MATHS

BOOKS - OBJECTIVE RD SHARMA ENGLISH

MEAN VALUE THEOREMS

Illustration

1. Rolle's theorem is not applicable to the function

$f(x) = |x| \text{for} - 2 \le x \le 2$ becase

A. f is continuus on [-2,2]

B. f is not derivable at x=0

C. $f(-2) = f(x)$

D. f is not a constant function

Answer: B

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2. A function is defined by $f(x) = 2 + (x-1)^{2/3} on [0, 2]$. Which of the following is not correct?

A. f is not derivable in (0,2)

B. f is not continuous in [0,2]

C. $f(0) = f(2)$

D. Rolle's theorem is applicable on [0,2]

Answer: D

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3. A function f is defined by $f(x) = e^x \sin x$ in $[0, \pi]$. Which of the following is not correct?

A. f is continuous in $[0, \pi]$

B. f is defferebtiable in $(0, \pi)$

C. $f(0) = f(\pi)$

D. Rolle's theorme is not applicable to f x on

$[0, \pi]$

Answer: D

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4. $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$

A. 0

B. -1

C. -2

D. -3

Answer: C

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5. If f(x) satisfies the condition for Rolle's heorem on [3,5] then $\int_3^5 f(x)$ dx equals

A. 2

B. -1

C. 0

D. $-4/3$

Answer: D

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6. If $2a + 3b + 6c = 0,$ then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

A. at least one root

B. at most one root

C. no root

D. none of these

Answer: A

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7. Let $f(x) = e^x, x \in [0, 1]$, then a number c of the Largrange's mean value theorem is

A. $\log_e(e - 1)$

B. $\log_e(e + 1)$

C. $\log_e e$

D. none of these

Answer: A

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8. If $0 < a < b < \frac{\pi}{2}$ and $f(a, b) = \frac{\tan b - \tan a}{b - a}$, then

A. $f(a, b) \geq 2$

B. $f(a, b) > 2$

C. $f(a, b) \leq 2$

D. none of these

Answer: D

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Section I Solved Mcqs

1. The value of c prescribed by Largrange's mean value . Theorem, when $f(x) = \sqrt{x^2 - 4}, a = 2 \text{ and } b = 3$ is

A. 2.5

B. $\sqrt{5}$

C. $\sqrt{3}$

D. $\sqrt{3} + 1$

Answer: B

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2. The value of c in Rolle's theorem when $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [1/2, 3]$ is

A. 2

B. $-\frac{1}{3}$

C. -2

D. $\frac{2}{3}$

Answer: A

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3. If $a+b+c=0$, then, the equation $3ax^2+2bx+c=0$ has , in the interval (0,1).

A. at least one root

B. at most one root

C. no root

D. none of these

Answer: A

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4. Let a, b, c be nonzero real numbers such that

$$\int_0^1 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

$$=\int_0^2 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx=0$$ Then

show that the equation $ax^2+bx+c=0$ will have one root between 0 and 1 and other root between 1 and 2.

A. one root between 0 and 1 and other root between 1 and 2

B. both roots between 0 and 1

C. both the roots between 1 and 2

D. none of these

Answer: A

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5. If $27a + 9b + 3c + d = 0$ then the equation $4ax^3 + 3bx^2 + 2cx + d$ has at leat one real root lying between

A. 0 and 1

B. 1 and 3

C. 0 and 3

D. none of these

Answer: C

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6. Which of the following is/are correct? Between any two roots of $e^x \cos x = 1$, there exists at least one root of $\tan x = 1$. Between any two roots of $e^x \sin x = 1$, there exists at least one root of $\tan x = -1$. Between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x = 1$. Between any two roots of $e^x \sin x = 1$, there exists at least one root of $e^x \cos x = 1$.

