MATHS

BOOKS - PATHFINDER MATHS (BENGALI ENGLISH)

LIMIT, CONTINUITY AND DIFFERENTIABILITY

Question Bank

1. Evaluate the right hand limit and left hand limit of the function

$$f(x) = \begin{cases} \frac{|x-4|}{x-4} & x \neq 4 \\ 0 & x = 4 \end{cases}$$

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2. Evaluate $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

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3. Solve

$$\lim_{x \to 1} \left[\sin^{-1} x\right]$$

(where [.] denotes greatest integer function.)

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4. Solve

$$\lim_{x \to 0^+} \left[\frac{\sin x}{x}\right]$$

(where [.] denotes greatest integer function.)

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5. Solve

$$\lim_{x \to 0^-} \left[\frac{\sin x}{x}\right]$$

(where [.] denotes greatest integer function.)

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6. Solve

$$\lim_{x \to 0^+} \left[\frac{\tan x}{x} \right]$$

(where [.] denotes greatest integer function.)

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7. Solve

$$\lim_{x \to 0^-} \left[\frac{\tan x}{x} \right]$$

(where [.] denotes greatest integer function.)

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8. Evaluate $\lim_{x \to 1} \frac{x^{P+1} - (P+1)x + P}{(x+1)^2}$

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9. Evaluate $\lim_{x \to 0} \frac{\tan(2x)}{x}$

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10. Evaluate $\lim_{x \to a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$

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11. The graph of function y=f(x) has a unique tangent at $(e^a, 0)$, through which the graph passes, then

$\lim_{x \to e^a} \frac{\log(1 + 7f(x)) - \sin(f(x))}{3f(x)}$ is equal to

A. 1

B. 2

C. 7

D. none of these

Answer: B

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12. Evaluate

$$\lim_{n \to \infty} \frac{[x] + [2x] + [3x] + \ldots\ldots + [nx]}{n^2}$$

where [.] denotes greatest integer function.

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13. Evaluate $\lim_{x \to 0} \frac{\int_0^{x^2} \cos t dt}{x \sin x}$

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14. Evaluate $\lim_{x \to 0} \frac{x - \int_0^x \cos t^2 dt}{x^3 - 6x}$

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15. Evaluate : $\lim_{x \to 0} \left\{ \tan\left(\frac{\pi}{4} + x \right) \right\}^{\frac{1}{x}}$

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16. If α and β be the roots of the quadratic equation $ax^2 + bx + c = 0$ then evaluate

$$\lim_{x \to \alpha} \frac{1 - \cos\left(ax^2 + bx + c\right)}{(x - \alpha)^2}$$

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17. Let a=min$\{x^2 + 2x + 3, x \in R\}$ and $b = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$. Then the value of $\sum_{r=0}^{n} a^r b^{n-r}$

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18. If $f(x) = \begin{cases} 2x + 3 & \text{when x} < 0 \\ 0 & \text{when x} = 0 \\ x^2 + 3 & \text{when x} > 0 \end{cases}$ Discuss continuity at x=0.

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19. Let y=f(x) be defined parametrically as $y = t^2 + t|t|, x = 2t - |t|, t \in R$, Discuss its continuity.

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20. Let f(x) be a continuous function defined for 1 le x le 3. If f(x) takes rational values for all x and f(2) = 10, then f(1,5) is equal to

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21. Find the points points of discontinuity of $y = \frac{1}{u^2 + u - 2}$ where $u = \frac{1}{x - 1}$

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22. If f(x)=[x], where [.] denotes greatest integer function. Then check the continuity on [1,2]

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23. Let

$$f(x) = \left\{\left(\{1+|\sin x|\}^{a/|\sin x|}, -\pi/6 < x < 0\right), (b, x = 0), \left(e^{\tan 2x/\tan 3}\right.\right.$$

Determine a and b such that f(x) is continuous at x=0

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24. Discuss the continuity of $f(x) = \left[\tan^{-1} x\right]$

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25. Examine the function, $f(x) = \begin{cases} x-1 & x < 0 \\ 1/4 & x = 0 \\ x^2 - 1 & x > 0 \end{cases}$

Discuss the continuity, and if discontinuous remove the discontinuity.

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26. Show the function,

$$f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1} & \text{when x} \leq 0 \\ 0 & \text{when x} = 0 \end{cases}$$

has non-removable discontinuity at x=0

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27. A function f(x) is defined by,

$$\Rightarrow f(x) = f(x) = \begin{cases} \frac{[x^2]-1}{x^2-1} \text{for} x^2 \neq 1 \\ 0 \text{for} x^2 = 1 \end{cases}$$

Discuss the continuity of f(x) at x=1.

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28. Discuss the continuity of the function,

$$f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n}\sin x}{1+x^{2n}}$$ at x=1

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29. Discuss the continuity of f(x) where $f(x) = \lim_{n \to \infty} \left(\sin\frac{\pi x}{2}\right)^{2n}$

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30. If f(x) be continuous function for all real values of x and satisfies, $x^2 + \{f(x) - 2\}x + 2\sqrt{3} - 3 - \sqrt{3}.\ f(x) = 0,\ \forall x \in R$.Then find the value of $f(\sqrt{3})$.

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31. Which of the following functions is differentiable at x=o ?

A. cos (|x|)+|x|

B. cos (|x|)-|x|

C. sin (|x|)+|x|

D. sin (|x|)-|x|

Answer: D

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32. Let

$$f(x) = \begin{cases} x \exp\left[-\left(\frac{1}{|x|} + \frac{1}{x} \right) \right] & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Test whether

f(x) is differentiable at x=0

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33. Let

$$f(x) = \begin{cases} x \exp\left[-\left(\frac{1}{|x|} + \frac{1}{x} \right) \right] & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Test whether

f(x) is differentiable at x=0

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34. The left hand derivative of f(x)=[x]sin (πx) at x=k, k is an integer, is:

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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35. A function $f: R \to R$ satisfies the equation f(x+y)=f(x)f(y) for all $x, y \in R$, $f(x) \neq 0$. Suppose that the function is differentiable at x=0 and f'(0)=2. Prove that f'(x)=2f(x).

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36. Let $f(x) = \begin{cases} -4 & -4 \leq x < 0 \\ x^2 - 4 & 0 \leq x \leq 4 \end{cases}$

Discuss the continuity and differentiability of g(x)=f(|x|)+|f(x)|

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37. Let f(x)=[n+p sin x], $x \in (0, \pi) n \in Z$ and p is a prime number, where [.] denotes the greatest integer function. Then find the number of points where f(x) is not differentiable.

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38. If f(x)=$\{|x| - |x-1|\}^2$, draw the graph of f(x) and discuss its continuity and differentiability of f(x)

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39. If f(x)=|x+1|{|x|+|x-1|}, then draw the graph of f(x) in the interval [-2,2] and discuss the continuity and differentiability in [-2, 2]

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40. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2)\, dt}{x \sin x}$ is

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41. The value of $\lim_{x \to 0} \frac{(1^x + 2^x + 3^x + \ldots + n^x)^{a/x}}{n}$, is:

A. $(n!)^{a/n}$

B. $n!$

C. $a^{n!}$

D. doesn't exists

Answer: A

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42. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{2/x} = e^3$, then the value of a and b, is :

A. $a = \frac{3}{2}, b \in R$

B. $a = \frac{1}{2}, b \in R$

C. $a = R, b \in R$

D. none of these

Answer: A

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43. A function is defined as,

$f(x) = \{(0, wherexisrational), (1, wherexisirrational)\}:.$ Then f(x) is

A. continuous for all $x \in R$

B. continuous for all $x \in R - \{0\}$

C. continuous for all $x \in R - \{0, 1\}$

D. discontinuous for all $x \in R$

Answer: D

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44. If $f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$ and $g(x) = x\left(1 - x^2\right), then$, f(g(x)) is continuous for,

A. R

B. R-{0}

C. R-{0,1}

D. R-{-1,0,1}

Answer: D

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45. If f(x)=-1+|x-2|, $0 \leq x \leq 4$

g(x)=2-|x|, $-1 \leq x \leq 3$

Then, fog(x) is continuous for x belonging to

A. [0,4]

B. [-1.3]

C. [0,3]

D. [-1,2]

Answer: D

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46. Let f(x)=[sinx + cosx], $0 < x < 2\pi$, (where [.] denotes the greatest integer function). Then the number of points of discontinuity of f(x) is :

A. 6

B. 5

C. 4

D. 3

Answer: C

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47. If $f(x) = \begin{cases} \frac{\sin\{\cos x\}}{x - \frac{\pi}{2}} & x \neq \frac{\pi}{2} \\ 1 & x = \frac{\pi}{2} \end{cases}$, where {.} denotes the fractional part of x, then f(x) is :

A. continuous at $x = \frac{\pi}{2}$

B. $\lim_{x \to \frac{\pi}{2}} f(x)$, but f(x) is not continuous at $x = \frac{\pi}{2}$

C. $\lim_{x \to \frac{\pi}{2}}$ does not exists

D. $\lim_{x \to \frac{\pi^-}{2}} f(x) = 1$

Answer: B

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48. If $f(x) = \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}, x \neq 2$

=k, x=2

and if f(x) is continuous at x=2, find the value of k.

