

## **PHYSICS**

### **BOOKS - DC PANDEY ENGLISH**

### SUPERPOSITION OF WAVES

### **Example**

- 1. In interference, two individual amplitudes are  $A_0$  each and the intensity is  $I_0$  each. Find resultant amplitude and intensity at a point, where:
- (a) phase difference between two waves is  $60^{\circ}$ .
- (b) path difference between two waves is  $\frac{\lambda}{3}$ .



2. Three waves from three coherent sources meet at some point.

Resultant amplitude of each is  $A_0$ . Intensity corresponding to  $A_0 is I_0$ .

Phase difference between first wave and second wave is  $60^\circ$ . Path difference between first wave and third wave is  $\frac{\lambda}{3}$ . The first wave lags behind in phase angle from second and third wave. Find resultant intensity at this point.



**3.** Two waves of equal frequencies have their amplitude in the ratio of 5:3. They are superimposed on each other. Calculate the ratio of the

maximum to minimum intensities of the resultant wave.



- **4.** In interference,  $\dfrac{I_{\max}}{I_{\min}}=lpha$  , find (a)  $\dfrac{A_{\max}}{A_{\max}}$  (b)  $\dfrac{A_1}{A_1}$  (c)  $\dfrac{I_1}{I_1}$  .
  - Match Video Calution

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**5.** The displacement of a standing wave on a string is given by

$$y(x,t) = 0.4\sin(0.5x)\cos(30t)$$

where x and y are in centimetres.

- (a) Find the frequency, amplitude and wave speed of the component waves.
- (b) What is the particle velocity at x=2.4cm at t=0.8s?



- **6.** The vibrations of a string of length 60 cm fixed at both ends are represented by the equation  $y=4\sin\Bigl(\frac{\pi x}{15}\Bigr)\cos(96\pi t)$ , where x and y are in cm and t in seconds.
- (a) What is the maximum displacement of a point at x=5cm?
- (b)Where are the nodes located along the string?

superposition gives the above wave.

- (c)What is the velocity of the particle at x=7.5cm and t=0.25s?
- (d)Write down the equations of the component waves whose

**7.** In terms of T,  $\mu$  and I, find frequency of

- (a) fourth overtone mode
- (b) third harmonic.

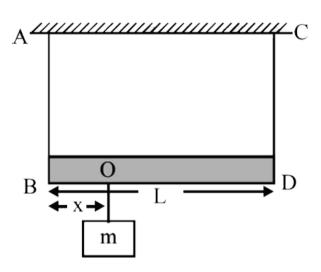


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 ${\it CD}$  of equal length. A block of mass m is suspended from point  ${\it O}$  such that BO is equal to 'x'. Further it is observed that the frequency of 1st

**8.** A massless rod of length L is suspened by two identical string AB and

harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is



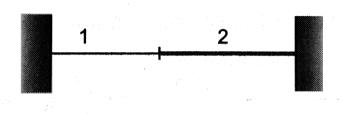


**9.** An object of specific gravity  $\rho$  is hung from a thin steel wire. The fundamental frequency for transverse standing waves in wire is 300Hz.

The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is



**10.** Two strings 1 and 2 are taut between two fixed supports (as shown in figure) such that the tension in both strings is same. Mass per unit length of 2 is more than that of 1. Explain which string is denser for a transverse travelling wave.





11. A sound wave and a light wave are reflected and refracted (or transmitted) from water surface. Show the changes in different physical quantities associated with a wave.



12. A harmonic wave is travelling on string 1. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string, and that the boundary between the two strings is at x=0. If the expression for the incident wave is

$$u_i = A_i \cos(k_1 x - \omega_1 t)$$

- (a) What are the expressions for the transmitted and the reflected waves in terms of  $A_i,\,k_1$  and  $\omega_1$ ?
- (b) Show that the average power carried by the incident wave is equal to the sum of the average power carried by the transmitted and reflected waves.



13. From energy conservation principle prove the relations,

$$A_r = igg(rac{v_2-v_1}{v_1+v_2}igg)A_i$$
 and  $A_t = igg(rac{2v_2}{v_1+v_2}igg)A_i$ 

Here, symbols have their usual meanings.



# **Example Type 1**

**1.** A stretched wire is oscillating in third overtone mode. Equation of transverse stationary wave produced in this wire is

$$y = A\sin(6\pi x)\sin(20\pi t)$$

Here, x is in metres. Find the length of the wire.



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## **Example Type 2**

**1.** Length of a stretched wire is 2m. It is oscillating in its fourth overtone mode. Maximum amplitude of oscillations is 2mm. Find amplitude of oscillation at a distance of 0.2m from one fixed end.



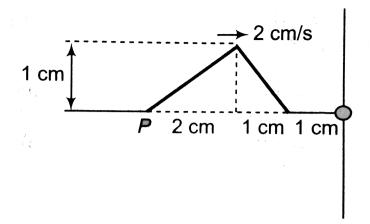
1. A standing wave is formed by two harmonic waves,  $y_1=A\sin(kx-\omega t)$  and  $y_2=A\sin(kx+\omega t)$  travelling on a string in opposite directions. Mass density is ' $\rho$ ' and area of cross section is S. Find the total mechanical energy between two adjacent nodes on the string.



