MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Application of dy/dx

Example

1. The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at $P(-2, 0)$ and cuts the y-axis at

the point Q where its gradient is 3. Find the equation of the curve completely.

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2. Tangents are drawn from the origin to the curve $y = \sin x$. Prove that their points of contact lie on the curve $x^2y^2 = \left(x^2 - y^2\right)$

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3. In the curve $x^a y^b = K^{a+b}$, prove that the potion of the tangent intercepted between the coordinate axes is divided at its points of contact into segments which are in a constant ratio. (All the constants being positive).

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4. if the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $x^m y^n = a^{m+n}$ prove that $p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n\alpha \cos^m\alpha$

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5. Show that the normal to the curve $5x^5 - 10x^3 + x + 2y + 6 = 0$ at $P(0, -3)$ meets the curve again at two points. Find the equations of the tangents to the curve at these points.

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6. Find the equation of the straight line which is a tangent at one point and normal at

another point to the curve $y = 8t^3 - 1, x = 4t^2 + 3.$

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7. A man is standing on a straight bridge over a river and another man on a boat is on the river just below lthe man on the bridge. If the first man starts walking at the uniform speed of 4 m/ min and the boat moves perpendicularly to the bridge at the speed of 5 m/min then at what rate are they separating

after 4 minutes if the height of the bridge above the boat is 3 m?

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8. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

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9. Without use of tables and any standard value of logarithm, find the value of $\log_e 7$.

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Exercise

1. Determine the quadratic curve $y = f(x)$ if it touches the line $y = x$ at the point $x = 1$ and passes through the point $(-1, 0)$.

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2. Find the value of a, b, c such that curves $y = x^2 = ax + bandy = cx - x^2$ will touch each other at the point $(1, 0)$

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3. Prove that the curves $2y^2 = x^3$ and $y^2 = 32x$ cut each other orthogonally at the origin

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4. Prove that the curves $xy = 4 and x^2 + y^2 = 8$ touch each other.

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5. Find the equation of the tangent to the curve $y = \left(x^3 - 1\right)(x - 2)$ at the points where the curve cuts the x-axis.

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6. Show that the line $\frac{d}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.

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7. The equation of tangents to the curve $y = \cos(x + y), \ -2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$, is

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8. Find the equations of the tangents drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$.

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9. If the tangent at (x_0, y_0) to the curve $x^3 + y^3 = a^3$ meets the curve again at (x_1, y_1), then $\frac{x_1}{x_0} + \frac{y_1}{y_0}$ is equal to

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10. Show that the length of the portion of the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at any point on it, intercepted between the coordinate axes is constant.

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11. The sum of the intercepts made on the axes of coordinates by any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is equal to

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12. Prove that the tangent at any point on the rectangular hyperbola $xy = c^2$, makes a triangle of constant area with coordinate axes.

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13. Show that the segment of the tangent to the curve $y = \frac{a}{2} In\left(\frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}}\right) - \sqrt{a^2-x^2}$ contained between the y=axis and the point of tangency has a constant length.

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14. If α, β are the intercepts made on the axes by the tangent at any point of the curve $x = a\cos^3\theta$ and $y = b\sin^3\theta$, prove that $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1.$

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15. Show that the segment of the tangent to the hyperbola $y = \frac{a^2}{x}$ intercepted between

the axes of the coordinates is bisected at the point of contact

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16. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b).

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17. Find the equation of the normal to the curve $y = (1+y)^y + \sin^{-1}\left(\sin^2 x\right) atx = 0.$

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18. Prove that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $y\cos\theta - x\sin\theta = a\cos 2\theta,$ where θ is the angle which the normal makes with the axis of x.

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19. Find the equation of the normal to the curve $x^2 = 4y$which passes through the point (1, 2).

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20. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.

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21. Prove that all normals to the curve

$$x = a\cos t + at\sin t, \quad y = a\sin t - at\cos t$$

are at a distance a from the origin.

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22. Show that for any point of the curve $x^2 - y^2 = a^2$ the segment of the normal from the point to the point of intersection of the normal with the x-axis is equal to the distance of the point from the origin.

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23. Use calculus to find the conditiion that the line $x\cos\theta + y\sin\theta = p$ is a normal to the curve $b^2x^2 + a^2y^2 = a^2b^2$.

