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## PHYSICS

## BOOKS - CENGAGE PHYSICS (ENGLISH)

## BASIC MATHEMATICS

Illustration

1. Calculate $(1001)^{1 / 3}$.
A. 10.00333
B. 10
C. 10.0333
D. 100

Answer: A

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2. Expand $(1+x)^{-3}$.

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## 3. The value of acceleration due to gravity $(g)$

at height $h$ above the surface of earth is given
by
$g^{\prime}=\frac{g R^{2}}{(R+h)^{2}}$. If $h \ll R$, then prove that
$g^{\prime}=g\left(1-\frac{2 h}{R}\right)$.

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4. Convert $45^{\circ}$ to radians.
A. $\frac{\pi}{4} r a d$
B. $\frac{\pi}{3} r a d$
C. $\frac{\pi}{6} \mathrm{rad}$
D. None of Above

Answer: A

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## 5. Convert $\pi / 6 \mathrm{rad}$ to degrees.

A. $30^{\circ}$
B. $80^{\circ}$
C. $60^{\circ}$

## D. $45^{\circ}$

## Answer: A

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6. Given that $\sin 30^{\circ}=1 / 2$ and
$\cos 30^{\circ}=\sqrt{3} / 2$. Determine the values of
$\sin 60^{\circ}, \sin 120^{\circ}, \sin 240^{\circ}, \sin 300^{\circ}, \quad$ and
$\sin \left(-30^{\circ}\right)$.

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7. Find the value of $\sin ^{-1} 1$.

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8. Find the value of $\cos ^{-1}(-1 / 2)$.

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9. Consider two points $P_{1}(2,7)$ and $P_{2}(6,15)$.

Write the equation and draw a straight line through these points.
A. $y=2 x+3$
B. $y=2 x+5$
C. $y=x+3$
D. $2 y=2 x+3$

Answer: A

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10. Plot the line $2 x-3 y=12$.
11. Plot the line $-3 x-5 y=15$.

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12. Plot a graph for the equation
$y=-x^{2}+4 x-1$.

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13. Plot a graph for the equation $y=x^{2}-4 x$.
14. A particle starts with uniform acceleration.

Draw a graph taking the displacement(s) of the particle along $y$-axis and time( t ) along x axis. What is the curve known as?

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15. A particle starts with some initial velocity with an acceleration along the direction of motion. Draw a graph depicting the variation
of velocity ( v ) along y -axis with the variation of displacement(s) along $x$-axis.

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16. Suppose that the function $F$ is defined for all real numbers $r$ by the formula $f(r)=2(r-1)+3$. Evaluate F at the input values $0,2, x+2$, and $f(2)$.
17. A function $f(x)$ is defined as
$f(x)=x^{2}+3$.
Find
$f(0), F(1), f\left(x^{2}\right), f(x+1)$ and $f(f(1))$.

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18. If $y=x^{5}$, then find $d y / d x$.
A. $5 x^{3}$
B. $4 x^{3}$
C. $4 x^{4}$
D. $5 x^{4}$

## Answer: D

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19. If $y=\frac{1}{x^{10}}=x^{-10}$, then find $d y / d x$.

$$
\begin{aligned}
& \text { A. }-\frac{1}{x^{11}} \\
& \text { B. }-\frac{9}{x^{11}} \\
& \text { C. }-\frac{10}{x^{10}} \\
& \text { D. }-\frac{10}{x^{11}}
\end{aligned}
$$

## Answer: D

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20. If $y=\frac{1}{\sqrt{x}}=(x)^{-1 / 2}$, then find $d y / d x$.
A. $-\frac{1}{2 x^{1 / 2}}$
B. $\frac{1}{2 x^{3 / 2}}$
C. $-\frac{1}{2 x^{3 / 2}}$
D. None of the Above
21. If $y=3 x^{2}+2 x$, then find $d y / d x$.
A. $6 x-2$
B. $6 x$
C. $6 x+1$
D. $6 x+2$

## Answer: D

22. If $y=4 x^{4}+2 x^{3}+\frac{5}{x}+9$, then find $d y / d x$.

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> 23. Find the derivatives of
> $y=\left(x^{2}+1\right)\left(x^{3}+3\right)$.