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49. In the function

`f(x)=[(x-2)^3/a]sin(x-2)+acos(x-2), (where [.] denotes the greatest integer function) is continuous and differentiable in (4,6), then

A. $a \in [8, 64]$

B. $a \in [0, 8]$

C. $a \in [64, \infty]$

D. none of these

Answer: C

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50. Let f(x) = |x|+|sin x|, $x \in \left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Then, f is :

A. continuous , $\forall x \in R - \{0\}$

B. continuous and differentiable everywhere

C. nowhere differentiable

D. not differentiable at x=0

Answer: D

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51. If f is a periodic function, then

A. f' and f'' are also periodic

B. f' is periodic but f'' is not periodic

C. f'' is periodic but f' is not periodic

D. none of the above

Answer: A

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52. If f(x)=$\left[\sin^2 x\right]$ (where [.] denotes the greatest integer function) then :

A. f is everywhere continuous

B. f is everywhere differentiable

C. f is a constant function

D. none of the above

Answer: D

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53. Let f(x) be a polynomial of degree one and f(x) be a function defined by

$$f(x) = \begin{cases} g(x) & x \leq 0 \\ \frac{1+x}{(2+x)^{1/x}} & x > 0 \end{cases}$$

If f(x) is continuous at x=0 and f(-1)=f'(1), then g(x) is equal to :

A. $-\frac{1}{9}(1+6\log_e 3)x$

B. $\frac{1}{9}(1+6\log_e 3)x$

C. $-\frac{1}{9}(1+6\log_e 3)x$

D. none of these

Answer: A

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54. $\lim_{x \to 0} \frac{e^{\cot x} - e^{\cos x}}{\cot x - \cos x}$ is

A. -1

B. 1

C. 0

D. none of these

Answer: B

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55. value of $\lim_{x \to \infty} \left(\frac{x+1}{x+2} \right)^{2x+1}$ is

A. e^2

B. e^{-2}

C. 1/e

D. e

Answer: B

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56. $\lim_{x \to \infty} \frac{3^x + 4^{x+1} + 5^x}{5^x + 3^{x+1}}$ is

A. $\frac{1}{2}$

B. $\frac{3}{5}$

C. $\frac{4}{3}$

D. 1

Answer: D

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57. $\lim_{x \to \infty} \left[\frac{(2+x)^{40}(4+x)^{5}}{(2-x)^{45}} \right]$ equals

A. -1

B. 1

C. 16

D. 32

Answer: A

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58. $\lim_{x \to 5} \frac{x-5}{|x-5|}$ equals to

A. 2

B. 0

C. -2

D. none of these

Answer: D

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59. $\lim_{x \to \frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3} - x\right)}{2\cos x - 1}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

Answer: B

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60. If $f(x) = \left(\frac{x^2 + 5x + 3}{x^2 + x + 2}\right)^x$ then $\lim_{x \to \infty} f(x)$=

A. e^4

B. e^3

C. e^2

D. 2^4

Answer: A

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61. If $\lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 1} - px - q\right) = 0$, then

A. $p = 0, q = 0$

B. $p = 1, q = -1$

C. $p = -1, q = 1$

D. $p = 2, q = -1$

Answer: B

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62. $The value of$ lim_(xrarrinfty)frac(x^3sin(1/x)-2x^2)(1+3x^2)`is

A. 0

B. $-\frac{1}{3}$

C. -1

D. $-\frac{2}{3}$

Answer: B

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63. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & \text{for}[x] \neq 0 \\ 0 & \text{for}[x] = 0 \end{cases}$ where [x] denotes greatest integer function, then $\lim_{x \to 0} f(x)$ is

A. 1

B. 0

C. -1

D. Limit does not exist

Answer: D

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64. The number of points at which the function $f(x) = \frac{1}{\log}|x|$ is discontinuous is

A. 1

B. 2

C. 3

D. 4

Answer: C

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65. The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is not defined at x =0. The value which should be assigned to f(x) at x=0, So that it is continuous at x=0 is

A. a-b

B. a+b

C. log a+ log b

D. none of these

Answer: B

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66. A function f(x) is defined as $f(x) = \begin{cases} \frac{1-\cos 3x}{x^2} & x \neq 0 \\ b^2+4 & x = 0 \end{cases}$ If f(x) is continuous at x=0, then b equals to

A. $\pm \frac{2}{\sqrt{3}}$

B. $\pm \frac{3}{\sqrt{2}}$

C. $\pm \frac{1}{\sqrt{2}}$

D. $\pm \frac{\sqrt{3}}{2}$

Answer: C

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67. If $f(x) = \begin{cases} (\cos x)^{\frac{1}{\sin x}} & \text{for} x \neq 0 \\ k & \text{for} x = 0 \end{cases}$ The value of K, so that f is continuous at x=0 is

A. 0

B. 1

C. $\frac{1}{2}$

D. none of these

Answer: B

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68. Points of discontinuities of the function $f(x) = 4x + 7[x] + 2\log(1 + x)$, where $[x]$ denotes the integral part of x, is

A. 0

B. 1

C. $-\frac{3}{2}$

D. all of these

Answer: D

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69. The function f(x)=|x|atx=0 is:

A. Continuous but non-differentiable

B. Discontinuous and differentiable

C. Discontinuous and non-differentiable

D. Continuous and differentiable

Answer: A

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70. $g(x) = x|x|then$g''(x)

A. does not exist at x=0

B. is always positive

C. is always non-negative

D. is always non-zero

Answer: A

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71. The function defined by $f(x) = \begin{cases} (\mid x-3) & x \geq 1 \\ \frac{1}{4}x^2 - \frac{3}{2}x + \frac{13}{4} & x < 1 \end{cases}$ is

A. continuous at x=1

B. Continuous at x=3

C. Differentiable at x=1

D. all of these

Answer: D

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72. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5} & f \text{ or } x \neq 1 \\ -\frac{1}{3} & f \text{ or } x = 1 \end{cases}$ then f(1)=

A. $-\frac{1}{9}$

B. $-\frac{2}{9}$

C. $-\frac{1}{3}$

D. $\frac{1}{3}$

Answer: B

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73. Let h(x)=max$\{-x, 1, x^2\}$ for every real x, then number of points of non-differentiability of h (x) is

A. 1

B. 2

C. 3

D. 4

Answer: B

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74. $\lim_{n \to \infty} \frac{n^k \sin^2 n!}{n+1} = 0$ for

A. all k

B. $o \leq k < 1$

C. k=1

D. kgt1

Answer: B

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75. If $f(x) = \frac{\sin\left(2\pi\left[\pi^2 x\right]\right)}{5 + [x^2]}$ denotes the greatest integer function), then f(x) is

A. discontinuous at some x

B. continuous at all x, but the derivative $f'(x)$ doesn't exist for some x

C. $f'(x) \exists f \text{ or } allx, butf''(x)`$ doesn't for some x

D. `f''(x) exists for all x.

Answer: D

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76. If $f(x) = |x - 25|$ and g(x)=f(f(x)) then for $x > 50, g'(x)$ is equal to

A. 0

B. 1

C. 25

D. None of these

Answer: B

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77. Let $f(x) = \left[\tan^2 x\right]$, where [.]denotes the greatest integer function. Then

A. $\lim_{x \to 0} f(x)$ doesn't exist

B. f(x) is continuous at x=0

C. f(x) is not differentiable at x=0

D. $f'(0) = 1$

Answer: B

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78. If f(x+y)=f(x). f(y) for all x and y and f(5)=2, f'(0)=4, then f '(5) will be

A. 2

B. 4

C. 6

D. 8

Answer: D

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79. if $f(x) = \left[\sqrt{2}\sin x\right]$, where [x] denotes the greatest integer function, then

A. f(x) is continuous at x=0

B. maximum value of f(x) is 1 in interval $[-2\pi, 2\pi]$

C. f(x) is discontinuous at $X = \frac{n\pi}{2} + \frac{\pi}{4}, n \in I$

D. f(x) is differentiable at $x = n\pi, n \in I$

Answer: B

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80. f(x)=[x]+|x-1| then f(x), where [.] denotes greatest integer function) is

A. Continuous at x=0

B. not differentiable at x=1/2

C. discontinuous at x=2

D. differentiable at x=-2

Answer: C

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81. If $f(x) = \begin{cases} -x & x \leq 1 \\ 3+x & x > 1 \end{cases}, g(x) = \begin{cases} x^2 & x \leq 1 \\ 2-x & x > 1 \end{cases}$ then $\lim_{x \to 1^+} f(g(x))$ equal to

A. 1

B. -1

C. 5

D. 2

Answer: B

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82. If {x} denotes the fractional part of x, then $\lim_{x \to 0} \frac{e^x - 1}{x}$ is

A. 0

B. 1

C. ∞

D. none of these

Answer: D

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83. Consider $f(x) = \begin{cases} \frac{x^2}{|x|} & x \neq 0 \\ 0 & x = 0 \end{cases}$