A. at least one root

B. at most one root

C. exuctly one root

D. no root

Answer: A

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7. If the functions f(x) and g(x) are continuous on [a,b] and differentiable on (a,b) then in the interval (a,b) the equation

$$\begin{vmatrix} f'(x) & f(a) \\ g'(x) & g(a) \end{vmatrix} = \frac{1}{a-b} = \begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix}$$

A. has at least one root

B. has exactly one root

C. has at most one root

D. no root

Answer: A

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8. Let f be a function which is continuous and differentiable for all real x. If $f(2) = -4$ and $f'(x) \geq 6$ for all $x \in [2, 4]$, then

A. $f(4) < 8$

B. $f(4) \geq 8$

C. $f(4) \geq 2$

D. none of these

Answer: B

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9. The value of C (if exists) in Lagrange's theorem for the function |x| in the interval [-1,1] is

A. 0

B. $1/2$

C. $-1/2$

D. non-existent in the internal

Answer: D

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10. The equation $\sin x + x \cos x = 0$ has at least one root in

A. $(-\pi/2, 0)$

B. $(0, \pi)$

C. $(-\pi/2, \pi/2)$

D. none of these

Answer: B

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11. Let $f(x)=ax^5+bx^4+cx^3+dx^2+ex,$ where a,b,c,d,e in R and $f(x)=0$ has a positive root. α. Then,

A. f'(x)=0 has a root α_1 such that $0 \leq \alpha_1 \leq \alpha_0$

B. f'(x)=0 has at leat one real root

C. f'(x)=0 has at least two real roots

D. all of the above

Answer: D

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12. If $f''(x) \leq 0$for all$x \in (a, b)$ then f'(x)=0

A. exactly once in (a,b)

B. at most once in (a,b)

C. at leat once

D. none of these

Answer: B

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13. In $[0, 1]$ Lagrange's mean value theorem is not applicable to

A. $f(x)\begin{cases} \frac{1}{2}-x & x<\frac{1}{2} \\ \left(\frac{1}{2}-x\right)^2 & x\geq\frac{1}{2} \end{cases}$

B. $f(x) = \{\left(\frac{\sin x}{x}, x \neq 0\right), (1, x = 0):\}$

C. f(x)=x|x|

D. f(x)=|x|

Answer: A

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14. It is given that the Rolles theorem holds for the function $f(x) = x^3 + bx^2 + cx, \quad x \in [1, 2]$ at the point $x = \frac{4}{3}$. Find the values of b and c.

A. $b = 8, c = -5$

B. $b = -5, c = 8$

C. $b = 5, c = -8$

D. $b = -5, c = -8$

Answer:

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15. Let (x) satisfy the required of Largrange's Meahn value theorem in [0,3]. If $f(0) = 0$ and $|f'(x)| \leq \frac{1}{2}$ for all $x \in [0, 2]$ then

A. $f(x) \leq 2$

B. $|f(x)| \leq 1$

C. $f(x) = 2x$

D. f(x)=3 for at least one 'x in [0.2]

Answer: B

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16. If $f(x)$ satisfies the condition of Rolles theorem in [1, 2] then $\int_1^2 f'(x)dx$ is equal to (A) 1 (B) 3 (C) 0 (D) none of these

A. 3

B. 0

C. 1

D. 2

Answer: B

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17. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval [1,3] and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

A. $a = -11$

B. $b = -6$

C. $a = 6$

D. $a = 11$

Answer: D

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18. If $f(x) = x^{\alpha} \log x$ and $f(0) = 0$, then the value of $'\alpha'$ for which Roole's theorem can be applied in [0, 1], is

A. -2

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D

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19. A value of C for which the coclusion of mean value theorem holds for the function f(x) $\log_e x$ on the interval $[1, 3]$ is

A. $2 \log_3 e$

B. $\frac{1}{2} \log_3$

C. $\log_3 e$

D. $\log_e 3$

Answer: A

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20. For a twice differentiable function $f(x), g(x)$ is defined as $g(x) = f'(x)^2 + f'(x) f(x) on [a, e]$. If for `a

A. 7

B. 4

C. 6

D. 3

Answer: C

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21. Let f be two differentiable function satisfying $f(1) = 1, f(2) = 4, f(3) = 9$, then

