$

D. $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

Answer: c

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21. if $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non- coplanar vectors and $\vec{b}_1 = \vec{b} - \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{b}_2 = \vec{b} + \frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\vec{a}, \vec{c}_1 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}.\vec{c}}{|\vec{c}|^2}\vec{b}_1, \vec{c}_2 = \vec{c} - \frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}}{|\vec{b}}$

, then the set of orthogonal vectors is

A. $(\vec{a}, \vec{b}_1, \vec{c}_3)$

B. $\left(\vec{c}a, \vec{b}_1, \vec{c}_2\right)$

C. $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$

D. $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

Answer: c

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22. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{=} \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c} is \frac{1}{\sqrt{3}}$ is (A) $4\hat{i} - \hat{j} + 4\hat{k}$ (B) $\hat{i} + \hat{j} - 3\hat{k}$ (C) $2\hat{i} + \hat{j} - 2\hat{k}$ (D) $4\hat{i} + \hat{j} - 4\hat{k}$

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $3\hat{i} + \hat{j} - 3\hat{k}$

C. $2\hat{i} + \hat{j} - 2\hat{k}$

D. $4\hat{i} + \hat{j} - 4\hat{k}$

Answer: a

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23. Lelt two non collinear unit vectors $\hat{a}$ and $\hat{b}$ form and acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$ Then (A) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{\frac{1}{2}}$ (B) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{\frac{1}{2}}$ (C) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{\frac{1}{2}}$ (D) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{\frac{1}{2}}$

A. , $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$

B. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + \hat{a}.\hat{b})^{1/2}$

C. $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$

D. , $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ and $M = (1 + 2\hat{a}.\hat{b})^{1/2}$

Answer: a

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24. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right).\left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a}.\vec{c} = \frac{1}{2}$ then (A) $\vec{a}, \vec{b}, \vec{c}$ are non coplanar (B) $\vec{b}, \vec{c}, \vec{d}$ are non coplanar (C) $\vec{b}, \vec{d}$ are non paralel (D) $\vec{a}, \vec{d}$ are paralel and $\vec{b}, \vec{c}$ are parallel

A. $\vec{a}, \vec{b}$ and $\vec{c}$ are non- coplanar

B. $\vec{b}, \vec{c}$ and $\vec{d}$ are non-coplanar

C. $\vec{b}$ and $\vec{d}$ are non- parallel

D. $\vec{a}$ and $\vec{d}$ are parallel and $\vec{b}$ and $\vec{c}$ are parallel

Answer: c

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25. Two adjacent sides of a parallelogram $ABCD$ are given by $\vec{A}B = 2\hat{i} + 10\hat{j} + 11\hat{k} and \vec{A}D = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side $AB,$ then the cosine of the angel α is given by a. $\frac{8}{9}$ b. $\frac{\sqrt{17}}{9}$ c. $\frac{1}{9}$ d. $\frac{4\sqrt{5}}{9}$

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: b

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26. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

A. Parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square.

Answer: a

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27. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vectors $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$ is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} + \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: c

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28. Let $\overline{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overline{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overline{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overline{PT}, \overline{PQ}$ and $\overline{PS}$ is

A. 5

B. 20

C. 10

D. 30

Answer: c

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JEE Previous Year (Multiple Question)

1. about to only mathematics

A. 0

B. 1

C. $\frac{1}{4}\left(a_1^2+a_2^2+a_2^2\right)\left(b_1^2+b_2^2+b_2^2\right)$

D. $\frac{3}{4}\left(a_1^2+a_2^2+a_2^2\right)\left(b_1^2+b_2^2+b_2^2\right)\left(c_1^2+c_2^2+c_2^2\right)$

Answer: c

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2. The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0) and \vec{b} = (0, 1, 1)$ is a. one b. two c. three d. infinite

A. one

B. two

C. three

D. infinite

Answer: b

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3. Let $\vec{a} = 2\hat{i} = \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors . A vector in the pland of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\left(\frac{2}{3}\right)}$ is (A) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (B) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (C) $-2\hat{i} - \hat{j} + 5\hat{k}$ (D) $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: a,c