A. f(x) is discontinuous everywhere

B. f(x) is continuous everywhere

C. f(x) exists in (-1,1)

D. f(x) exists in (-2,2)

Answer: B

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84. Value of $\lim_{x \to 3} \frac{\sin\left(e^{x-3} - 1\right)}{\log(x-2)}$ is

A. 2

B. 1

C. -1

D. -2

Answer: B

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85. $\lim_{x \to 1} x^{1/1-x}$ is

A. e

B. 1/e

C. 1

D. None of these

Answer: B

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86. $\lim_{x \to 0} \left(\frac{1 + \tan x}{1 - \tan x} \right)^{\frac{1}{x}}$ is

A. 0

B. 1

C. e^2

D. None of these

Answer: C

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87. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is :

A. 2

B. 1

C. 0

D. 3

Answer: B

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88. The value of $\lim_{x \to 0} \frac{|\cos x + \sin x|}{x^2}$ is equal to

A. 0

B. 1

C. $\frac{\pi}{2}$

D. None of these

Answer: A

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89. $\lim_{x \to \infty} \frac{\sqrt[2]{x^2+1} - \sqrt[3]{x^2+1}}{\sqrt[4]{x^2+1} - \sqrt[5]{x^4-1}}$ is equal to

A. 1

B. -1

C. 0

D. None of these

Answer: A

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90. The value of a, b and c such that

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2 \text{ are}$$

A. a=1,b=-2, c=1

B. a=1,b=2, c=-1

C. a=1,b=2, c=1

D. a=-1,b=2, c=1

Answer: C

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91. Let f(x) = lim_(n rarr infty)(sinx)^(2n)`, then f is

A. continuous at $x = \frac{\pi}{2}$

B. discontinuous at $x = (2k+1)\frac{\pi}{2}, k \in Z$

C. continuous at $x = -\frac{\pi}{2}$

D. continuous at infinite number of points

Answer: B

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92. The set of all points where the function f(x)= x/(1+|x|)` is differentiable, is

A. `(- infty, infty)

B. $(0, \infty)$

C. $(-\infty, 0) \cup (0, \infty)$

D. None of these

Answer: A

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93. Let $f(x) = \frac{x(1 + a\cos x) - b\sin x}{x^2}$, $x \neq 0$ and f(0) = 1

A. 5/2, 3/2

B. 5/2, -3/2

C. -5/2, -3/2

D. None of these

Answer: C

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94. If $x + |y| = 2y$, then y as a function of x is :

A. defined for all real x

B. continuous at x = 0

C. differentiable for all x

D. such that $\frac{dy}{dx} = \frac{1}{3}f$ or $x < 0$

Answer: A::B::D

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95. Let [x] denotes the greatest integer less than or equal to x, If (x) = $[x \sin \pi x]$, then f(x) is :

A. continuous at x = 0

B. continuous in (-1, 0)

C. differentiable at x = 1

D. differentiable in (-1, 1)

Answer: A::B::D

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96. The function

$$f(x) = \begin{cases} |x-3| & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$$

A. continuous at x = 1

B. differentiable at x = 1

C. discontinuous at x = 1

D. differentiable at x = 3

Answer: A::B

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97. Which of the following functions are continuous on $(0, \pi)$:

A. tan x

B. $\int_0^x t \sin \frac{1}{t} dt$

C. $f(x) = \begin{cases} 1 & 0 \leq x \leq \frac{3\pi}{2} \\ 2\frac{\sin 2}{9}x & \frac{3\pi}{4} < x < \pi \end{cases}$

D. $f(x) = \begin{cases} x \sin x & 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x) & \frac{\pi}{2} < x\pi \end{cases}$

Answer: B::C

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98. If $f(x) = \min\{1, x^2, x^3\}$ then :

A. f(x) is continuous everyone

B. f(x) is continuous and differentiable everywhere

C. f(x) is not differentiable at two points

D. f(x) is not differentiable at one point

Answer: A::D

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99. Let f(x) be defined in [-2, 2] by

$f(x) = \text{maximum}\left(\sqrt{4-x^2}, \sqrt{1+x^2}, -2 \le x \le 0\right.$

minimum $\left(\sqrt{(4-x^2)}, \sqrt{1+x^2}, 0 < x \le 2\right.$, then f(x)

A. is continuous at all points

B. has a point of discontinuity

C. is not differentiable only one point

D. is not differentiable at more than one point

Answer: B::D

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100. If $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{\lambda}{x}}, (a, b, c, \lambda > o)$ is equal to

A. 1, if $\lambda = 1$

B. abc, if $\lambda = 1$

C. abc, if $\lambda = 3$

D. $(abc)^{\frac{2}{3}}$, if $\lambda = 1$

Answer: C::D

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101. If $f(x) = \left(\frac{|x|}{2+|x|}\right)^{2x}$, then

A. $\lim_{x \to \infty} f(x) = e^{-4}$

B. $\lim_{x \to -\infty} f(x) = e^{4}$

C. $\lim_{x \to -\infty} f(x) = \infty$

D. $\lim_{x \to -\infty} f(x) = 1$

Answer: A::B

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102. Let $f(x) = \begin{cases} 1 + \frac{2x}{\lambda} & o \leq x \leq 1 \\ \lambda x & 1 \leq x \leq 2 \end{cases}$

if $\lim_{x \to 1} f(x)$ exists, then λ is

A. -2

B. -1

C. 1

D. 2

Answer: B::D

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103. If m, n in N, lim x tends to 0 '(sin(x^m)/sin(x^n))' (m, n∈N)=0, If

A. 1, if n = m

B. 0 if $n > m$

C. ∞, if$n < m$

D. $\frac{n}{m}$, if$n < m$

Answer: A::B::C

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104. Let $f(x) = \frac{1-\cos 4x}{x^2}, g(x)\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$ and

$$\phi(x) = \begin{cases} f(x) & x < 0 \\ a & x = 0 \\ g(x) & x > \end{cases}$$

Answer the following question based on above passage :

$\lim_{x \to 0}$ f(x) is equal to

A. 1/2

B. 2

C. 1/8

D. 8

Answer: D

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105. Let $f(x) = \frac{1-\cos 4x}{x^2}, g(x)\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}$ and

$$\phi(x) = \begin{cases} f(x) & x < 0 \\ a & x = 0 \\ g(x) & x > \end{cases}$$

Answer the following question based on above passage :

$\lim_{x \to 0}$ g(x) is equal to

A. 1/8

B. 8

C. 2

D. 1/2

Answer: B

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106. Let $f(X) = \lfloor x \rfloor - \lceil x \rceil$ for all $x \in R$

Answer the following question based on above passage :

$\lim_{x \to 0} f(x)$=

A. 0

B. 1

C. -1

D. none of these

Answer: C

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107. Let $f(X) = \lfloor x \rfloor - \lceil x \rceil$ for all $x \in R$

Answer the fllowing question based on above passage :

Domain of continuity of f(x) is

A. R

B. R-1

C. 1

D. none of these

Answer: B

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108. Match List - I with List-II

List - I		List - II	
(1)	Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(P)	4
(2)	If an edge of a cube increases by 1%, then percentage increase in volume is	(Q)	0.6π
(3)	If the rate of decrease of $\frac{x^2}{2}-2x+5$ is twice the rate of decrease of x, then x is equal to (rate of decreases is non-zero)	(R)	3
(4)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(S)	$\frac{3\sqrt{3}}{4}$

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109. Match List - I with List-II

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(4)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(S)	$\frac{3\sqrt{3}}{4}$

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110. Let f and g be two continuous and let h be defined as

$$h(x) = \lim_{n \to \infty} \frac{x^{2n} f(x)_x^{2m} g(x)}{1 + x^{2n}}$$

where m is a fixed positive integer

If lim_(x rarr 1) h(x)` exists then a real root of f(x) - g(x) = 0 is

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111. If $\lim_{x \to 5} \left(\frac{x^k - 5^k}{x - 5} \right) = 500$, then k is equal to

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112. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$\lim_{x \to 0} \left\{ \frac{\sin x}{x} \right\}$

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113. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$\lim_{x \to 0} \frac{e^x - e^{x \cos x}}{x + \sin x}$

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114. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$$\lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$$

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115. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$$\lim_{n \to \infty} \left(1 + \frac{1}{a_1}\right)\left(1 + \frac{1}{a_2}\right) \ldots \left(1 + \frac{1}{a_n}\right), \quad \text{where} \quad a_1 = 1 \quad \text{and}$$

$a_n = n(1 + a_{n-1}) \forall n \geq 2$

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116. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$$\lim_{x \to 4} \frac{(\cos \alpha)^x - (\sin \alpha)^x - \cos 2\alpha}{(x-4)}, a \in (0, \pi/2)$$

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117. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$$\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} + e(x-1)}{x}$$

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118. Evaluate the following limits (if exists), where {,} denotes the fractional part of x and [,] denotes the greatest integer part

$$\lim_{x \to 0} \frac{\cos^2\left(1 - \cos^2\left(1 - \cos^2 \ldots\ldots \cos^2(x) \ldots\ldots\right)\right)}{\sin\left[\pi\left(\frac{\sqrt{x+4}-2}{x}\right)\right]}$$

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119. If x is a real number in [0, 1]. Then find the value of $\lim\ (m \to \infty)\ \lim\ (n \to \infty)\left[1 + \cos^{2m}(n!\pi x)\right]$

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120. Let $\alpha \in R$,Prove that a function $f: R \to R$ is differentiable at α , if and only if there is a function $g: R \to R$ which is continuous at α and satisfies $f(x) - f(\alpha) - g(x)(x - \alpha)$ for all $x \in R$

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121. Suppose $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n.$ If $|p(x)| \leq \left|e^{x-1} - 1\right| \leq 1$ then prove |a1 + 2a2 +..... + n an| ≤ 1.