A. f''(x)=2 for all `x in R

B. $f'(x)5 = f''(x,)f$ or $some$ x in[1,3]`

C. there exists at least one $x \in (1, 3)$ such that

f''(x)=2

D. none of these

Answer: C

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22. Let $f: [0, 4] \in R$ be acontinuous function such that $|f(x)| \leq 2$for all$x \in [0, 4]$ and $\int_0^4 f(t) = 2.$

Then, for all $x \in [0, 4]$, the value of $\int_0^k f(t)$ dt lies

in the in the interval

A. $[-6 + 2x, 10 - 2x]$

B. $[-12 + 2x, \ -7 + 2x]$

C. $[11 - 2x, 17 + 2x]$

D. $[-8 - 2x, 6 - 2x]$

Answer: A

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23. If $f(x) = (x-p)(x-q)(x-r)$ where $p < q < \ < r,$ are real numbers, then the application, of Rolle's theorem on f leasds to

A. $(p+q+r)(pq+qr+rp) = 3$

B. $(p+q+r)^2 = 3(pq+qr+rp)$

C. $(p+q+r)^2 > 3(pq+qr+rp)$

D. $(p+q+r)^2 < 3(pq+qr+rp)$

Answer: C

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24. Let $f, g: [-1, 2]\overrightarrow{R}$ be continuous functions which are twice differentiable on the interval $(-1, 2)$. Let the values of $f and g$ at the points $-1, 0 and 2$ be as given in the following table: , $x = -1$, $x = 0$, $x = 2$ $f(x)$, 3, 6, 0 $g(x)$, 0, 1, -1 In each of the intervals $(-1, 0) and (0, 2)$ the function $(f - 3g)''$ never vanishes. Then the correct statement(s) is (are) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)$. $f'(x) - 3g'(x) = 0$ has exactly one solutions in $(-1, 0)$. $f'(x) - 3g'(x) = 0$ has exactly one solutions in $(-1, 2)$. $f'(x) - 3g'(x) = 0$ has exactly two solutions in $(-1, 0)$ and exactly two solutions in $(0, 2)$.

A. $f'(x) - 3g, (x) = 0$ has exctly three solution in $(-1, 0) \cup (0, 2)$

B. $f(x) - 3g'(x)$=0 has exactly one solution in (-1,0)

C. f'(x)-3g'(x)=0 has exactly one solution in (0,2)

D. f'(x)-3g'(x)=0 has exactly one solution in (-1,0) and exactly one solution in (0,2)

Answer: D

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Section Ii Assertion Reason Type

1. Prove that the equation $3x^5 + 15x - 18 = 0$ has exactly one real root.

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2. Statement-1 : If f is differentiable on an open interval (a,b) such that $|f'(x) \leq M \text{for all} x \in (a, b)$, then $|f(x) - f(y)| \leq M|x - y| \text{for all} \in (a, b)$

Satement-2: If f(x) is a continuous function defined on [a,b] such that it is differentiable on (a,b) then exists $c \in$ (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

A. Statement-1 is True, Statement-2 is Ture, Statement-2 is a correct explanation for statement-1

B. Statement-1 is True, Statement-2 is Ture, Statement-2 is not a correct explanation for statement-1

C. Statement-1 is True, Statement-2 is False

D. Statement-1 is False, Statement-2 True.

Answer: A

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3. Statement-1 : There is no value ofb k for which the equaiton $x^3 - 3x + k = 0$ has two distinct roots between 0 and 1 .

Statement-2: $x > \sin x \text{for all} x > 0$

A. Statement-1 is True, Statement-2 is Ture, Statement-2 is a correct explanation for statement-1

B. Statement-1 is True, Statement-2 is Ture, Statement-2 is not a correct explanation for statement-1

C. Statement-1 is True, Statement-2 is False

D. Statement-1 is False, Statement-2 True.

Answer: B

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4. Statement-1: The equation $e^{x-1} + x - 2 = 0$ has only one real root.