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### **Example Type 4**

**1.** A triangular wave pulse moving at 2 cm/s on a rope approached an end at which it is free to slide on a vertical pole.



- (a) Draw the pulse at  $\frac{1}{2}s$  interval until it is completely reflected.
- (b) What is the particle speed on the trailing edge P at the instant depicted?



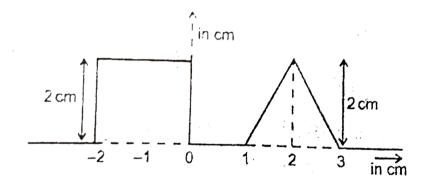
# Miscellaneous Example

- **1.** A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 18 cm and 16 cm, respectively.
- (a) What is the minimum possible length of the string?

(b) If the tension is 10 N and the linear mass density is 4 g/m, what is the fundamental frequency?



**2.** The figure shown at time t=0 second, a rectangular and triangular pulse on a uniform wire are approcaching other. The pulse speed is  $0.5cm\,/s$  . The resultant pulse at t=2 second is





**3.** Two wires are fixed in a sanometer. Their tension are in the ratio 8:1 The lengths are in the ratio 36:35 The diameter are in the ratio 4:1 Densities of the materials are in the ratio 1:2 if the lower frequency in

the setting is 360Hz. The beat frequency when the two wires are sounded together is



**4.** In a stationary wave pattern that forms as a result of reflection pf waves from an obstacle the ratio of the amplitude at an antinode and a node is  $\beta=1.5$ . What percentage of the energy passes across the obstacle?



- **5.** A string of linear mass density  $5.0 \times 10^{-3}$  kg//m is stretched under a tension of 65 N between two rigid supports 60 cm apart.
- (a) If the string is vibrating in its second overtone so that the amplitude at one of its antinodes is 0.25 cm, what are the maximum transverse
- (b) What are these quantities at a distance 5.0 cm from an node?

speed and acceleration of the string at antinodes?

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**6.** An aluminium wire of cross-sectional area  $(10^{-6})m^2$  is joined to a steel wire of the same cross-sectional area. This compound wire is stretched on a sonometer pulled by a weight of 10kg. The total length of the compound wire between the bridges is 1.5m of which the aluminium wire is 0.6m and the rest is steel wire. Transverse vibrations are setup in the wire by using an external source of variable frequency. Find the lowest frequency of excitation for which the standing waves are formed such that the joint in the wire is a node. What is the total number of nodes at this frequency? The density of aluminium is  $2.6 \times (10^3) kg/m^3$  and that of steel is  $1.04 \times 10^4 kg/m^2 (g=10m/s^2)$ 



7. Find the resultant amplitude and phase of a point at which N sinusoidal waves interfere. All the waves have same amplitude A and their phases increase in arithmetic progression of common difference  $\phi$ .



# Exercise 18 1

1. The ratio of intensities of two waves is 9:16. If these two waves interfere, then determine the ratio of the maximum and minimum possible intensities.



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2. In interference, two individual amplitudes are 5 units and 3 units. Find

(a)
$$rac{A_{
m max}}{A_{
m min}}$$
 (b)  $rac{I_{
m max}}{I_{
m min}}$  .



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3. Three waves due to three coherent sources meet at one point. Their amplitudes are  $\sqrt{2}A_0,\,3A_0$  and  $\sqrt{2}A_0$  . Intensity corresponding to  $A_0$  is  $l_0$ . Phasse difference between first and second is  $45^{\circ}$ . Path difference

between first and third is  $\frac{\lambda}{4}$ . In phase angle, first wave lags behind from

the other two waves. Find resultant intensity at this point.



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# Exercise 18 2

1. In stationary wave, phase difference between two particles can't be  $\frac{\pi}{2}$ .

Is this statement true or false?



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2. A stationary wave is given by

$$y = 5 \frac{\sin(\pi x)}{3} \cos 40\pi t$$

where x and y are in cm and t is in second.

- a. What are the amplitude and velocity of the component wave whose superposition can give rise to this vibration?
- b. What is the distance between the nodes?

c. What is the velocity of a particle of the string at the position

x = 1.5 cmwhent = 9/8s?



**3.** If two wave of the same frequency differ in amplitude and are propagated in opposite directions through a medium, will they produce standing waves? Is energy transported? Are there any nodes?



**4.** Two sinusoidal waves travelling in opposite directions interfere to produce a standing wave described by the equation

 $y = (1.5m)\sin(0.400x)\cos(200t)$ 

where, x is in meters and t is in seconds. Determine the wavelength, frequency and speed of the interfering waves.



**1.** Sound waves of frequency  $660H_Z$  fall normally on perfectly reflecting wall. The distance from the wall at which the air particles have the maximum amplitude of vibration is (speed of sound in air = 330m/s)



**2.** If the frequencies of the sound second and fifth harmonics of a string differ by 54 Hz. What is the fundamental frequency of the string?



**3.** A wire is attached to a pan of mass 200 g that contains a 2.0 kg mass as shown in the figure. When plucked, the wire vibrates at a fundamental frequency of 220 Hz. An additional unknown mass M is then added to the pan and a fundamental frequency of 260 Hz is detected. What is the value





- **4.** A wire fixed at both ends is 1.0 m long and has a mass of 36g. One of its oscillation frequencies is 250Hz and the next higher one is 300 Hz.
- (a) Which harmonics do these frequencies represent?
- (b) What is the tension in the wire?