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122. Let $f(x) = \begin{cases} \frac{a(1-\sin x)+b.\cos x+5}{x^2} & x<0 \\ 3 & x=0 \\ \left(1+\left(\frac{cx+dx^3}{x^2}\right)\right)^{1/x} & x>0 \end{cases}$ find a and b

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123. about to only mathematics

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124. Let f(x) be a continuous function in [-1, 1] and satisfies $f\left(2x^2-1\right)=2xf\,\forall x\in[-1,1]$. Prove that f(x) is identically zero for all $x\in[-1,1]$

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125. Let $g(x)=\int_0^x f(t)dt$ where f is such that $1/2\le f(t)\le 1$ for $t\in[0,1]$ and $0\le f(t)\le 1/2$ for `tin[1, 2]

Then the interval in which g(2) lies.

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126. Determine the values of x for which the following functions fails to be continuous or differentiable

$$f(x) = \begin{cases} (1-x) & x < 1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ (3-x) & x > 2 \end{cases}$$

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127. In a function $f:[-2a, 2a] \to R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left hand derivative at x = a is 0, then find the left hand derivative at x = -a,

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128. $\lim_{x \to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is equal to

A. 1

B. -1

C. 0

D. none of these

Answer: A

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129. $\lim_{x \to 2} (-1)^x$ (where [x] is the greatest integer function), is equal to

A. 1

B. -1

C. 1

D. none of these

Answer: D

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130. $\lim_{x \to 1([x]+[x])}$, (where [.] denotes the greatest integer function)

A. is equal to 0

B. is equal to 1

C. does not exist

D. none of these

Answer: C

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131. If $z_r = \cos\left(r\frac{\alpha}{n^2}\right) + i\sin\left(r\frac{\alpha}{n^2}\right)$, where r=1,2,3,...n, then $\lim_{n \to \infty} z_1 z_2 \ldots z_n$ is equal to

A. $\cos\alpha + i\sin\alpha$

B. $\cos\left(\frac{\alpha}{2}\right) - i\sin\left(\frac{\alpha}{2}\right)$

C. $e^{i\alpha/2}$

D. $r \infty t(3)\left(e^{i\alpha}\right)$

Answer: C

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132. $\lim_{(x \to \infty)} \left((x+1) \right)^{x/3}$ is given by

A. 1

B. e^3

C. e

D. e^4

Answer: D

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133. If $\lim_{x \to \infty} \left(\sqrt{x^2 - x + 1} - ax - b\right) = 0$ then the values of a and b are given by

A. a = -1, `b = frac(1)(2)

B. a = 1,$b = \frac{1}{2}$

C. a = 1 b = -frac(1)(2)

D. none of these

Answer: A

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134. $\lim_{x \to 0} \left[\frac{In \cos x}{(1 + x^2)^{\frac{1}{4}} - 1}\right]$ is equal to

A. 2

B. -2

C. 1

D. -1

Answer: B

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135. If $\lim_{n \to \infty} \left(an - \frac{1+n^2}{1+n} \right) = b$ a finite number then

A. a = 1, b= 1

B. a =1 , b= 0

C. a = -1 ,b= 1

D. none of these

Answer: A

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136. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is :

A. 1

B. 2

C. 3

D. 4

Answer: C

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137. The value of $\lim_{x \to \infty} \frac{x + \cos x}{x + \sin x}$ is

A. -1

B. 0

C. 1

D. none of these

Answer: C

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138. The value of f(0) so that the function $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain is equal to

A. 2

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $-\frac{1}{3}$

Answer: B

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139. Which of the following function has finite number of points of discontinuity ?

A. tan x

B. x[x]

C. $\left(\frac{[x]}{x}\right)$

D. $\sin[n\pi x]$

Answer: C

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140. Let $f(x) = [3 + 2\cos x], x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ where [.] denotes the greatest integer function The number of points of discontinuity of f (x) is

A. 3

B. 2

C. 5

D. none of these

Answer: A

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141. If $y = \frac{1}{t^2 - t - 6}$ and $t = \frac{1}{x - 2}$ then the values of x which make the function y discontinuous, are

A. $2, \frac{2}{3}, \frac{7}{3}$

B. $2, \frac{3}{2}, \frac{7}{3}$

C. $2, \frac{3}{2}, \frac{3}{7}$

D. none of these

Answer: B

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142. The function $f(x) = [x]^2 - [x^2]$ (where [.] denotes the greatest integer function) is discontinuous at

A. all integers

B. all integers except 0 and 1

C. all integers except 1

D. all integers except 0

Answer: C

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143. f + g may be continuous function, if

A. f is continuous and g is discontinuous

B. f is discontinuous and g is continuous

C. f and both are continuous

D. none of these

Answer: C

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144. If f(x) is continuous function $\forall x \in R$ and the range of $f(x) = \left(2, \sqrt{26}\right)$ and $g(x) = \left[\frac{f(x)}{a}\right]$ is continuous $\forall x \in R$ Then least positive integral value of a is

A. 2

B. 3

C. 6

D. 5

Answer: C

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145. Let f(x) be a continuous function defined for 1 le x le 3. If f(x) takes rational values for all x and f(2) = 10, then f(1,5) is equal to

A. 0

B. 10

C. not defined

D. any constant

Answer: B

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146. If alpha, beta (alpha < beta) are the points of discontinuity of the function f(f(x)), where $f(x) = \frac{1}{1-x}$ then the set of values of a for which the points (alpha , beta) and (a , a^2) lie on the same side of the line x+2y-3 =0 is

A. $-\left(\frac{3}{2,}, 1\right)$

B. $-\left(\frac{3}{2,}, 1\right)$

C. $[1, \infty]$

D. $\left[-\infty, \frac{3}{2}\right]$

Answer: A

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147. If f(x) = sec 2x + cosec 2x, then f(x) is discontinuous at all points in

A. $\{n\pi | n \in N|\}$

B. $\left\{(2n+1)\frac{\pi}{4} \mid n \in I\right\}$

C. $\left\{\frac{n\pi}{4}, n \in I\right\}$

D. All olf these

Answer: D

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148. For a real number y, let [y] denotes the greatest integer less than or equal to y. Then the function $f(x) = \frac{\tan[(x-\pi)\pi]}{1+[x]^2}$ is :

A. discontinuous at some x

B. continuous at all x, but the derivative f'(x) does not exist for some x

C. f'(x) exist for all x, but the derivative f''(x) does not exist for some x

D. f'' (x) exists for all x.

Answer: D

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149. If $f(x) = \begin{cases} x^3 & x > 0 \\ 0 & x = 0 \\ -x^3 & x < 0 \end{cases}$

A. f(x) is differentiable at x= 0

B. f(x) is continuous but not differentiable at x 0

C. Left hand derivative of f(x) at X = 0 is 1

D. none of these

Answer: A

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150. Let $h(x) = \min\{x, x^2\}$, x in R then h(x) is

A. differentiable everywhere

B. non-differentiable at three values of x

C. non-differentiable at two values of x

D. none of these

Answer: C

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151. The set of all points where the function $f(x) = \frac{x}{1+|x|}$ is differentiable is :

A. $(-\infty, \infty)$

B. $[0, \infty)$

C. $(-\infty.0) \cup (0, \infty)$

D. $(0, \infty)$

Answer: A

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152. Let $f(x) = a + b|x| + c|x|^4$ where a, b and continuous are real constants . Then, f(x) is differentiable at x = 0, if

A. a = 0

B. b = 0

C. c = 0

D. none of these

Answer: B

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153. If $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$, then at x = 0

A. f(x) is differentiable as well as continuous

B. f(x) is differentiable but not continuous

C. f(x) is continuous but not differentiable

D. f(x) is neither continuous nor differentiable

Answer: A

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154. If $f(x) = 2x + \left|x - x^2\right|, \ -1 \leq x \leq 1$ then f(x) is

A. continuous but not differentiable in [-1 , 1]

B. continuous as well as differentiable in [-1 ,1]

C. differentiable but not continuous in [-1 ,1]

D. neither differentiable nor continuous in [-1 , 1]

Answer: A

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155. Let f(x+y) = f(x) . f(y) for all x,y where f(0) $\neq$ 0. If f(5) =2 and f'(0) =3 , then f'(5) is equal to