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24. The volume of the cylindrical trunk of a tree is proportional to the cube of its diameter and the diameter increases uniformly from year to year,prove that the rate of increase in the volume when the diameter is

equal to $90cm$ is 25 times the ratewhen it is $18cm$.

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25. A stick of length $100cm$ rests against a vertical walland the horizontal floor. If the foot of the stick slides with the constant velocity of $10cm/s$ then find the magnitude of the velocity of the middle point of the stick when it is inclined at with the floor.

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26. A man running along a circular track has the speedof 10 km per hour. A source of light is at the centre of the circular track. A wall is along the tangent to the circular track at the point from which he starts.What is the speed of the shadow of the man on the wall when he covers $\frac{1}{8}th$ of the track? (Assume thetrack to be a line.)

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27. A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa chnages at alower rate than ordinate, is (a, b). Then $(a + b)$ is .....

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28. A particle is moving along the parabola $y^2 = 12x$ at the uniform rate of $10cm/s$. Find the components of velocity parallel to

each of the axes when the particle is at the point $(3, 6)$.

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29. A spherical balloon is being inflated so that itsvolume increases uniformly at the rate of $40cm3/\ \min$. How fast is its surface area increasing when the radius is$8cm$? Find how much approximately the radius will increase during the next $\frac{1}{2}$ minute.

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30. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

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31. The time t of a complete oscillation of a simple pendulum of length l is given by the

equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

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32. A spherical iceball is melting, such that the radius is decreasing at a constant rate of $0.1cm/s$. Find the amount of water formed in one second when the radius of the sphere is $7cm$. (Given $\pi = \frac{22}{7}$, sp. gr. ofice = 0.9.)

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33. Use differentials to obtain a resonable approximation to $(8.01)^{\frac{4}{3}} + (8.01)^2$.

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34. Let C be the curve $y^3 - 3xy + 2 = 0$. If H is the set of points on the curve C, where the tangent is horizontal and V is the set of points on the curve C, where the tangent is vertical, then H = ... and V = ...

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35. A spherical rain drop evaporates at a rate proportional to its surface area at any instant t. The differential equation giving the rate of change of the radius of the rain drop is _____

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36. The law of rectilinear motion of a body of mass 10kg is given by $s = 2t^2 + 3t + 4$. Then

the kinetic energy $\frac{1}{2}mv^2$ of the body after 5s from the start is .....

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37. The slope of the tangent to the curve $r^2 = a^2 \cos 2\theta$, where $x = r\cos\theta$ and $y = r\sin\theta$, at the point $\theta = \frac{\pi}{6}$ is

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38. A balloon is pumped at the rate of 10 cu cm/min. The rate of increase of its radius will beits radius is $15cm$.

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39. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then $a > 0, b > 0$ $a > 0, b < 0$ $a\langle 0, b\rangle 0$ (d) $a < 0, b < 0$ none of these

A. a ge 0, b ge 0

B. a ge 0, b le 0

C. a le 0,b ge 0

D. a le 0, b le 0

Answer:

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40. The angle between the curves $y = \sin x$ and $y = \cos x$ is

A. pi/4

B. tan^-1(2sqrt(2))

C. tan^-1(frac{1}{2sqrt(2)})

D. None of these

Answer:

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41. The point on the ellips $9x^2 + 16y^2 = 400$ at which the abscissa and ordinate decrease at the same rate is

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42. IF slope of tangent to curve $y = x^3$ at a point is equal to ordinate of point , then point is

A. (0,0)

B. (27,3)

C. (3,27)

D. none of these

Answer:

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43. Find the equations of the tangent and the normal to the curve $x = 3\cos\theta - \cos^3\theta,\ \ y = 3\sin\theta - s \in^3 \theta$

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44. Let $f(x)$ be a differentiable function and $f(x) > 0$ for all x. Prove that the curves $y = f(x)$ and $y = \sin kx \cdot f(x)$ touch each

other at the points of intersection of the two curves.

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45. Prove that the curve $y = e^{|x|}$ cannot have a unique tangent line at the point $x = 0$. Find the angle between the one-sided tangents to the curve at the point $x = 0$.

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