**5.** Two different stretched wires have same tension and mass per unit length. Fifth overtone frequency of the first wire is equal to second harmonic frequency of the second wire. Find the ratio of their lengths.



# Exercise 18 4

**1.** Two wave pulses identical in shape but inverted with respect to each other are produced at the two ends of a stretched string. At an instant when the pulses reach the middle, the string becomes completely straight. What happens to the energy of the two pulses?



**2.** The pulse shown in figure has a speed of  $10cm\,/\,s$  .



- (a) If the linear mass density of the right string is 0.25 that of the left string, at what speed does the transmitted pulse travel?
- (b) Compare the heights of the transmitted pulse and the reflected pulse to that of the incident pulse.



**3.** The harmonic wave  $y_i=\left(2.0\times 10^{-3}\right)\cos\pi(2.0x-50t)$  travels along a string towards a boundary at x=0 with a second string. The wave speed on the second string is 50m/s. Write expressions for reflected and transmitted waves. Assume SI units.



**1.** Assertion: Two waves  $y_1=A\sin(\omega t-kx)$  and  $y_2=A\cos(\omega t-kx)$  are superimposed, then x=0 becomes a node.

Reason: At node net displacement due to waves should be zero.

A. (a)Both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. (b)Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. (c)Assertion is true, but the Reason is false.

D. (d)Assertion is false but the Reason is true.

### **Answer: D**



**2.** Assertion: Stationary waves are so called because particles are at rest in stationary waves.

Reason: They are formed by the superposition of two identical waves travelling in opposite directions.

A. (a)Both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. (b)Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. (c)Assertion is true, but the Reason is false.

D. (d)Assertion is false but the Reason is true.

### **Answer: D**



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**3.** Assertion: When a wave travels from a denser medium to rarer medium, then amplitude of oscillation increases.

Reason: In denser medium, speed of wave is less compared to a rarer medium.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

#### Answer: B



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**4.** Assertion: A wire is stretched and then fixed at two ends. It oscillates in its second overtone mode. There are total four nodes and three antinodes.

Reason: In second overtone mode, length of wire should be  $l=rac{3\lambda}{2}$ , where  $\lambda$  is wavelength.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

### **Answer: B**



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**5.** Assertion: If we see the oscillations of a stretched wire at higher overtone mode, frequency of oscillation increases but wavelength decreases.

Reason: From  $v=f\lambda,\,\lambda\proptorac{1}{f}$  as v = constant.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

#### **Answer: A**



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**6.** Assertion: Standing waves are formed when amplitudes of two constituent waves are equal.

Reason: At any point net displacement at a given time is resultant of displacement of constituent waves.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

### **Answer: D**



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7. Assertion: In a standing wave x=0 is a node. Then, total mechanical energy lying between x=0 and  $x=\left(\frac{\lambda}{8}\right)$  is not equal to the energy lying between  $x=\frac{\lambda}{8}$  and  $x=\frac{\lambda}{4}$ .

Reason: In standing waves different particles oscillate with different amplitudes.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

#### **Answer: A**



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**8.** Assertion: Ratio of maximum intensity and minimum intensity in interference is 25:1. The amplitudes ratio of two waves should be 3:2.

Reason: 
$$rac{I_{
m max}}{I_{
m min}} = \left(rac{A_1+A_2}{A_1-A_2}
ight)^2$$
 .

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

Answer: A

**9.** Assertion: Three waves of equal amplitudes interfere at a point. Phase difference between two successive waves is  $\frac{\pi}{2}$ . Then, resultant intensity is same as the intensity due to individual wave.

Reason: Two different light sources are never coherent.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

### **Answer: B**



10. Assertion: For two sources to be coherent phase difference between

two waves at all points should be same.

Reason: Two different light sources are never coherent.

A. If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

B. If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C. If Assertion is true, but the Reason is false.

D. If Assertion is false but the Reason is true.

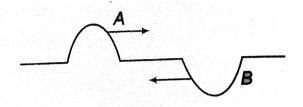
## Answer: D



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Level 1 Objective

1. Two identical harmonic pulses travelling in opposite directions in a taut string approach each other. At the instant when they completely overlap, the total energy of the string will be



A. zero

B. partly kinetic and partly potential

C. purely kinetic

D. purely potential

### Answer: C



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**2.** Three coherent waves having amplitudes 12mm, 6mm and 4mm arrive at a given point with successive phase difference of  $\frac{\pi}{2}$ . Then, the

amplitude of the resultant wave is A. 7mm B. 10mm C. 5mm D. 4.8 mm **Answer: B** Watch Video Solution **3.** A wave  $y=a\sin(\omega t-kx)$  on a string meets with another wave producing a node at x=0. Then the equation of the unknown wave is  $A + a \cos(kx - \omega t)$  $B. -a\cos(kx + \omega t)$  $\mathsf{C.} - a\cos(kx - \omega t)$ 

 $D. + a\cos(\omega t - kx)$ 

# Answer: C



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- 4. In a stationary wave system, all the particles of the medium
  - A. have zero displacement simultaneously at some instant
  - B. have maximum displacement simultaneously at some instant
  - C. are at rest simultaneously at some instant
  - D. all of the above