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24. If $y=[3 x+2][2 x-1]$, then find $\frac{d y}{d x}$.

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25. If $y=\left[2 x^{3}+3\right]\left[2 x^{-3}+1\right]$, then find $\frac{d y}{d x}$.

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26. Find the derivative of $y=\frac{t^{2}-1}{t^{2}+1}$.

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27. If $y=\left[\frac{x^{2}+1}{x+1}\right]$, then find $\frac{d y}{d x}$.

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28. If $y=\frac{\left(x^{2}+2 x\right)}{(3 x-4)}$, then find $\frac{d y}{d x}$.

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29. If $y=\frac{\sin x}{x+\cos x}$, then find $\frac{d y}{d x}$.

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30. Find the derivative of $y=\sin \left(x^{2}-4\right)$.

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31. Find the derivative of $y=\sqrt{x^{2}+1}$.

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32. If $y=\cos ^{2} x$, then find $\frac{d y}{d x}$.

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33. If $y=\cos x^{3}$, then find $\frac{d y}{d x}$.

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34. If $x=a t^{4}, y=b t^{3}$, then find $\frac{d y}{d x}$.

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35. If $f(x)=x \cos x$, find $f^{\prime \prime}(x)$.

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36. Find the minimum and maximum values of
the funciton $y=x^{3}-3 x^{2}+6$. Also find the values of $x$ at which these occur.

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37. The particle's position as a funciton of time
is given as $x=5 t^{2}-9 t+3$. Find out the maximum value of position co-ordinate? Also, plot the graph.
38. The velocity v of a particle is given by the equation $v=6 t^{2}-6 t^{3}$, where v is in $m s^{-1}, t$ is the instant of time in seconds while 6 and 6 are suitable dimensional constants. At what values of $t$ will the velocity be maximum and minimum? Determine these maximum and minimum values of the velocity.

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39. Integrate the following w.r.t. x.
40. $x^{3}$
41. $x-\frac{1}{x}$

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40. Solve:
41. $\int_{0}^{3}\left(a x^{2}+b x+c\right) d x$
42. $\int_{-1}^{1} e^{x} d x$
43. $\int_{-\pi / 2}^{\pi / 2} \cos x d x$
44. $\int_{0}^{10} \sec ^{2}(3 x+6) d x$

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41. Solve the integral $I=\int_{0}^{\pi} \sin ^{2} x d x$.

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42. Solve the integral $I=\int_{\infty}^{R} \frac{G M m}{x^{2}} d x$.

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43. Evaluate $\int \sqrt{1+y^{2}} \cdot 2 y d y$

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44. Evaluate $\int \frac{2 z d z}{\sqrt[3]{z^{2}+1}}$

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45. Calculate the area enclosed under the
curve $f(x)=x^{2}$ between the limits $x=2$
and $x=3$ (figure)

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46. At $t=0$, a body starts from origin with some initial velocity. The displacement $x(m)$
of the body varies with time $t(s)$ as $x=-(2 / 3) t^{2}+16 t+2$. Find the initial velocity of the body and also find how long does the body take to come to rest? What is the acceleration of the body when it comes to rest?

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47. Sita is driving along a staight highway in
her car. At time $t=0$, when Sita is moving at
$10 \mathrm{~ms}^{-1}$ in the positive $x$-direction, she passes
a signpost at $x=50 \mathrm{~m}$. Here acceleration is a
function of time:
$a=2.0 m s^{-2}-\left(\frac{1}{10} m s^{-3}\right) t$
a. Derive expressions for her velocity and position as functions of time.
b. At what time is her velocity greatest?
c. What is the maximum velocity?
d. Where is the car when it reaches the maximum velocity?

