MATHS

BOOKS - PATHFINDER MATHS (BENGALI ENGLISH)

BINOMIAL THEOREM AND PRINCIPLE OF MATHEMATICAL INDUCTION

Question Bank

1. The number of terms in the expansion of $\left(x+\frac{1}{x}\right)^{29}$ is

A. a) 31

B. b) 30

C. c) 29

D. d) 27

Answer: B

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2. The index of a in the 12th term of the expansion of $(a+2b)^{20}$ is

A. a) 8

B. b) 7

C. c) 9

D. d) 10

Answer: C

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3. The number of terms in the expansion of $(x+y)^5$ is

A. a) 5

B. b) 4

C. c) 6

D. d) 7

Answer: C

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4. The total number of terms in the expansion of $\left(1+2x+x^2\right)^2$ is

A. a) 2

B. b) 3

C. c) 4

D. d) 5

Answer: D

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5. In the expansion of $(1+x)^n$, coefficient of rth term from end is

A. $\hat{}\ nC_r$

B. $\hat{}\ nC_{n-r}$

C. $\hat{}\ nC_{r+1}$

D. $\hat{}\ nC_{n-r+1}$

Answer: D

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6. The middle term in $(2x - 3y)^{12}$ is

A. 6th term

B. 7th term

C. 5th term

D. 8th term

Answer: B

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7. If n is a positive integer, then $\hat{} \ {}^{n}C_1 + {}^{n}C_2 + \ldots + {}^{n}C_n$ is equal to

A. 2^n

B. $2^n - 1$

C. 2^{n-1}

D. $1 - 2^n$

Answer: B

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8. if the coefficients of x^7 & x^8 in the expansion $\left(2+\frac{x}{3}\right)^n$ are equal then n is equal to

A. 56

B. 55

C. 15

D. 45

Answer: B

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9. The coefficient of x^{-10} in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{10}$ is

A. -252

B. 210

C. $-(5!)$

D. -210

Answer: B

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10. If a_1, a_2 are the coefficients of x^n in the expansion of $(1+x)^{2n}$ & $(1+x)^{2n-1}$ respectively then $a_1 : a_2$ will be

A. 2:1

B. 1:2

C. 1:1

D. 1:3

Answer: A

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11. The coefficient of x^{10} in the expansion of $1+(1+x)+(1+x)^2+\ldots..+(1+x)^{20}$ is

A. $\hat{}\ 19C_9$

B. $\hat{}\ 20C_{10}$

C. $\hat{}\ 21C_{11}$

D. $\hat{}\ 22C_{12}$

Answer: C

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12. The sum of the coefficients of the terms of the expansion of $(3x - 2y)^n$ is

A. 2^n

B. 1

C. $2^n - 1$

D. 2^{n-1}

Answer: B

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13. The coefficient of the middle term of the expansion of $\left(1-2x+x^2\right)^n$ is

A. $\frac{2n!}{n^2!}$

B. $\frac{2n!}{n!^2}(-1)^n$

C. $\frac{(2n+1)!}{(n+1)!(n-1)!}(-1)^{n+1}$

D. $\frac{(2n+1)!}{(n+1)^2!}$

Answer: B

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14. The sum of the coefficients in the expansion of $\left(1-2x+2x^2\right)^{2014}$ is

A. 1

B. 0

C. -1

D. 2

Answer: A

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15. The middle term of the expansion of $(4x + 5y)^{18}$ is

A. a) 9th term

B. b) 10th term

C. c) 11th term

D. d) 12th term

Answer: B

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16. The value of $\left(\hat{} \ {}^{8}C_1 + {}^{8}C_2 + {}^{8}C_3 + \ldots. + {}^{8}C_8 \right)$ is

A. 256

B. 255

C. 257

D. 254

Answer: B

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17. The value upto 3 decimal place of $(0.999)^3$ is (applying Binomial Theorem)

A. 0.999

B. 0.998

C. 0.997

D. 0.995

Answer: C

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18. Find the 10th term of $\left(2x^2 + \frac{1}{x}\right)^{12}$.

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19. If rth term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$ is independent of x, then find the value of r.

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20. Find the middle term in the expansion of $\left(x - \frac{1}{2y}\right)^{10}$.

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21. find the value of $(99)^4$.

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22. Find a, if 17th and 18th terms in the expansions of $(2+a)^{50}$ are equal.

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23. If the 3rd term in the expansion of $\left(\frac{1}{x}+x^{\log_{10} x}\right)^5$ is 1000, then find x.

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24. Determine the constant term in the expansion of $\left(x^3-\frac{1}{x^2}\right)^{15}$.

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25. Find the coefficient of x^{20} in the expression of $(1+x^2)^{40}\left(x^2+2+\frac{1}{x^2}\right)^{-5}$.

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26. Find the term independent of x in the expansion of $\left(\sqrt{x}+\frac{1}{3x^2}\right)^{10}$.

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27. Find the coefficient of x in $(1-2x^3+3x^5)\left(1+x+\frac{1}{x}\right)^{10}$.

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28. In the expansion of $(1+x)^{m+n}$, where m & n are +ve integers, prove that the coefficients of x^m and x^n are equal.

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29. Determine the term independent of x in the expansion of $\left(3x^2 - \frac{1}{2x^3}\right)^{10}$.