### Answer: D



- **5.** In a standing transerse wave on a string :
  - A. In one time period all the particles are simultaneously at rest twice

once in a time period

C. All the particles may be at their positive extremes simultaneously twice in a time period

B. All the particles must be at their positive extremes simultaneously

D. All the particles are never at rest simultaneously

### Answer: A



- **6.** If  $\lambda_1, \lambda_2$  and  $\lambda_3$  are the wavelength of the waves giving resonance to the fundamental, first and second overtone modes respectively in a string fixed at both ends. The ratio of the wavelengths  $\lambda_1:\lambda_2:\lambda_3$  is
  - A. 1:2:3
  - B. 1:3:5
  - C. 1:  $\frac{1}{2}$ :  $\frac{1}{3}$
  - D. 1:  $\frac{1}{3}$ :  $\frac{1}{5}$

### **Answer: C**



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**7.** For a certain stretched string, three consecutive resonance frequencies are observed as 105, 175 and 245 Hz respectively. Then, the fundamental frequency is

A. 30 Hz

B. 45 Hz

C. 35 Hz

D. None of these

## **Answer: C**



**8.** A string 1m long is drawn by a 300 Hz vibrator attached to its end. The string vibrates in three segments. The speed of transverse waves in the string is equal to

- A. 100m/s
- B. 200m/s
- $\mathsf{C.}\,300m\,/s$
- D. 400m/s

#### **Answer: B**



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**9.** Three resonant frequencies of a string are 90, 150 and 210 Hz. (a) Find the highest possible fundamental frequency of vibration of this string. (b) Which harmonics of the fundamental are the given frequencies? (c) Which overtones are these frequencies? (d) If the length of the string is 80 cm, what would be the speed of a transverse wave on this string?

A. 45m/s

B. 75m/s

C. 48m/s

D. 80m/s

### **Answer: C**



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10. The period of oscillations of a point is 0.04 s and the velocity of propagation of oscillation is 300 m/s. The difference of phases between the oscillations of two points at distance 10 m and 16 m respectively from the source of oscillations is

A. 
$$2\pi$$

$$\operatorname{B.}\frac{\pi}{2}$$

C. 
$$\frac{n}{4}$$

D. 
$$\pi$$

#### **Answer: D**



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11. A transverse wave described by equation  $y=0.02\sin(x+30t)$  (where x and t are in meters and seconds, respectively) is travelling along a wire of area of cross-section  $1mm^2$  and density  $8000kg/m^2$ . What is the tension in the string?

- A. (a)20 N
- B. (b)7.2 N
- C. (c)30 N
- D. (d)14.4 N

## **Answer: B**



# **Objective Questions**

- 1. In a standing wave, node is a point of
  - A. maximum strain
  - B. maximum pressure
  - C. maximum density
  - D. All of these

#### Answer: D



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# **Level 1 Subjective**

1. Two waves are travelling in same direction along a stretched string. The waves are  $90^{\circ}$  out of phase. Each wave has an amplitude of 4.0 cm. Find the amplitude of the resultant wave.

- 2. Two wires of different densities are soldered together end to end then stretched under tension T. The waves speed in the first wire is twice that in the second wire.
- (a) If the amplitude of incident wave is A, what are amplitudes of reflected and transmitted waves?
- (b) Assuming no energy loss in the wire, find the fraction of the incident power that is reflected at the junction and fraction of the same that is transmitted.



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3. A wave is represented by

$$y_1 = 10\cos(5x + 25t)$$

where,  $\boldsymbol{x}$  is measured in meters and  $\boldsymbol{t}$  in seconds. A second wave for which

$$y_2=20\cos\Bigl(5x+25t+rac{\pi}{3}\Bigr)$$

interferes with the first wave. Deduce the amplitude and phase of the resultant wave.



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waves passing through a region are represented by  $y = (1.0cm)\sin[(3.14cm^{-1})x - (157s^{-1}x - (157s^{-1})t]$  $y = (1.5cm)\sin[(1.57cm^{-1})x - (314s^{-1})t].$ and Find the displacement of the particle at x = 4.5 cm at time t = 5.0 ms.

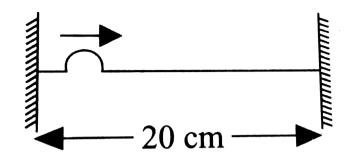


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**5.** A string of length 20cm and linear mass density 0.40g/cm is fixed at both ends and is kept under a tension of 16N. A wave pulse is produced at t=0 nearj an end as shown in figure which travels towards the other end.

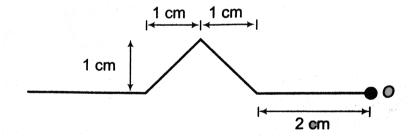
when will the string have the shape shown in the figure again?

$$\left( \ \in \ imes 10^{-2} s 
ight)$$





**6.** A wave pulse on a string has the dimensions shown in figure. The wave speed is  $v=1cm\,/\,s.$ 



- (a) If point O is a fixed end, draw the resultant wave on the string at t=3 s and t=4 s.
- (b) Repeat part (a) for the case in which O is a free end.

**7.** Two sinusoidal waves combining in a medium are described by the equations

$$y_1 = (3.0cm)\sin\pi(x + 0.60t)$$

and 
$$y_2 = (3.0cm)\sin \pi (x - 0.06t)$$

where, x is in centimetres and t is in seconds. Determine the maximum

- displacement of the medium at
- (b)x=0.500 cm and
- (c) x=1.50 cm.