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48. A particle starts moving and its displacement after $t$ seconds is given in meter by the relation $\quad x=5+4 t+3 t^{2}$. Calculate the magnitude of its
a. Initial velocity
b. Velocity at $t=3 s$
c. Acceleration
49. Let the instantaneous velocity of a rocket,
just after launching, be given by the expression $v=2 t+3 t^{2}$ (where v is in $m s^{-1}$ and $t$ is in seconds). Find out the distance travelled by the rocket from $t=2 s$ to $t=3 s$.

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50. A particle moves with a constant acceleration $a=2 m s^{-2}$ along a straight line.

If it moves with an initial velocity of $5 \mathrm{~ms}^{-1}$,
then obtain an expression for its
instantaneous velocity.

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51. In the previous problem, if the particle occupies a position $x=7 m$ at $t=1 s$, then obtain an expression for the instantaneous displacement of the particle.
52. A police jeep, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the jeep is
0.6 km north of the intersection and the car is
0.8 km to the east, the police determine with radar that the distance between them and the car is increasing at $20 \mathrm{kmh}^{-1}$. If the jeep is moving at $60 \mathrm{kmh}^{-1}$ at the instant of measurement, what is the speed of the car?
53. From point $A$ located on a highway (figure) one has to get by car as soon as possible to point $B$ located in the field at a distance $l$ from the highway. It is known that the car moves in the field $\eta$ times slower than on the highway.

At what distance from point $D$ one must turn

3. Two particles, 1 and 2 , move with constant velocities $v_{1}$ and $v_{2}$ along two mutually perpendicular straight lines toward the intersection point O . At the moment $t=0$ the particles were located at the distances $l_{1}$ and $l_{2}$ from the point O . How soon will the distance between the particles become the smallest? What is it equal to?

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4. Two bodies start moving in the same straight line at the same instant of time from
the same origin. The first body moves with a constant velocity of $40 \mathrm{~ms}^{-1}$, and the second starts from rest with a constant acceleration of $4 m s^{-2}$. Find the time that elapses before the second catches the first body. Find the also the greatest distance between them prior to it and time at which this occurs.
5. Water pours out rate of $Q$ from a tap, into a cylindrical vessel of radius $r$. The rate at which
the height of water level rises the height is $h$, is

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6. Using the method of integration show that
the area of triangle of base $b$ and altitude $h$ is $\frac{1}{2} b h$.
7. Using the method of integration, show that the volume of a right circular cone of base radius r and height h is $V=\frac{1}{3} \pi r^{2} h$.

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8. An experiment on the take off performance
of an aeroplane shows that the acceleration
vales as shown in (figure) and that it takes $12 s$
to take off from a rest position.
a. Write the acceleration vs. time, velocity vs.
time and position vs. time relations for
complete journey.
b. Plot velocity vs. time relation for the motion.
c. Find the distance along the run way covered by the aeroplane.

9. You are given a rod of length L. The linear mass density is $\lambda$ such that $\lambda=a+b x$. Here
$a$ and $b$ are constants and the mass of the rod increases as $x$ decreases. Find the mass of the rod

10. Expand $(1+x)^{-2}$.

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2. Using binomial expansion, simplify the
expression $Q\left[\left(1+\frac{\Delta x}{x}\right)^{3}-1\right]$, assuming
$\Delta x$ to be small in comparison to x .

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1. Plot the lines: (a) $3 x+2 y=0$,
$x-3 y+6=0$

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2. If a particle starts moving with initial velocity $\quad u=1 m s^{-1} \quad$ and $\quad$ acceleration $a=2 m s^{-2}$, the veloctiy of the particle at any
time is given by $v=u+a t=1+2 t$. Plot the velocity-time graph of the particle.
3. A particle starts moving with initial velocity
$u=25 m s^{-1}$ and retardation $a=-2 m s^{-2}$.