A. 6

B. 0

C. 1

D. none of these

Answer: A

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156. If f is an even function such that $\lim_{h \to 0} \frac{f(h) - f(0)}{h}$ has some finite non-zero value, then

A. f is continuous and derivable at h= 0

B. f is continuous but not derivable at x = 0

C. f may be discontinuous at x = 0

D. none of these

Answer: A

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157. The values of constants a and b so as to make the function $f(x) = \begin{cases} \frac{1}{|x|} & |x| \geq 1 \\ ax^2 + b & |x| < 1 \end{cases}$, continuous as well as differentiable for all x, are

A. $a = -\frac{1}{2}, b = \frac{3}{2}$

B. $a = \frac{1}{2}, b = \frac{3}{2}$

C. $a = -\frac{1}{2}, b = -\frac{3}{2}$

D. none of these

Answer: A

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158. If $f(x) = \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \to \infty} f(x)$ is

A. 0

B. ∞

C. 1

D. none of these

Answer: C

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159. Number of points at which $f(x) = \left|x^2 + x\right| + |x - 1|$ is non-differentiable is

A. 0

B. 1

C. 2

D. 3

Answer: D

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160. If $\lim_{x \to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is non zero real number, then a is equal to:

A. $\frac{1}{n}$

B. $\frac{n+1}{n}$

C. n

D. $n + \frac{1}{n}$

Answer: D

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161. $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)dt}{x^2 - \frac{\pi^2}{16}}$

A. $\frac{8}{\pi} f(2)$

B. $\frac{2}{\pi} f(2)$

C. $\frac{2}{\pi} f\frac{1}{2}$

D. 4f(2)

Answer: A

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162. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is

A. $\frac{3}{2}$

B. 1

C. -1

D. none of these

Answer: B

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163. If $f(x) = \frac{1}{3}\left\{f(x+1) + \frac{5}{f(x+2)}\right\}$ and $f(x) > 0$ for all $x \in R$, then $\lim_{x \to \infty} f(x)$ is

A. $\sqrt{\frac{2}{5}}$

B. $\sqrt{\frac{5}{2}}$

C. ∞

D. none of these

Answer: B

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164. If $\phi(x) = \lim_{n \to \infty} \frac{x^{2n} f(x) + g(x)}{1 + x^{2n}}$, then

A. $\phi(x) = g(x)$ for all $x \in R$

B. $\phi(x) = f(x)$ for $x \in R$

C. $\phi(x) = \begin{cases} g(x) & \text{for -1} < x < 1 \\ f(x) & \text{for} |x| \geq 1 \end{cases}$

D. $\phi(x) = \begin{cases} g(x) & \text{for} |x| < 1 \\ f(x) & \text{for} |x| > 1 \\ \frac{f(x) + g(x)}{2} & \text{for } |x| = 1 \end{cases}$

Answer: D

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165. Let $f(x) = \left[x^3 - 3\right]$ where [.] denotes the greatest integer function Then the number of points in the interval (1, 2) where the function is discontinuous, is

A. 4

B. 2

C. 6

D. none of these

Answer: C

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166. If $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1)[x] & 2 \leq x \leq 3 \end{cases}$, where [.] denotes the greatest integer function, then

A. both f'(1) and f'(2) do not exist

B. f'(1) exists but f'(2) does not exist

C. f'(2) exists but f'(1) does not exist

D. both f' (1) and f'(2) exists

Answer: A

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167. If `f(x)={(4, -3ltxlt-1),(5+x, -1lexlt0),(5-x, 0 lexlt2),(x^2+x-3, 2lexlt3):}` then f(Ixl) is

A. differentiable but not continuous in (-3 ,3)

B. continuous but not differentiable in (-3,3)

C. continuous as well as differentiable in (-3,3)

D. neither continuous nor differentiable in (-3,3)

Answer: B

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168. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a & x < 4 \\ a + b & x = 4 \\ \frac{x-4}{|x-4|} + b & x > 4 \end{cases}$ Then , f (x) is continuous at x = 4, when

A. a = 0, b = o

B. a = 1, b = 1

C. a = -1 , b = 1

D. a = 1, b = -1

Answer: D

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169. Let $f(x) = \begin{cases} a\frac{|x^2-x-2|}{2+x-x^2} & x < 2 \\ b & x = 2 \\ \frac{x-[x]}{x-2} & x > 2 \end{cases}$ If f(x) is continuous at x = 2 , (where [.] denotes greatest integer function) : then (a , b) is ,

A. (1 , 1)

B. (1, 2)

C. (2, !)

D. (2, 2)

Answer: A

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170. The value of f (0), so that function

$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$ becomes continuous for all x, is given by

A. $a^{\frac{3}{2}}$

B. $a^{\frac{1}{2}}$

C. $-a^{\frac{1}{2}}$

D. $-a^{\frac{3}{2}}$

Answer: C

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171. Let $f(x) = |x| - 1|$, then points where f(x), is not differentiable is/are :

A. 0, +-1

B. -1

C. o

D. 1

Answer: A

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172. Let f be twice differentiable function satisfying f(1) = 1, f(2) = 4, f(3) =9 then :

A. f''(x) = 2, AA x in R

B. f'(x) = 5 = f'' (x), for some x in (1, 3)

C. There exists at least one x in (1,3) such that f''(x) = 2

D. none of these

Answer: C

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173. $f(x) = \min\{1, \cos x, 1 - \sin x\}, \ -\pi \leq x \leq \pi$ then

A. f(x) is not differentiable at '0'

B. f(x) is differentiable at $\frac{\pi}{2}$

C. f(x) is not differentiable at π

D. none of these

Answer: A

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174. Let $f: R \to R$ be any function. Define $g: R \to R$ by g(x)=|f(x)| for all x. Then g is:

A. onto if onto

B. one-one if is one-one

C. continuous if f is continuous

D. differentiable if f is differentiable

Answer: C

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175. If $f(x) = \lim_{n \to \infty} (\sin x)^{2n}$ where $n \in l^+$ then f(x) is

A. continuous at$x = \frac{\pi}{2}$

B. discontinuous at $x = \frac{\pi}{2}$

C. discontinuous at $x = (\pi)$

D. none of these

Answer:

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176. The function $f(x) = (\sin 2x)^{\tan^2 2x}$ is not defined at $x = \frac{\pi}{4}$. The value of $f\left(\frac{\pi}{4}\right)$ so that f is continuous at $x = \frac{\pi}{4}$

A. $\sqrt{e}$

B. $\frac{1}{\sqrt{e}}$

C. 2

D. none of these

Answer: B

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177. Let $f: R \to R$ be a differentiable function having f(2) = 6, $f'(2) = \left(\frac{1}{48}\right)$. Then $\lim_{x \to 2} \frac{\int_2^{f(x)} f\left(4t^3\right) dt}{x - 2}$ is equals

A. 18

B. 12

C. 36

D. 24

Answer: A

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178. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \log(1+t)}{t^4+4} dt$ is

A. 0

B. $\frac{1}{12}$

C. $\frac{1}{24}$

D. $\frac{1}{64}$

Answer: B

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179. Let $f: R \to R$ be such that f (1) = 3 and f' (1) = 6 Then, $\lim_{x \to 0} \left(\frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}}$ equals

A. 1

B. $e^{\frac{1}{2}}$

C. e^2

D. e^3

Answer: C

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180. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{2/x} = e^3$ then

A. a = 3, b= 0

B. a=$\frac{3}{2}$, b=1

C. $\frac{3}{2}$, b=4

D. a=2 , b=3

Answer: B::C

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181. If x+|y| = 2y, then y as a function of x is

A. defined for all real x

B. continuous at x = 0

C. differentiable all x

D. such that $\frac{dy}{dx} = \frac{1}{3}$ for x < 0

Answer: A::B::D

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182. On the interval I = [-2, 2], the function

$$f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

A. is continuous for all $x \in I$

B. is continuous for all$x \in I - \{0\}$

C. assumes all intermediate values from f(-2) to f(2)

D. has a maximum value equal to $(3/e)$

Answer: B::C::D

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183. In the interval $0 < x < 2\pi$, the function f(x)= |sin2x| is not differentiable at

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer: B::C::D

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184. If $\lim_{x \to 0} 4\left(\frac{x+1}{2x+1}\right) = y^2 + 4y + 5$ then y can be equal to

A. 1

B. -1

C. -4

D. -3

Answer: B::D

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185. At the pointx = 0, for the function $f(x) = \frac{1}{2 - |x|}$ the following are true ?