(a)x=0.250 cm,

(d) Find the three smallest values of x corresponding to antinodes.



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**8.** A standing wave is formed by the interference of two travelling waves, each of which has an amplitude  $A=\pi cm$  , angular wave number  $k=(\pi/2)$  per centimetre.

- (a) Calculate the distance between two successive antinodes.

  (b) What is the amplitude of the standing wave at x = 0.50cm from a
- (b) What is the amplitude of the standing wave at x=0.50cm from a node?



**9.** Find the fundamental frequency and the next three frequencies that could cause a standing wave pattern on a string that is 30.0 m long, has a mass per unit length of  $9.00\times 10^{-3}kg/m$  and is stretched to a tension of 20.0 N.



- 10. A string vibrates in its first normal mode with a frequency of 220 vibrations /s. The vibrating segment is 70.0 cm long and has a mass of 1.20g.
- (a) Find the tension in the string.
- (b) Determine the frequency of vibration when the string vibrates in three segments.



11. A 60.0 cm guitar string under a tension of 50.0 N has a mass per unit length of 0.100 g/cm. What is the highest resonance frequency of the string that can be heard by a person able to hear frequencies upto 20000 Hz?



12. A wire having a linear density of 0.05g/cm is stretched between two rigid supports with a tension of 450N. It is observed that the wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.



**13.** The vibrations from an 800 Hz tuning fork set up standing waves in a string clamped at both ends. The wave speed in the string is known to be

400m/s for the tension used. The standing wave is observed to have four antinodes. How long is the string?



- 14. A string vibrates in 4 segments to a frequency of 400 Hz.
- (a) What is its fundamental frequency?
- (b) What frequency will cause it to vibrate into 7 segments?



15. A sonometer wire has a total length of 1m between the fixed ends. Where should the two bridges be placed below the wire so that the three segments of the wire have their fundamental frequencies in the ratio 1:2



:3?

**16.** A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamatal frequecy of 186 Hz?



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17. Adjacent antinodes of a standing wave on a string are 15.0 cm apart. A particle at an antinode oscillates in simple harmonic motion with amplitude 0.850 cm and period 0.0750 s. The string lies along the  $\pm$ x-axis and is fixed at x=0.

- (a) find the displacement of a point on the string as a function of position and time.
- (b) Find the speed of propagation of a transverse wave in the string.
- (c) Find the amplitude at a point 3.0 cm to the right of an antinode.



**18.** A 1.50 m long rope is stretched between two supports with a tension that makes the speed of transverse waves 48.0m/s. What are the wavelength and frequency of

- (a) the fundamental?
- (b) the second overtone?
- (c) the fourth harmonic?



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- **19.** A wire with mass 40.0 g is stretched so that its ends are tied down at points 80.0 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the antinodes of 0.300 cm.
- (a) What is the speed of propagation of transverse wave in the wire?

  Compute the tension in the wire.
- (c) Find the maximum transverse velocity and acceleration of particles in the wire.



20. Two harmonic waves are represented in SI units by

$$y_1(x,t) = 0.2\sin(x-3.0t)$$
 and  $y_2(x,t) = 0.2\sin(x-3.0t+\phi)$ 

- (a) Write the expression for the sum  $y=y_1+y_2f$  or  $\phi=\frac{\pi}{2}rad$ .
- (b) Suppose the phase difference  $\phi$  between the waves is unknown and the amplitude of their sum is 0.32 m, what is  $\phi$ ?



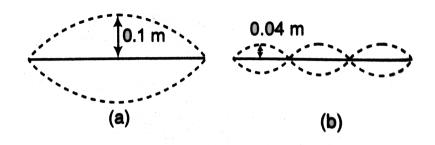
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**21.** Figure shows different standing wave patterns on a string of linear mass density  $4.0 \times 10^{-2} kg/m$  under a tension of 100 N. The amplitude of antinodes is indicated in each figure. The length of the string is 2.0m. (i) Obtain the frequencies of the modes shown in figures (a) and (b).

each mode. (Take the initial configuration of the wire in each mode to be

(ii)Write down the transverse displacement y as a function of x and t for

as shown by the dark lines in the figure).





**22.** A string fastened at both ends has successive resonances with wavelengths of 0.54 m for nth harmonic and 0.48 m for the (n+1) th harmonic and 0.48 m for the (n+1) th harmonic.

- (a) Which harmonics are these?
- (b) What is the length of the stirng?
- (c) What is the wavelength of the fundamental frequency?



**23.** A wave  $y_i=0.3\cos(2.0x-40t)$  is travelling along a string toward a boundary at x=0. Write expressions for the reflected waves if .

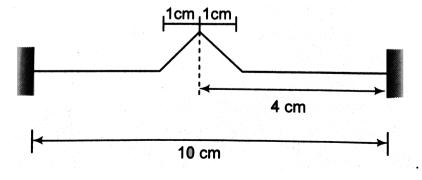
- (a) the string has a fixed end at x=0 and
- (b) The string has a free end at x=0.

Assume SI units.



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**24.** A string that is 10 cm long is fixed at both ends. At t=0, a pulse travelling from left to right at 1cm/sis4.0cm from the right end as shown in figure. Determine the next two times when the pulse will be at that point again. State in each case whether the pulse is upright or inverted .