Statement-2 : Between any two root of an equation f(x)=0 there is a root of its derivative f'(x)=0

A. Statement-1 is True, Statement-2 is Ture, Statement-2 is a correct explanation for statement-1

B. Statement-1 is True, Statement-2 is Ture, Statement-2 is not a correct explanation for statement-1

C. Statement-1 is True, Statement-2 is False

D. Statement-1 is False, Statement-2 True.

Answer: A

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Exercise

1. Let a and b be two distinct roots of a polynomial equation f(x) =0 Then there exist at least one root lying between a and b of the polynomial equation

A. f(x)

B. f'(x)

C. f''(x)

D. none of these

Answer: B

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2. If $2a + 3b + 6c = 0$, then prove that at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval (0,1).

A. (0,1)

B. (1,2)

C. (2,3)

D. none of these

Answer: A

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3. Let $f(x) and g(x)$ be two functions which are defined and differentiable for all $x \geq x_0$. If $f(x_0) = g(x_0) and f'(x) > g'(x)$ for all $x > x_0$, then prove that $f(x) > g(x)$ for all $x > x_0$.

A. $f(x) < g(x)$ for some $x > x_0$

B. $f(x) = g(x)$ for some $x > x_0$

C. $f(x) > g(x)$ for some $x > x_0$

D. none of these

Answer: C

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4. Let f be differentiable for all x, If $f(1) = -2$ and $f'(x) \geq 2$ for all $x \in [1, 6]$, then find the range of values of $f(6)$.

A. $f(6) = 5$

B. $f(6) < 5$

C. $f(6) < 5$

D. $f(6) > 8$

Answer: D

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5. If the function $f(x) = x^3 - 6x^2 + ax + b$ defined on [1,3] satisfies Rolles theorem for $c = \frac{2\sqrt{3} + 1}{\sqrt{3}}$ then find the value of $a and b$

A. $a = 11, b = 6$

B. $a = -11, b = 6$

C. $a = 11, b \in R$

D. none of these

Answer: C

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6. Let

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + + \frac{a_{n-1}}{2} + a_n = 0.$$

Show that there exists at least real x between 0 and 1 such that

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \quad + a_n = 0$$

A. at least one zero

B. at most one zero

C. only 3 zeros

D. only 2 zeros

Answer: A

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7. The number of values of k for which the equation $x^3 - 3x + k = 0$ has two distinct roots lying in the interval $(0, 1)$ is (a) three (b) two (c) infinitely many (d) zero

A. three

B. two

C. infinitely many

D. no value of k satifies the requirement

Answer: D

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8. if f(x)=(x -4) (x-5) (x-6) (x-7) then, (A) f'(x) =0 has four roots (B) three roots f'(x) =0 lie in (4,5) $\cup (5, 6) \cup (6, 7)$ (C) the equation $f'(x) = 0$ has only one real root. (D) three roots of f'(x) =0 lie in (3,4) $\cup (4, 5) \cup (5, 6)$

A. f'(x)=0 has four roots

B. three roots of $f'(x) = 0$ line in (4,5) $\cup (5, 6) \cup (6, 7)$

C. the equation $f'(x) = 0$ hs only one root

D. three roots of $f'(x) = 0$ line $\in (3, 4) \cup (4, 5) \cup (5, 6)$

Answer: B

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9. Let $f and g$ be differentiable on [0,1] such that $f(0) = 2, g(0), f(1) = 6 and g(1) = 2.$ Show that there exists $c \in (0, 1)$ such that $f'(c) = 2g'(c)$.

A. 1

B. 2

C. -2

D. -1

Answer: B

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10. If the equation $a_n x^n + a_{n-1}x^{n-1} + .. + a_1 x = 0, a_1 \neq 0, n \geq 2$, has a positive root $x = \alpha$ then the equation $n a_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root which is

A. a positive root less than α

B. a positive root larger than α

C. a negative root

D. no positive root

Answer: A

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11. The equation $x \log x = 3 - x$ has, in the interval (1,3) :

A. exactly one root

B. at most one root

C. at least one root

D. no root

Answer: C

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12. If f(x) and g(x) ar edifferentiable function for $0 \leq x \leq 1$ such that $f(0) = 2, g(0) = 0, f(1) = 6, g(1) = 2,$ then in the interval (0,1)