Draw the velocity-time graph.

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Exercise 23

1. Find the vertex of the following quardratic equations:
a. $y=x^{2}-8 x$
b. $y=-2 x^{2}+3$
c. $y=x^{2}-6 x+4$

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2. If a particle starts moving along $x$-axis from
the origin with initial velocity $u=1 \mathrm{~ms}^{-1}$ and acceleration $a=2 m s^{-2}$, the relationship between displacement and time is
$x=u t+\frac{1}{2} a t^{2}=1 \times t+\frac{1}{2} \times 2 \times t^{2}=t+r^{2}$
Draw the displacement (x)-time (t) graph.

## Exercise 24

1. Differentiate the following w.r.t.x.
a. 9
b. $\pi^{4}$
c. $2 e^{3}$ d. $x^{2}+5$
e. $(x+5)^{-1 / 2}$ f. $5 x^{3 / 2}$
g. $\sqrt{x+3}$
h. $\left(2 x^{2}+9\right)^{3}$
2. Differentiate the following w.r.t. $x$.
a. $\left(x^{2}+3 x\right)(2 x+7)$
b. $\left(3 x^{2}+2\right)\left(4 x-3 x^{3}\right)$
c. $\sqrt{x}\left(x^{3}+x^{2}-3 x\right)$
d. $\sin x \log x$

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3. Differentiate the following w.r.t. x.
a. $\tan ^{3} x$ b. $\tan x^{2}$ c. $\sin ^{2} \sqrt{x}$

# 4. <br> Find <br> $\frac{d y}{d x}$, <br> when 

$x=a(\theta+\sin \theta) a n d y=a(1-\cos \theta)$

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5. A particle starts from rest and its angular displacement (in rad) is given $\theta=\frac{t^{2}}{20}+\frac{t}{5}$.

Calculate the angular velocity at the end of $t=4 s$.
6. A metallic disc is being heated. Its area A (in
$m^{2}$ ) at any time t (in second) is given by
$A=5 t^{2}+4 t+8$. Calculate the rate of increase in area at $t=3 s$.

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Exercise 25

1. Calculate:
a. $\int\left(x^{2}+2\right) d x$, b. $\int\left(x^{3}-\frac{1}{x}+3 x\right) d x$
c. $\int\left(x^{3 / 2}-x^{1 / 2}+5\right) d x$, d. $\int\left(e^{2 x}+5\right) d x$

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2. Evaluate:
a. $\int_{1}^{2} x^{3} d x$, b. $\int_{u}^{v} m v d v$
c. $\int_{3}^{4}\left(\frac{1}{x}\right) d x$, d. $\int_{4}^{9} \sqrt{x} d x$
e. $\int_{0}^{\pi / 4} \cos 2 x d x$

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1. The displacement of a particle is given by
$y=\left(6 t^{2}+3 t+4\right) m$, where t is in seconds.

Calculate the instantaneous speed of the particle.

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2. The velocity of a particle is given by $v=12+3\left(t+7 t^{2}\right)$. What is the acceleration of the particle?
3. A particle starts from origin with uniform acceleration. Its displacement after t seconds
is given in meter by the relation
$x=2+5 t+7 t^{2}$. Calculate the magnitude of its
a. Initial velocity
b. Velocity at $t=4 \mathrm{~s}$
c. Uniform acceleration
d. Displacement at $t=5 \mathrm{~s}$
4. The acceleration of a particle is given by $a=t^{3}-3 t^{2}+5$, where $a$ is in $m s^{-2}$ and t is in second. At $t=1 s$, the displacement and velocity are 8.30 m and $6.25 \mathrm{~ms}^{-1}$, respectively.

Calculate the displacement and velocity at $t=2 s$.