## Subjective Questions

- 1. A thin taut string tied at both ends and oscillating in its third harmonic
- has its shape described by the equation

$$y(x,t)=(5.60cm)\sin\left[\left(0.0340\frac{\mathrm{rad}}{c}m\right)x\right]\sin\left[\left(50.0\frac{\mathrm{rad}}{s}\right)t\right]$$
, where the origin is at the left end of the string, the x-axis is along the string and the

- y-axis is perpendicular to the string.
- (a) Draw a sketch that shows the standing wave pattern.
- (b)Find the amplitude of the two travelling waves that make up this standing wave.
- (c) What is the length of the string?
- (d) Find the wavelength, frequency, period and speed of the travelling wave.
- (e) Find the maximum transverse speed of a point on the string.
- (f) What would be the equation y(x,t) for this string if it were vibrating in its eighth harmonic?

- **2.** A 160 g rope 4 m long is fixed at one end and tied to a light string of the same length at the other end. Its tension is 400 N.
- (a) What are the wavelength of the fundamental and the first two overtones?
- (b) What are the frequencies of these standing waves?

[Hint: In this case, fixed end is a node and the end tied with the light string is antinode.]



**3.** Two pulses travelling in opposite directions along a string are shown for t=0 in the figure. Plot the shape of the string at `t= 1.0, 2.0, 3.0, 4.0 and 5.0s respectively .





**4.** Sources separated by 20 m vibrate according to the equation  $y_1=0.06\sin\pi t$  and  $y_2=0.02\sin\pi t$ . They send out waves along a rod with speed 3m/s What is the equation of motion of a particle 12 m from the first source and 8 m from the second,  $y_1, y_2$  are in m?



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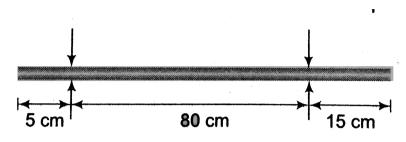
**5.** Three component sinusoidal waves progressing in the same directions along the same path have the same period byt their amplitudes are  $A, \frac{A}{2}$  and  $\frac{A}{3}$ . The phases of the variation at any position x on their path at time t=0 are  $0, -\frac{\pi}{2}$  and  $-\pi$  respectively. Find the amplitude and phase of the resultant wave.



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**6.** A metal rod of length 1m is clamped at two points as shown in the figure. Distance of the clamp from the two ends are 5 cm and 15 cm, respectively. Find the minimum and next higher frequency of natural

longitudinal oscillation of the rod. Given that Young's modulus of elasticity and density of aluminium are  $Y=1.6\times10^{11}Nm^{-2}$  and  $ho=2500kgm^{-3}$  , respectively.





## **Level 2 Single Correct**

**1.** When tension of a string is increased by 2.5 N, the initial frequency is altered in the ratio of 3:2. The initial tension in the string is

A. 6N

B. 5N

C. 4N

#### **Answer: D**



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- 2. The lengths of two wires of same material are in the ratio 1:2, their tensions are in the ratio 1:2 and their diameters are in the ratio 1:3. the ratio of the notes they emits when sounded together by the same source is
  - A.  $\sqrt{2}$
  - B.  $\sqrt{3}$
  - $\mathsf{C.}\,2\sqrt{3}$
  - D.  $3\sqrt{2}$

### **Answer: D**



**3.** When a string is divided into three segments of length  $l_1$ ,  $l_2$  and  $l_3$  the fundamental frequencies of these three segments are  $f_1,\,f_2$  and  $f_3$ 

respectively. The original fundamental frequency f of the string is

A. (a) 
$$f=f_1+f_2+f_3$$

B. (b) 
$$\dfrac{1}{f}=\dfrac{1}{f_1}+\dfrac{1}{f_2}+\dfrac{1}{f_3}$$
 C. (c)  $\dfrac{1}{\sqrt{f}}=\dfrac{1}{\sqrt{f}_1}+\dfrac{1}{\sqrt{f}_2}+\dfrac{1}{\sqrt{f}_3}$ 

D. (d) None of these

### Answer: B



4. Three one-dimensional mechanical waves in an elastic medium is given as

 $y_1 = 3A\sin(\omega t - kx), y_2 = A\sin(\omega t - kx + \pi)$  and  $y_3 = 2A\sin(\omega t + kx)$ are superimposed with each other. The maximum displacement amplitude

of the medium particle would be

Α.	4/

B. 3A

C. 2A

D. A

#### **Answer: A**



- **5.** A string is under tension so that its length is increased by 1/n times its original length . The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be
  - A.  $\eta$ : 1
  - B.  $1:\eta$
  - C.  $\sqrt{\eta}$ : 1
  - D.  $1:\sqrt{\eta}$

### **Answer: D**



## **Watch Video Solution**

**6.** A sting of length 1m and linear mass density 0.01 kg/m is stretched to a tension of 100 N. When both ends of the string are fixed, the three lowest frequencies for standing wave are  $f_1$ ,  $f_2$  and  $f_3$ . When only one end of the string is fixed, the three lowest frequencies for standing wave are  $n_1$ ,  $n_2$  and  $n_3$ . Then,

A. 
$$n_3 = 5n_1 = f_3 = 125Hz$$

B. 
$$f_3=5f_1=n_2=125Hz$$

C. 
$$f_3 = n_2 = 3f_1 = 150Hz$$

D. 
$$n_2=rac{f_1+f_2}{2}=75Hz$$

### Answer: D



**7.** A sonometer wire has a length of 114 cm between its two fixed ends. Where should the two bridges be places so as to divide the wire into three segments, whose fundamental frequencies are in the ratio 1:3:4?