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30. Find the coefficient of x^5 in the expression of $\left(1+x^2\right)^5(1+x)^4$.

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31. Which term in the expansion of $(1+x)^p\left(1+\frac{1}{x}\right)^q$ is independent of x, where p,q

are positive integers ?

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32. Determine the coefficient of x^4 in the expansion of $\left(1+x+x^2+x^3\right)^n$.

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33. Determine the x-free term in $\left(\sqrt{x}-\frac{\sqrt{c}}{\sqrt{x}}\right)^{10}$.

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34. Show that the sum of the coefficients of all odd terms in the expansion of $(1+x)^{2p}$ is 2^{2p-1}.

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35. Find the fifth term form the end in the expansion of $\left(\frac{\sqrt{x}}{3}+\frac{2}{5}x\right)^{11}$.

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36. Show that the middle term in the expansion of $(x+1)^{2n}$ is $\frac{1.3.5.\ldots..(2n-1)}{n!}2^n.x^n$.

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37. The first three terms in the binomial expansion of $(x+y)^n$ are 1,56 and 1372 respectively. Find the values of x and y.

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38. If n be a positive integer, then by using binomial theorem show that $3^{2n+2}-8n-9$ is always divisible by 64.

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39. Find the greatest value of the term independent of x in the expansion of $\left(x \sin \alpha + \frac{\cos \alpha}{x}\right)^{10}$, where $\alpha \in R$.

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40. Find the coefficient of x in the expansion $(1 - x^2 + 2x^4)\left(1 - \frac{1}{x}\right)^6$.

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41. If $\left(1+x+x^2\right)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots + a_{2n} x^{2n}$, then prove that $a_0 + a_2 + a_4 + \ldots. + a_{2n} = \frac{1}{2}(3^n + 1)$.

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42. If the coefficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.p., then prove that $2n^2 - 9n + 7 = 0$.

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43. If the coefficients of four consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3 and a_4 respectively. then prove that `a_1/(a_1+a_2)+a_3/(a_3+a_4)=2a_2/(a_2+a_3).

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44. The 3rd,4th and fifth terms in the expansion of $(x+a)^n$ are 252, 1512, and 5670 respectively. Find the values of x,a & n.

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45. The coefficient of three consecutive terms in the expansion of $(1+x)^n$ are a, b, c respectively prove that $\frac{2ac+b(a+c)}{b^2-ac}=n.$

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46. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$.

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47. Show that the integral part of the value of $\left(9+4\sqrt{5}\right)^n$ is odd for positive integer .

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48. If the 3rd, 4th. 5th and sixth term in the expansion of $(x+\alpha)^n$ are a,b,c,d respectively, then prove that $\left(\frac{b^2-ac}{c^2-bd}\right)=\frac{5a}{3c}$.

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49. If coefficient of x^2 and x^{11} are 27 and -192 respectively of $\left(1+ax+2x^2\right)^6$ then show that a=-1.

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50. Find the coefficient of x^5 in the expansion of $(1+x)^{21}+(1+x)^{22}+\ldots+(1+x)^{30}$.

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51. Determine the x-independent term in the expansion of $(1+4x)^p\left(1+\frac{1}{4x}\right)^q$ where p & q are positive integers.

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52. For $n \in N$, $2^{3n} - 1$ is divisible by

A. a) 7

B. b) 8

C. d) 6

D. d) 16

Answer: A

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53. For $n \in N$, $n^3 + 2n$ is divisible by

A. a) 6

B. b) 5

C. c) 4

D. d) 3

Answer: D

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54. For $n \in N$, $3^{2n-1} + 2^{n+1}$ is always divisible by

A. a) 5

B. d) 6

C. c) 7

D. d) 9

Answer: C

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55. For $n \in N 2^{3n} - 7n - 1$ is always divisible by

A. a) 49

B. b) 64

C. c) 36

D. d) 81

Answer: A

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56. The greatest positive integer divides (n+1) (n+2).....(n+r) is

A. a) r

B. b) r!

C. c) (n+r)

D. d) (r+1)

Answer: B

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57. Applying the principle of mathematical induction (P.M.I.) prove that $1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

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58. Using mathematical induction show 7+77+777+.....+n terms $= \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$

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59. Applying P.M.I. prove that $x^n - y^n$ is always divisible by x+y where n is a pos. even integer

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60. Applying the principle mathematical induction (P.M.I.) show that $5^{2n+2} - 24n - 25$ is always divisible by 576 where n is a natural number.

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61. Applying P.M.I. prove that $(1+x)^n$ gt 1+nx $wherenisapos\int e \geq r$ and nge2 :xgt(-1)`

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62. Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ by P.M.I. where n is a pos integer.

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63. For which natural numbers n the inequality $2^n > 2n + 1$is true?

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64. For n being a natural number prove that $1.1! + 2.2! + 3.3! + \ldots . + n.\, n! = (n+1)! - 1$ by applying P.M.I

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65. For $n \in N$, prove that $\left(\frac{n+1}{2}\right)^n >$ n!

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66. Show that $101^{50} > 99^{50} + 100^{50}$

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67. Using P.M.I. prove that $2^n > n$ for all $n \geq 1 : n \in N$

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68. If $n \geq 3$ is an integer prove that $2n + 1 < 2^n$

by P.M.I.

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