A. f(x) is continuous

B. f(x) discontinuous

C. f(x) is differentiable

D. f(x) is not differentiable

Answer: A::D

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186. If $f(x) = |\log_{10} x|$ then at x=1

A. f(x) is continuous and $f'\left(1^+\right) = \log_{10} e$

B. f(x) is continuous and $f'\left(1^+\right) = \log_e 10$

C. f(x) is continuous and $f'\left(1^+\right) = \log_e 10$

D. f(x) is continuous and $f'\left(1^{+}\right) = \log_{10} e$

Answer: A::D

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187. A function is defined as f(x) = $\begin{cases} e^{x} & x \leq 0 \\ |x-1| & x > 0 \end{cases}$, then f(x) is

A. continuous at x=0

B. continuous at x = 1

C. differentiable all x=0

D. differentiable at x= 1

Answer: A::B

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188. If $f(x) = |x-a|\phi(x)$, where $\phi(x)$ is continuous function, then

A. $f'(a^+) = \phi(a)$

B. $f'(a-) = -\phi(a)$

C. $f'(a^+) = f'(a^-)$

D. None of these

Answer: A::B

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189. If $f(x) = \min\{1, x^2, x^3\}$ then

A. f (x) is continuous everywhere

B. f (x) is continuous and differentiable everywhere

C. f (x) is not differentiable at two points

D. f (x) is not differentiable at one point

Answer:

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190. Let L= $\lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

A. a=2

B. a=1

C. $a = 4$

D. $a = 3$

Answer:

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191. For function $f(x) = x \cos \frac{1}{x}, x \geq 1$

A. for at least one x in interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

B. $\lim_{x \to \infty} f'(x) = 1$

C. for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

D. f'(x) is strictly decreasing in the interval $[1, \infty)$

Answer:

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192. মান নির্ণয় করো $\cos \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

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193. The function $y = \sin^{-1}(\cos x)$ is not differentiable at

A. $x = \pi$

B. $x = -2\pi$

C. $x = 2\pi$

D. $x = \frac{\pi}{2}$

Answer: A::B::C

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194. If f(x) = 0 for $x < 0$ and f(x) is differentiable at x = 0 then for $x > 0$, f(x) may be

A. x^2

B. x

C. $-x$

D. $-x^3$

Answer: A

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195. Let f be a polynomial function such that $f(x)f(y) + 2 = f(x) + f(y) + f(xy)\,\forall x, y \in R^+ \cup \{0\}$and f(x) is one-one $\forall x, y \in R^+$ with f(0)=1 and f(1)=2

Answer the following questions based on above passage:

The function y=f(x) is given by

A. $x^{1/3} - 1$

B. $1 + \frac{2x^3}{3}$

C. $1 + x^2$

D. $1 - x^2$

Answer: C

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196. Let f be a polynomial function such that $f(x)f(y) + 2 = f(x) + f(y) + f(xy)\,\forall x, y \in R^+ \cup \{0\}$and f(x) is one-one $\forall x, y \in R^+$ with f(0)=1 and f(1)=2

Answer the following questions based on above passage:

The function y=f(x) is given by

A. $\frac{\pi}{2} - \frac{1}{3}$ sq. units

B. $\pi - \frac{1}{3}$ sq. units

C. $\frac{\pi}{2} - \frac{1}{6}$ sq. units

D. $\pi - \frac{2}{3}$ sq. units

Answer: D

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197. Let f be a polynomial function such that $f(x)f(y) + 2 = f(x) + f(y) + f(xy)\,\forall x, y \in R^+ \cup \{0\}$and f(x) is one-one $\forall x, y \in R^+$ with f(0)=1 and f(1)=2

Answer the following questions based on above passage:

A. 3

B. 4

C. 5

D. 6

Answer: D

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198. Newton-Leinbnitz's formula states that

$$\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)dt\right) = f(\psi(x))\left\{\frac{d}{dx}\psi(x)\right\} - f(\phi(x))\left\{\frac{d}{dx}\phi(x)\right\}$$

Answer the following questions based on above passage:

Let f(x) be a differentiable function and f(1)=2, If $\lim_{x\to 1}\int_2^{f(x)} \frac{2t}{x-1}dt = 4$, then the value of f'(1) is equal to

A. 1

B. 2

C. 3

D. 4

Answer: A

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199. Newton-Leinbnitz's formula states that

$$\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)dt\right) = f(\psi(x))\left\{\frac{d}{dx}\psi(x)\right\} - f(\phi(x))\left\{\frac{d}{dx}\phi(x)\right\}$$

Answer the following questions based on above passage:

If $\lim_{x\to 0}\int_0^x \frac{t^2dt}{(x-\sin x)\sqrt{a+t}} = 1$, then the value of a is

A. 1

B. 2

C. 3

D. 4

Answer: D

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200. Newton-Leinbnitz's formula states that

$$\frac{d}{dx}\left(\int_{\phi(x)}^{\psi(x)} f(t)dt\right) = f(\psi(x))\left\{\frac{d}{dx}\psi(x)\right\} - f(\phi(x))\left\{\frac{d}{dx}\phi(x)\right\}$$

Answer the following questions based on above passage:

$$\lim_{x \to 0} \int_0^{x^2} \frac{(\cos t)^2 dt}{(x \sin x)} =$$

A. 0

B. 1

C. 2

D. 3

Answer: B

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201. f(x) is not differentiable at x=c if either $Lf'(c) \neq Rf'(c)$ [if they both exist]

or Lf'(c) exists but Rf'(c)does not exist,

or Lf'(c) does not exists, Rf' exists, then f(x) is not continuous x=c.

Answer the following questions based on above passage:

Let f(x)=sin|x|-|x|, then Lf'(0) is equal to

A. continuous as well as differentiable at x=0

B. continuous at x=0, but not differentiable at x=0

C. neither continuous at x=0 nor differentiable at x=0

D. none of these

Answer: B

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202. If $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1)[x] & 2 \leq x \leq 3 \end{cases}$, then f(x) is

A. both f'(1) and f'(2) do not exist

B. f'(1) exists but f'(2) does not exist

C. f'(2) exists but f'(1) does not exist

D. both f' (1) and f'(2) exists

Answer: A

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203. f(x) is not differentiable at x=c if either $Lf'(c) \neq Rf'(c)$ [if they both exist]

or Lf'(c) exists but Rf'(c)does not exist,

or Lf'(c) does not exists, Rf' exists, then f(x) is not continuous x=c.

Answer the following questions based on above passage:

Let f(x)=sin|x|-|x|, then Lf'(0) is equal to

A. 1

B. 2

C. 0

D. None of these

Answer: C

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204. Let $f(x)$ be a polynomial satisfying $f(0)=2$, $f'(0)=3$ and $f''(x)=f(x)$.

Answer the following the question based on above passage:

$f(x)$ is given by

A. 4

B. 5

C. 7

D. 6

Answer: D

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205. Match List - I with List-II

List - I		List - II	
(1)	Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(P)	4
(2)	If an edge of a cube increases by 1%, then percentage increase in volume is	(Q)	0.6π
(3)	If the rate of decrease of $\frac{x^2}{2}-2x+5$ is twice the rate of decrease of x, then x is equal to (rate of decreases is non-zero)	(R)	3
(4)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(S)	$\frac{3\sqrt{3}}{4}$

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206. If [x] denotes greatest integer $\leq x$, then match the following limits and their values:

Match List-I with List-II

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207. Match List - I with List-II

List - I		List - II	
(1)	Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(P)	4
(2)	If an edge of a cube increases by 1%, then percentage increase in volume is	(Q)	0.6π
(3)	If the rate of decrease of $\frac{x^2}{2}-2x+5$ is twice the rate of decrease of x, then x is equal to (rate of decreases is non-zero)	(R)	3
(4)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(S)	$\frac{3\sqrt{3}}{4}$

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208. The value of $\lim_{x \to 1} \frac{x + x^2 + x^3 + \ldots + x^{97} - 97}{(x - 1)}$ must be M. The last digit of M is :

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209. If $\lim_{x \to 0} \frac{1}{x^2}(e^{ax} - e^x - x) = \frac{3}{2}$, then the value of $|\alpha|$must be

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210. If $f(x) = \begin{cases} \frac{(1 - \sin^3 x)}{3\cos^2 x} & x < \frac{\pi}{2} \\ a & x = \frac{\pi}{2} \\ \frac{b(1 - \sin x)}{(\pi - 2x)^2} & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then the value of $\left(\frac{b}{a}\right)^{\frac{1}{3}}$ is

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211. Given, $f(x) = \begin{cases} \frac{x^4 - 256}{x - 4} & x \neq 4 \\ \lambda & x = 4 \end{cases}$. If f is continuous at x = 4, then the value of $\hat{} \sqrt[4]{\lambda}$ must be

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212. If $f(x) = \begin{cases} \frac{8^x - 4^x - 2^x + 1^x}{x^2} & x > 0 \\ e^x \sin x + \pi x + \lambda In 4 & x \leq 0 \end{cases}$ is continuous at x = 0, then the value of e^{λ} must be

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213. If $f(x) = \begin{cases} \frac{(\exp\{(x+3)\ln 27\})^{\frac{1}{27}[x]} - 9}{3^x - 27} & x < 3 \\ \lambda. \frac{(1 - \cos(x-3))}{(x-3)\tan(x-3)} & x > 3 \end{cases}$ is continuous at x = 3, then the value of 9λ must be

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214. If $f(x) = \begin{cases} 5 + x^2 & x < 1 \\ x - 4 & x \geq 1 \end{cases}$ is jump discontinuous, then the number of jumps must be

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215. A function $f: R \to R$ satisfies the equation f(x + y) = f(x), f(y) for all $x, y \in R$, $f(x) \neq 0$. Suppose that the function is differentiable at x = 0 and f' (0) = 2. Find $\frac{f'(x)}{f(x)}$

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216. If f(x) is continuous and f(9/2) = 2/9, then find $\lim_{x \to 0} 9f\left(\frac{1 - \cos 3x}{x^2}\right)$.