A. 
$$l_1 = 72cm, l_2 = 24cm, l_3 = 18cm$$

$$\mathrm{B.}\,l_1=60cm,\,l_2=40cm,\,l_3=14cm.$$

$$\mathsf{C.}\,l_1=52cm,\,l_2=30cm,\,l_3=32cm.$$

$$\mathrm{D.}\,l_1=65cm,\,l_2=30cm,\,l_3=19cm.$$

#### **Answer: A**



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**8.** The frequency of sonometer wire is f. The frequency becomes f/2 when the mass producing the tension is completely immersed in water and on immersing the mass in a certain liquid, frequency becomes f/3. The relative density of the liquid is

A. 0.32

- B. 1.67
- C. 1.41
- D. 1.18

### Answer: D



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fundamental mode. Amplitude at the centre of the string is 4 mm.

9. A string of length 1.5 m with its two ends clamped is vibrating in

Minimum distance between the two points having amplitude 2 mm is:

- A. 1m
- B. 75 cm
- C. 60 cm
- D. 50 cm

Answer: A

10. A man generates a symmetrical pulse in a string by moving his hand up and down. At t=0 the point in his hand moves downward. The pulse travels with speed of 3m/s on the string & his hands passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t=

A. 1.25 s

B. 1s

 $\mathsf{C.}\ 11 \, / \, 12s$ 

D. 23/24 s

**Answer: C** 



11. Among two interfering sources, let  $S_1$  be ahead of the phase by  $90^\circ$  relative to  $S_2$ . If an observation point P is such that  $PS_1-PS_2=1.5\lambda$ , the phase difference between the waves from  $S_1$  and  $S_2$  reaching P is

- A.  $(a)3\pi$
- $\mathsf{B.}\left(b\right)\frac{5\pi}{2}$
- $\mathrm{C.}\left(c\right)\frac{7\pi}{2}$
- D.  $(d)4\pi$

#### **Answer: B**



## **Watch Video Solution**

**12.** A wire having a linear density 0.1kg/m is kept under a tension of 490N. It is observed that it resonates at a frequency of 400 Hz. The next higher frequency is 450 Hz. Find the length of the wire.

A. (a) 0.4 m



C. (c) 0.6m

D. (d) 0.49m

## **Answer: B**



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frequency that will cause it to vibrate in 2 segments will be

13. A string vibrates in 5 segments to a frequency of 480 Hz. The

A. (a) 96 Hz

B. (b) 192 Hz

C. (c) 1200 Hz

D. (d) 2400 Hz

## **Answer: B**



**14.** A wave travels on a light string. The equation of the waves is  $y=A\sin(kx-\omega t+30^\circ)$ . It is reflected from a heavy string tied to an end of the light string at x = 0. If 64% of the incident energy is reflected, then the equation of the reflected wave is

A. 
$$y=0.8A\sin(kx-\omega t+30^{\circ}+180^{\circ})$$

B. 
$$y=0.8A\sin(kx+\omega t+30^\circ+180^\circ)$$

C. 
$$y=0.8A\sin(kx-\omega t-30^\circ)$$

D. 
$$y=0.8A\sin(kx-\omega t+30^\circ)$$

#### Answer: B



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**15.** The tension, length, diameter and density of a string B are double than that of another string A. Which of the following overtones of B is same as the fundamental frequency of A?

A. (a)1st B. (b)2nd C. (c)3rd D. (d)4th **Answer: C Watch Video Solution Level 2 More Than One Correct** 1. If the tension in a stretched string fixed at both ends is changed by  $21\,\%$  , the fundamental frequency is found to increase by 15Hz, then the A. original frequency is 150 Hz B. velocity of propagation of the transverse wave along the string increases by 5%.

C. velocity of propagation of the transverse wave along the string increases by 10% .

D. fundamental wavelength on the string does not change.

## Answer: A::C::D



## 2. For interference to take place

A. sources must be coherent

B. sources must have same amplitude

C. waves should travel in opposite directions

D. sources must have same frequency

## Answer: A::D



A. (a) This is an example of interference
B. (b) Amplitudes of waves may be different
C. (c) particles at nodes are always at rest
D. (d) Energy is conserved
Answer: A::B::D
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4. When a wave travels from a denser to rarer medium, then
A. speed of wave increases
B. wavelength of wave decreases
C. amplitude of wave increases
D. there is no change in phase angle

**3.** Regarding stationary waves, choose the correct options.



## **Watch Video Solution**

**5.** A wire is stretched and fixed at two ends. Transverse stationary waves are formed in it. It oscillates in its third overtone mode. The equation of stationary wave is

$$y = A\sin kx \cos \omega t$$

Choose the correct options.