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4. For three vectors $\vec{u}, \vec{v} and \vec{w}$ which of the following expressions is not equal to any of the remaining three ? a.$\vec{u}.\left(\vec{v} \times \vec{w}\right)$ b. $\left(\vec{v} \times \vec{w}\right).\vec{u}$ c. $\vec{v}.\left(\vec{u} \times \vec{w}\right)$ d. $\left(\vec{u} \times \vec{v}\right).\vec{w}$

A. $\vec{u}.\left(\vec{v} \times \vec{w}\right)$

B. $\left(\vec{v} \times \vec{w}\right).\vec{u}$

C. $\vec{v}.\left(\vec{u} \times \vec{w}\right)$

D. $\left(\vec{u} \times \vec{v}\right).\vec{w}$

Answer: c

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5. Which of the following expressions are meaningful? a.$\vec{u}\left(\vec{v} \times \vec{w}\right)$ b. $\left(\vec{u}.\vec{v}\right).\vec{w}$ c. $\left(\vec{u}.\vec{v}\right)\vec{w}$ d. $\vec{u} \times \left(\vec{v}.\vec{w}\right)$

A. $\vec{u}.\left(\vec{v} \times \vec{w}\right)$

B. $\left(\vec{u}.\vec{v}\right).\vec{w}$

C. $(\vec{u}.\vec{v})\vec{w}$

D. $\vec{u} \times (\vec{v}. Vecw)$

Answer: a,c

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6. If $\vec{a}$ and $\vec{b}$ are two non collinear vectors and vecuveca0(veca.vecb)vecb and vecv=vecaxxvecb$then$|vecv|$is(A)$|vecu|(B)|vecu|+|vecu.vecb|(C) |vecu|+|vecu.veca|` (D) none of these

A. $|\vec{u}|$

B. $|\vec{u}| + |\vec{u}. Veca|$

C. $|\vec{u}| + |\vec{u}.\vec{b}|$

D. $|\vec{u}| + \vec{u}.(\vec{a} + \vec{b})$

Answer: a,c

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7. Vector $\frac{1}{3}\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is

A. a unit vector

B. makes an angle $\pi/3$ with vector $\left(2\hat{i} - 4\hat{j} + 3\hat{k}\right)$

C. parallel to vector $\left(- \hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$

D. perpendicular to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Answer: a,c,d

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8. Let $\vec{A}$ be a vector parallel to the line of intersection of planes $P_1 and P_2$. Plane P_1 is parallel to vectors $2\hat{j} + 3\hat{k} and 4\hat{j} - 3k and P_2$ is parallel to $\hat{j} - \hat{k} and 3\hat{i} + 3\hat{j}$. Then the angle betweenvector $\vec{A}$ and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is $\pi/2$ b. $\pi/4$ c. $\pi/6$ d. $3\pi/4$

A. $\pi/2$

B. $\pi/4$

C. $\pi/6$

D. $3\pi/4$

Answer: b,d

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9. The vectors which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is /are (A) $\hat{j} - \hat{k}$ (B) $-\hat{i} + \hat{j}$ (C) $\hat{i} - \hat{j}$ (D) $-\hat{j} + \hat{k}$

A. $\hat{j} - \hat{k}$

B. $-\hat{i} + \hat{j}$

C. $\hat{i} - \hat{j}$

D. $-\hat{j} + \hat{k}$

Answer: a,d

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10. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$ if $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

A. $\vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z} - \vec{x}\right)$

B. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right)$

C. $\vec{a}.\vec{b} = -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

D. $\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{z} - \vec{y}\right)$

Answer: a,b,c

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11. Let $ti\angle PQR$ be a triangle . Let $\vec{a} = \overline{QR}, \vec{b} = \overline{RP}$ and $\vec{c} = \overline{PQ}$. if $\left|\vec{a}\right| = 12, \left|\vec{b}\right| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$ then which of the following is (are) true ?

A. $\frac{|\vec{c}|^2}{2}$ - $|\vec{a}|$ = 12

B. $\frac{|\vec{c}|^2}{2}$ - $|\vec{a}|$ = 30

C. $|\vec{a} \times \vec{v}b + \vec{c} \times \vec{a}| = 48\sqrt{3}$

D. $\vec{a}.\vec{b}$ = - 72

Answer: a,c,d

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