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5. A particle starts moving along the $x$-axis
from $t=0$, its position varying with time as
$x=2 t^{3}-3 t^{2}+1$.
a. At what time instants is its velocity zero?
b. What is the velocity when it passes through
the origin?

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6. A particle moves along the x-axis obeying
the equation $x=t(t-1)(t-2)$, where x is
in meter and t is in second
a. Find the initial velocity of the particle.
b. Find the initial acceleration of the particle.
c. Find the time when the displacement of the
particle is zero.
d. Find the displacement when the velocity of
the particle is zero.
e. Find the acceleration of the particle when
its velocity is zero.

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7. The speed of a car increases uniformly from
zero to $10 \mathrm{~ms}^{-1}$ in $2 s$ and then remains
constant (figure)

a. Find the distance travelled by the car in the
first two seconds.
b. Find the distance travelled by the car in the next two seconds.
c. Find the total distance travelled in $4 s$.

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8. A car accelerates from rest with $2 m s^{-2}$ for
$2 s$ and then decelerates constantly with
$4 m s^{-2}$ for $t_{0}$ second to come to rest. The graph for the motion is shown in figure.

a. Find the maximum speed attained by the
car.
b. Find the value of $t_{0}$.

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9. A stationary particle of mass $m=1.5 \mathrm{~kg}$ is acted upon by a variable force. The variation of force with respect to displacement is plotted in figure.
a. Calculate the velocity acquired by the particle after getting displaced through $6 m$.
b. What is the maximum speed attained by the
particle and at what time is it attained?


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10. The displacement of a body at any time $t$ after starting is given by $s=15 t-0.4 t^{2}$. Find the time when the velocity of the body will be $7 m s^{-1}$.

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11. A particle moves along a straight line such
that its displacement at any time $t$ is given by
$s=\left(t^{3}-6 t^{2}+3 t+4\right) m$
The velocity when the acceleration is zero, is

## - Watch Video Solution

12. The displacement x of a particle moving in one dimension under the action of a constant
force is related to time $t$ by the equation
$t=\sqrt{x}+3$, where x is in meter and t is in second. Find the displacement of the particle when its velocity is zero.

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13. The position $x$ of a particle varies with time t according to the relation $x=t^{3}+3 t^{2}+2 t$.

Find the velocity and acceleration as functions of time.
14. The displacement of a particle along the $x$ axis is given by $x=3+8 t+7 t^{2}$. Obtain its velocity and acceleration at $t=2 s$.

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15. The acceleration a in $m s^{-2}$ of a particle is given by $a=3 t^{2}+2 t+2$, where t is the time.

If the particle starts out with a velocity
$v=2 m s^{-1}$ at $t=0$, then find the velocity at
the end of $2 s$.
16. The displacement x of a particle along the x -axis at time $t$ is given by $x=\frac{a_{1}}{2} t+\frac{a_{2}}{3} t^{2}$. Find the acceleration of the particle.

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17. A particle moves along a straight line such
that its displacement at any time $t$ is given by
$s=\left(t^{3}-6 t^{2}+3 t+4\right) m$
The velocity when the acceleration is zero, is

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18. The acceleration of $a$ bus is given by $a_{x}(t)=a t$, where $a=1.2 m s^{-2}$.

If the bus's velocity at time $t=1.0 s$ is $5.0 m s^{-1}$, what is its velocity at time $t=2.0 s$ ?
19. The acceleration of a motorcycle is given by
$a_{x}(t)=A t-B t^{2}$, where $A=1.50 m s^{-3}$ and
$B=0.120 \mathrm{~ms}^{-4}$. The motorcycle is at rest at
the origin at time $t=0$.
a. Find its position and velocity as funcitons of time.
b. Calculate the maximum velocity it attains.

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20. The acceleration of a particle varies with
time $t$ seconds according to the relation
$a=6 t+6 m s^{-2}$. Find velocity and position
as funcitons of time. It is given that the particle starts from origin at $t=0$ with velocity $2 m s^{-1}$.