- A. Amplitude of constituent waves is  $rac{A}{2}$
- B. The wire oscillates in three loops
- C. The length of the wire is  $\frac{4\pi}{k}$
- D. Speed of stationary wave is  $\frac{\omega}{k}$

### Answer: A::C



6. Which of the following equations can form stationary waves?

(i) $y = A\sin(\omega t - kx)$  (ii)  $y = A\cos(\omega t - kx)$ 

(iii)  $y = A \sin(\omega t + kx)$  (iv)  $y = A \cos(\omega t - kx)$  .

A. (i) and (ii)

B. (i) and (iii)

C. (iii) and (iv)

D. (ii) and (iv)

## Answer: B::D



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## **More Than One Correct Options**

1. Two waves

 $y_1 = A\sin(\omega t - kx)$ 

and  $y_2 = A \sin(\omega t + kx)$ 

superimpose to produce a stationary wave, then

A. x=0 is a node

B. x=0 is an antinode.

C.  $x = \frac{\pi}{k}$  is a node.

D.  $\pi=rac{2\pi}{k}$  is an antinode

## Answer: B::D



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# Level 2 Comprehension Based

1. Incident wave  $y=A\sin\left(ax+bt+\frac{\pi}{2}\right)$  is reflected by an obstacle at x = 0 which reduces intensity of reflected wave by 36%. Due to superposition, the resulting wave consists of a standing wave and a travelling wave given by

 $y = -1.6 \sin ax \sin bt + cA \cos(bt + ax)$ 

where A, a, b and c are positive constants.

1. Amplitude of reflected wave is

A. 0.6A

B. 0.8A

C. 0.4A

D. 0.2A

#### **Answer: B**



## **Watch Video Solution**

- **2.** Incident wave  $y=A\sin\Bigl(ax+bt+\frac{\pi}{2}\Bigr)$  is reflected by an obstacle at x = 0 which reduces intensity of reflected wave by 36%. Due to
- superposition, the resulting wave consists of a standing wave and a

where A, a, b and c are positive constants.

 $y = -1.6\sin ax \sin bt + cA\cos(bt + ax)$ 

2. Value of c is

- A. 0.2
- B. 0.4
- C. 0.6
- D. 0.3

#### **Answer: A**



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- **3.** Incident wave  $y = A \sin\!\left(ax + bt + \frac{\pi}{2}\right)$  is reflected by an obstacle at x
- = 0 which reduces intensity of reflected wave by 36%. Due to

superposition, the resulting wave consists of a standing wave and a

travelling wave given by

$$y = -1.6 \sin ax \sin bt + cA \cos(bt + ax)$$

where A, a, b and c are positive constants.

3. Position of second antinode is

A. 
$$x=rac{\pi}{3a}$$

B. 
$$x=rac{3\pi}{a}$$

$$\mathsf{C.}\,x = \frac{3\pi}{2a}$$

D. 
$$x=rac{2\pi}{3a}$$

## **Answer: C**



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## Level 2 Subjective

to make a combined string of length 3L. The first piece of string has mass per unit length  $\mu_1$ , the second piece has mass per unit length  $\mu_2=4\mu_1$ 

1. Three pieces of string, each of length L, are joined together end-to-end,

and the third piece has mass per unit length  $\mu_3=\mu_1/4$  .

(a) If the combined string is under tension F, how much time does it take a transverse wave to travel the entire length 3L? Give your answer in terms of L,F and  $\mu_1$  .

(b) Does your answer to part (a) depend on the order in which the three piece are joined together? Explain.



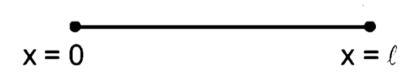
**2.** In a stationary wave that forms as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode to the amplitude at node is 6. What percentage of energy is transmitted?



3. A standing wave  $\xi=a\sin kx.\cos\omega t$  is maintained in a homogeneous rod with cross — sectional area S and density  $\rho$ . Find the total mechanical energy confined between the sections corresponding to the adjacent displacement nodes.



**4.** A string tied between x=0 and x=l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length  $\mu$  is given. Find the total energy of the string.

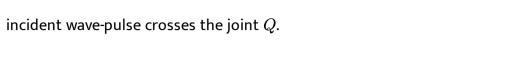




**5.** A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8m and mass 0.06kg. QR has length 2.56m and mass 0.2kg. The wire PQR is under a tension of 80N. A sinusoidal wavepulse of amplitude 3.5cm is sent along the wire PQ from end P. No power is dissipated during the propagation of the wave-pulse. Calculate,

(a) the time taken by the wave-pulse to reach the other end  ${\cal R}$  of the wire,

and
(b) the amplitude of the reflected and transmitted wave-pulse after the



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**6.** A light string is tied at one end to a fixed support and to a heavy string of equal length L at the other end as shown in figure. A block of mass m is tied to the free end of heavy string. Mass per unit length of the strings are  $\mu$  and  $9\mu$  and the tension is T. Find the possible values of frequencies such that junction of two wire point A is a node.





**7.** A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum vibration . The note emitted has a frequency of 100Hz. What

will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum vibratin?



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- **8.**  $y_1=8\sin(\omega t-kx)$  and  $y_2=6\sin(\omega t+kx)$  are two waves travelling in a string of area of cross-section s and density rho. These two waves are superimposed to produce a standing wave.
- (a) Find the energy of the standing wave between two consecutive nodes.
- (b) Find the total amount of energy crossing through a node per second.