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217. If f(x)f(y) + 2 = f(x) + f(y) + f(xy) and f(1) = 2, f'(1) = 2, then find f(x).

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218. Let f(x) = x + cos x + 2 and g(x) be the inverse function of f(x), then g'(3) equals _

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219. Number of point of discontinuity of $f(x) = \tan^2 x - \sec^2 x$ in $(0, 2\pi)$ is _

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220. Evaluate the following limits :

$$\lim_{x \to 0} \frac{5^x - 4^x}{3^x - 1}$$

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221. Evaluate the following limits :

$$\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{2n^3}$$

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222. Evaluate the following limits :

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$

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223. Evaluate the following limits :

$$\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$$

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224. Evaluate the following limits :

$$\lim_{x \to 0} \sqrt{\frac{\frac{1}{2}(1 - \cos 2x)}{x}}$$

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225. Evaluate the following limits :

$$\lim_{x \to 0} (1 + |\sin x|)^{\frac{1}{x}}$$

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226. Evaluate the following limits :

$$\lim_{x \to 1} (1 - x) \tan \frac{\pi x}{2}$$

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227. Evaluate the following limits :

$$\lim_{x \to \infty} x\left(e^{\frac{1}{x}} - 1\right)$$

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228. Evaluate the following limits :

$$\lim_{x \to \frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3} - x\right)}{2\cos x - 1}$$

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229. Evaluate the following limits :

$$\lim_{n \to \infty} n\left(x^{\frac{1}{n}} - 1\right), x > 0$$

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230. Find the set of points where the following functions are discontinuous

f(x) = tan 2x

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231. Find the set of points where the following functions are discontinuous

f(x) = {3x}

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232. Find the set of points where the following functions are discontinuous

f(x) = x/sinx

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233. Find the set of points where the following functions are discontinuous

$f(x) = \tan x$

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234. Find the set of points where the following functions are discontinuous

$$f(x) = \frac{1}{1 - e^{\frac{x-1}{x-2}}}$$

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235. Find the set of points where the following functions are discontinuous

$$f(x) = \frac{3\sin^2 x + \cos^2 x + 1}{2\cos^2 x - 1}$$

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236. Let $f(x) = \begin{cases} \frac{x^2-4}{x+2} & x < -3 \\ Ina & x = -3 \\ a + bx & x > -3 \end{cases}$. For what value of a and b is f(x) continuous on the real line?

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237. Let $f(x) = \begin{cases} ax+1 & x<1 \\ 3 & x=1 \\ bx^2+1 & x>1 \end{cases}$. For what values of a and b is f(x) continuous at x = 1 ?

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238. Let f be a twice differentiable function such that f"(x) = -f(x) , and f'(x) = g(x) , $h(x) = [f(x)]^2 + [g(x)]^2$ Find h(10), if h(5) = 11

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239. If $|c| \leq \frac{1}{2}$ and f(x) is a differentiable function at x = 0 given by

$$f(x) = \begin{cases} b\sin^{-1}\left(\frac{c+x}{2}\right) & -\frac{1}{2} < x < 0 \\ \frac{1}{2} & x = 0 \\ \frac{e^{\frac{ax}{2}}-1}{x} & 0 < x < \frac{1}{2} \end{cases}$$

Find the value of 'a' `

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240. Let $f(x) = \begin{cases} x + a & \text{if } x < 0 \\ |x - 1| & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ (x - 1)^2 + b & \text{if } x \geq 0 \end{cases}$, where a and b are non-negative real numbers. Determine the composite function gof. If (gof) (x) is continuous for all real x, determine the values of a and b. Further, for these values of a and b, is gof differentiable at x = 0?

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241. f is a function such that f(x + y) = f(x) .f(y), $\forall x, y \in R$, if f(1) = 3, then find $\sum_{r=1}^{n} f(r)$.

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242. A real valued function satisfies the relation $f(x + y) = f(x) + f(y) + (e^x - 1)(e^y - 1)$, $\forall x, y \in R$. If f'(0) = 2, find f(x).

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243. If f(x)f(y) + 2 = f(x) + f(y) + f(xy) and f(1) = 2, f'(1) = 2, then find f(x).

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244. Let $f(x) = \begin{cases} 1+x & 0 \leq x \leq 2 \\ 3-x & 2 < x < 3 \end{cases}$

Determine the form of g(x) = f[f(x)] and hence find the points of discontinuity of g, if any.

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245. Let $f(x) = \begin{cases} 1+x & 0 \leq x \leq 2 \\ 3-x & 2 < x < 3 \end{cases}$

Discuss the continuity of f, f' on [0, 2].

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246. Let

$$f(x) = \begin{cases} x \exp\left[-\left(\frac{1}{|x|} + \frac{1}{x} \right) \right] & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Test whether

f(x) is differentiable at x=0

A. f is differentiable at x=0

B. h is differentiable at x=0

C. $(f \circ h)$ is differentiable at x=0

D. $(h \circ f)$ is differentiable at x=0

Answer: A::D

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247. Let a and b be real numbers such the the function

$$f(x) = \begin{cases} -3ax^2 - 2 & x < 1 \\ bx + a^2 & x \geq 1 \end{cases}$$

is differentiable for all $x \in R$. The positive value(s) of a is (are)

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248. Let m and n be two positive integers greater than 1. if $\lim_{\alpha \to 0}\left(\frac{e^{\cos(a^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$,then the value of m/n is

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249. The correct statement(s) is (are)

A. $f(1)<0$

B. $f(2)<0$

C. $f(x)\neq 0$for any $x\in(1,3)$

D. $f(x)=0$for some$x\in(1,3)$

Answer: A::B::C

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250. Let f(x) be a polynomial of degree four having extreme values at x=1 and x=2. $\lim_{x \to 0} \left[1 + \frac{f(x)}{x^2}\right] = 3$,then f(2) is equal to:

A. -4

B. 0

C. 4

D.

Answer: B

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251. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to:

A. 3

B. 2

C. 44198

D.

Answer: B

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252. If the function .

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx+2 & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of k+m is

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253. Let $f: R \to R$ be defined as .

$f(x) = \begin{cases} 0 & \text{x is irrational} \\ \sin|x| & \text{x is irrational} \end{cases}$ Then which of the following is true?

A. f is discontinuous for all x

B. f is continuous for all x

C. f is discontinuous at $x = k\pi$, where kis an integer

D. f is continuous at $x = k\pi$, where kis an integer

Answer: D

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254. If $\lim_{x \to 0} \frac{axe^x - b\log(1+x)}{x^2} = 3$,then the values of a, b are respectively

A. 2,2

B. 1,2

C. 2,1

D. 2,0

Answer: A

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255. Let $[x]$denote the greatest integer less than or equal to x . Then the value of α for which the function $f(x) = \begin{cases} \frac{\sin[-x^2]}{x^2} & x \neq 0 \\ \alpha & x = 0 \end{cases}$ is continuous at $x = 0$ is

A. $\alpha = 0$

B. $\alpha = \sin(-1)$

C. $\alpha = \sin(1)$

D. $\alpha = 1$

Answer: C

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256. Let $f: R \to R$ be differentiable at $x = 0$. If $f(0) = 0$ and $f'(0) = 2$,then the value of

$$\lim_{x \to 0} \frac{1}{x}[f(x) + f(2x) + f(3x) + \ldots . + f(2015x)]$$ is

A. 2015

B. 0(zero)

C. 2015×2016

D. 2015×2014

Answer: C

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257. Let $f: R \to R$ be such that $f(2x-1) = f(x)$ for all $x \in R$. If f is continuous at $x = 1$ and `f(1)=1,Then

A. $f(2) = 1$

B. $f(2) = 2$

C. f(x) is continuous at x=1

D. `f is continuous at all points

Answer: A::D

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258. Let $f$be any continuously differentiable function on [a,b] and twice differentiable on (a, b) such that $f(a) = f'(a) = 0$. Then

A. x

B. $f'(x) = 0$ for some $x \in (a, b)$

C. $f''(x) = 0$ for some $x \in (a, b)$

D. $f'''(x) = 0$ for some $x \in (a, b)$

Answer: B::C

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259. Let $f: [a, b] \to [1, \infty)$ be a continuous function and lt $g: R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if x} \leq a \\ \int_a^x f(t)dt & \text{if a} \leq x \leq b \\ \int_a^b f(t)dt & \text{if x} > b \end{cases}$$

Then

A. g(x) is continuous but not differentiable at a

B. g(x) is differentiable on R

C. g(x) is continuous but no differentiable at b

D. g(x) is continuous and differentiable at either a or b but not both

Answer: A::C

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260. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: R \to R$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