A. $f'(x) = 0$ for all x

B. $f'(x) = 2g'(x)$ for at leaset one x

C. $f'(x) = 2g'(x)$ for at most one x

D. none of these

Answer: B

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13. If $\alpha\beta(\alpha < \beta)$ are two distinct roots of the equation. $ax^2 + bx + c = 0$, then

A. $\alpha > -\frac{b}{2a}$

B. $\beta < -\frac{b}{2a}$

C. $\alpha < -\frac{b}{2a} < \beta$

D. $\beta < -\frac{b}{2a} < \alpha$

Answer: C

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14. If (x) is a function given by

$$f(x) = \begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix} \text{ where} 0 < a < b < \frac{\pi}{2}$$

Then the equatiion f'(x)=0

A. has at least one root in (a,b)

B. has at most one root in (a,b)

C. has exactly one root in (a,b)

D. has no root in (a,b)

Answer: A

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15. The value of c in Lagrange's theorem for the functin f(x)=log sin x in the interval $[\pi/6, 5\pi/6]$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. none of these

Answer: B

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16. n is a positive integer. If the value of c presecribed in Rolle's theorem for the function $f(x) = 2x(x-3)^n$ on the interaval [0,3] is 3/4, then the value of n, is

A. 5

B. 2

C. 3

D. 4

Answer: C

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17. The distance travelled by a particle upto tiem x is given by $f(x) = x^3 - 2x + 1$. The time c at which at velocity of the particle is equal to its average velocity between times x=-1 and x=2 sec. is

A. 15 sec

B. $\sqrt{\frac{3}{2}}$ sec

C. $\sqrt{3}$ sec

D. $\sqrt{\frac{7}{3}}$ sec

Answer: C

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18. The number of real roots of the equation $e^{x-1} + x - 2 = 0$ is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: A

19. If the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_0 = 0, n$ being a positive integer, has two different real roots a and b. then between a and b the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots\ldots + a_1 = 0$ has

A. exactly one root

B. almost one root

C. at least one root

D. no root

Answer: C

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20. If $4a + 2b + c = 0$, then the equation $3ax^2 + 2bx + c = 0$ has at least one real lying in the interval

A. (0,1)

B. (1,2)

C. (0,2)

D. none of these

Answer: C

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21. For the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$, the value of c for mean value theorem is

A. 1

B. $\sqrt{3}$

C. 2

D. none of these

Answer: B

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22. If from Largrange's mean value theorem, we have $f'(x(1)) = \frac{f'(b) - f(a)}{b - a}$ then,

A. $a < x_1 \leq b$

B. $a \leq x_1 < b$

C. $a < x_1 < b$

D. $a \leq x_1 \leq b$

Answer: C

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23. Rolle's theorem is applicable in case of $\phi(x) = a^{\sin x}, a > 0$ in

A. any interval

B. the interval $[0, \pi]$

C. the interval $(0, \pi / 2)$

D. none of these

Answer: B

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24. The value of c in Rolle's theorem when $f(x) = 2x^3 - 5x^2 - 4x + 3, x \in [1/2, 3]$ is

A. 2

B. $-1/3$

C. -2

D. $2/3$

Answer: A

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25. When the tangent the curve y=x log (x) is parallel to the chord joining the points (1,0) and (e,e) the value of x , is

A. $1/1-e$

B. $e^{(e-1)(2e-1)}$

C. $e^{\frac{2e-1}{e-1}}$

D. $\dfrac{e-1}{e}$

Answer: A

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26. The value of c in Rolle's theorem for the function $f(x)=\frac{x(x+1)}{e^{x}}$ defined on $[-1,0]$ is

A. 0.5

B. $\frac{1+\sqrt{5}}{2}$

C. $\frac{1-\sqrt{5}}{2}$

D. -0.5

Answer: C

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27. The value of c in Lgrange's Mean Value theorem for the function f(x) =x(x-2) in the interval [1,2] is

A. 1

B. $1/2$

C. $2/3$

D. $3/2$

Answer: D

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28. In Rolle's theorem the value of c for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

A. 1

B. -1

C. $3/2$

D. $1/3$

Answer: A

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