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261. The largest value of the non-negative integer a for which `lim_(xrarr1)

{(-ax+sin(x-1)+a)/(x+sin(x-1)-1)}^((1-x)/(1-sqrtx))= 1/4

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262. Let $f_1 : R \to R, f_2 : [0, \infty) \to R, f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

and $f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$

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263. $\lim_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to :

A. $-\pi$

B. π

C. $\frac{\pi}{2}$

D. 1

Answer: B

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264. If g is the inverse of a function $f$and $f(x)=\frac{1}{1+x^5}$, Then $g'(x)$ i equal to :

A. $\frac{1}{1+\{g(x)\}^5}$

B. $1+\{g(x)\}^5$

C. $1+x^5$

D. $5x^4$

Answer: B

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265. The function $f(x)=\frac{\tan\left\{\pi\left[x-\frac{\pi}{2}\right]\right\}}{2+[x^2]}$, where [x] denotes the greatest integer $\leq x,$ is

A. continuous for all values of x

B. discontinuous at $x = \frac{\pi}{2}$

C. not differentiable for some values of x

D. discontinuous at x=-2

Answer: A

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266. Let $f(x)$ be a differentiable function and $f'(4) = 5$. Then $\lim_{x \to 2} \frac{f(4) - f(x^2)}{x - 2}$ equals

A. 0

B. 5

C. 20

D. -20

Answer: D

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267. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \cos\left(t^2\right) dt}{x \sin x}$ is

A. 1

B. -1

C. 2

D. $\log_e 2$

Answer: A

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268. If lim_(xrarr0)(2asinx-sin2x)/tan^3x` exists and is equal to 1, then thee value of a is

A. 2

B. 1

C. 0

D. 1

Answer: B

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269. The function $f(x) = a\sin|x| + be^{|x|}$ is differentiable at $x = 0$ when

A. $3a + b = 0$

B. $3a - b = 0$

C. $a + b = 0$

D. a-b=0

Answer: C

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270. let $f(x) = \begin{cases} \int_0^x |1-t| dt & x > 1 \\ x - \frac{1}{2} & x \leq 1 \end{cases}$

then

A. $f(x)$ is continuous at $x = 1$

B. $f(x)$ is not continuous at $x = 1$

C. $f(x)$ is differentiable $x = 1$

D. $f(x)$ is not differentiable at $x = 1$

Answer: A::D

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271. The number of points in $(-\infty, \infty)$, for which $x^3 - x\sin x - \cos x = 0$, is

A. 6

B. 4

C. 2

D. 0

Answer: C

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272. $\lim_{x \to 0} \frac{(1 - \cos 2)(3 + \cos x)}{x \tan 4x}$ is equal to

A. 44198

B. 1

C. 2

D. $-\frac{1}{4}$

Answer: C

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273. The limit of $x$sin (e^(1//x))asxrarr0`

A. is equal to 0

B. is equal to 1

C. is equal to $e/2$

D. does not exist

Answer: A

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274. Let $f(x) = \{$f(x={(x^3-3x+2, xlt2),(x^3-6x^2+9x+2, xge2):}`

Then

A. $\lim_{x \to 2} f(x)$ does not exist

B. $f$is not continuous at $x = 2$

C. $fiscont \in uousbut \neg d \Leftrightarrow erentiab \leq at$x=2`

D. $fiscont \in uous$ and $d \Leftrightarrow erentiab \leq at$x=2`

Answer: C

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275. The limit of $\sum_{n=1}^{1000}(-1)^n x^n$ as $x \to \infty$

A. does not exist

B. exists and equals to 0

C. exists and approaches $+\infty$

D. exists and approaches $-\infty$

Answer: C

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276. The limit of $\left[\frac{1}{x^2}+\frac{(2013)^x}{e^x-1}-\frac{1}{e^x-1}\right]$ as $x \to 0$

A. approaches $+\infty$

B. approaches-∞

C. is equal to $\log_e(2013)$

D. does not exist

Answer: A

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277. The limit of $\left\{\frac{1}{x}\sqrt{1+x} - \sqrt{1+\frac{1}{x^2}}\right.$ as $x \to 0$

A. does not exist

B. is equal to 1/2

C. is equal to 0

D. is equal to 1

Answer: A

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278. If $\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$,then

A. $a = 1, b = 4$

B. $a = 1, b = -4$

C. $a = 2, b = -3$

D. $a = 2, b = 3$

Answer: B

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279. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right| & x \neq 0 \\ 0 & x = 0 \end{cases}, x \in R$, then f is

A. differentiable both at x= 0 and at x = 2

B. differentiable at x = 0 but not differentiable at x= 2

C. not differentiable at x = 0 but differentiable at x = 2

D. differentiable neither at x= 0 nor at x = 2

Answer: B

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280. Let $\alpha(a)$ and $\beta(b)$ be the roots of the equation ,

$\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where

$a>-1$. Then $\lim_{x\to 0^+}\alpha(a)$ and $\lim_{x\to 0^+}\beta(a)$ are

A. -5/2 and 1

B. -1/2 and -1

C. $-\frac{7}{2}$ and 2

D. $-\frac{9}{2}$ and 3

Answer: B

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281. For every integer n, let a_n and b_n be real numbers Let function $f: R \to R$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x & \text{for} x \in [2n, 2n+1] \\ b_n + \cos \pi x & \text{for} x \in (2n-1, 2n) \end{cases}$$

for all integers n If f is continuous, then which of the following hold(s) for all n ?

A. $a_{n-1} - b_{n-1} = 0$

B. $a_n - b_n = 1$

C. $a_n - b_{n+1} = 1$

D. $a_{n-1} - b_n = 1$

Answer: B::D

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282. If $f: R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is

A. continuous for every real x

B. discontinuous only at x = 0

C. discontinuous only at non-zero integral values of x

D. continuous only at x= 0

Answer: A

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283. Consider the function f(x) = |x-2|+|x-5|,

Statement 1: f'(4)=0

Statement 2: differentiable in (2,5) and f(2)=f(5)

A. Statement 1 is false, Statement 2 is true

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for statement 1

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for statement 1

D. Statement 1 is true, Statement 2 is false

Answer: B

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284. Let $R \to R$ be a function such that $f(x+y) = f(x) + f(y)\,\forall x, y \in R$. If f(x) is differentiable at x= 0, then

A. f(x) is differentiable only in a finite interval containing zero

B. f(x) is continuous $\forall x \in R$

C. f'(x) is constant $\forall x \in R$

D. f(x) is differentiable except a finitely many points

Answer: B::C

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285. If $\lim_{x \to 0} \left[1 + x \ln\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b \sin^2 \theta, b > 0 \text{and} \theta \in (-\pi, \pi].$

then the value of θ is

A. $\pm \frac{\pi}{4}$

B. $\pm \frac{\pi}{3}$

C. $\pm \frac{\pi}{6}$

D. $\pm \frac{\pi}{2}$

Answer: D

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286. If $f(x) = \begin{cases} -x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < c \leq 1 \\ \ln x & x > 1 \end{cases}$

A. f(x) is continuous at $x - -\frac{\pi}{2}$

B. f(x) is not differentiable at x = 0

C. f(x) is differentiable at x = 1

D. f(x) is differentiable at $x = -\frac{3}{2}$

Answer: A::B::C::D

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287. $\lim_{x \to 2} \left(\frac{\sqrt{1 - \cos\{2(x-2)\}}}{x-2} \right)$ =?

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288. The value of p and q for which the function

`f(x)={((sin(p+1)x+sinx)/x, x lt 0),(q, x = 0),((sqrt(x+x^2)-sqrtx)/x^(3/2), xgt0):}

is continuous for all x in R, is

A. p=5/2, q=1/2

B. 1=-3/2, q=1/2

C. p=1/2, q=3/2

D. p=1/2, q=-3/2

Answer: B

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289. The value of $\lim_{x \to 1} \frac{x + x^2 + \ldots + x^n - n}{x - 1}$ is

A. n

B. $\frac{n+1}{2}$

C. $\frac{n(n+1)}{2}$

D. $\frac{n(n-1)}{2}$

Answer: C

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290. $\lim_{x \to 0} \frac{\sin\left(\pi \sin^2 x\right)}{x^2} =$

A. π^2

B. 3π

C. 2π

D. π

Answer: D

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291. If the function

$$f(x) = \begin{cases} \frac{x^2 - (A+2)x + A}{x-2} & \text{for} x \neq 2 \\ 2 & \text{for} x = 2 \end{cases}$$ is continuous at x = 2, then

A. A=0

B. A=1

C. A=-1

D. A=2

Answer: A

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292. $f(x) = \begin{cases} [x] + [x] & \text{when} x \neq 2 \\ \lambda & \text{when} x = 2 \end{cases}$

If f(x) is continuous at x = 2, the value of λ will be

A. -1

B. 1

C. 0

D. 2

Answer: A

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293. Let IR be the set of real numbers andf : IR be such that for all $x, y \in IR$

$|f(x) - f(y) \leq |x - y|^3$ Prove that f is a constant function.

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