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India's Number 1 Education App

## MATHS

## BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

## Binomial Theorem for Positive Integrel Index

## Example

1. Find the number of terms in the expansion of $\left(x+\sqrt{x^{2}-1}\right)^{6}+\left(x-\sqrt{x^{2}-1}\right)^{6}$

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2. Prove that $13^{99}-19^{93}$ is divisible by 162 .
3. In $\left(33+\frac{1}{33}\right)^{n}$ if the ratio of 7th term from the beginning to the 7th term from the end is $1 / 6$, then find the value of $n$.

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4. The sixth term in the expansion of $\left(\sqrt{2^{\log \left(10-3^{x}\right)}}+\left(2^{(x-2) \log 3}\right)^{\frac{1}{5}}\right)^{m}$ is equal to 21 , if it is known that the binomial coefficient of the 2 nd 3 rd and 4 th terms in the expansion represent, respectively, the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10) The value of $m$ is

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5. Find the coefficient of $x^{3}$ in the expansion of
$\left(1+x+2 x^{2}\right)\left(2 x^{2}-\frac{1}{3 x}\right)^{9}$

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6. The coefficient of $x^{r}[0 \leq r \leq(n-1)]$ in lthe expansion of $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^{2}++(x+2)^{n-1}$ is ${ }^{\wedge} n C_{r}\left(3^{r}-2^{n}\right)$ b. ${ }^{\wedge} n C_{r}\left(3^{n-r}-2^{n-r}\right)$ c. ${ }^{\wedge} n C_{r}\left(3^{r}+2^{n-r}\right)$ d. none of these

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7. Determine the term independent of $a$ in the expansion of $\left(\frac{a+1}{a^{\frac{2}{3}}-a^{\frac{1}{3}}+1}-\frac{a-1}{a-a^{\frac{1}{2}}}\right)^{10}$

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8. If $a, b, c$ and $d$ are any four consecutive coefficients in the expansion of $(1+x)^{n}$, then prove that
(i) $\frac{a}{a+b}+\frac{c}{b+c}=\frac{2 b}{b+c}$
(ii) $\left(\frac{b}{b+c}\right)^{2}>\frac{a c}{(a+b)(c+d)}$, if $x>0$.
9. Let $R=(5 \sqrt{5}+11)^{2 n+1}$ and $f=R-[R]$ where [] denotes the greatest integer function, prove that $R f=4^{2 n+1}$

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10. Find the greatest term in the expansion of $(x+y)^{18}$ when $x=2, y=1$.

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11. Given that the 4th term in the expansion of $[2+(3 / 8 x)]^{10}$ has the maximum numerical value. Then find the range of value of $x$.

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12. Find the coefficient of $x^{n}$ in the expansion of $\left(1+\frac{x}{1!}+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\ldots+\frac{x^{n}}{n!}\right)^{2}$.

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13. If $\sum_{r=0}^{2 n} a_{r}(x-2)^{r}=\sum_{r=0}^{2 n} b_{r}(x-3)^{r}$ and $a_{k}=1$ for all $k \geq n$, then show that $b_{n}={ }^{2 n+1} C_{n+1}$.

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14. Sum the series : ${ }^{1000} C_{50}+2 \cdot{ }^{999} C_{49}+3 \cdot{ }^{998} C_{48}+\ldots+51 \cdot{ }^{950} C_{0}$

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15. 

Prove
that
$n C_{1} \sin x \cdot \cos (n-1) x+n C_{2} \sin 2 x \cdot \cos (n-2) x+n C_{3} \sin 3 x \cdot \cos (n-3)$

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16. find the sum of the series $\sum_{r=0}^{n}(-1)^{r} \quad \cdot{ }^{n} C_{r}$ $\left[\frac{1}{2^{r}}+\frac{3^{r}}{2^{2 r}}+\frac{7^{r}}{2^{3 r}}+\frac{15^{r}}{2^{4 r}} \ldots\right.$ up to m terms $]$

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17. If $s=a+(a+d)+(a+2 d)+\ldots+(a+n d) \quad$ and $S=a+(a+d) \cdot{ }^{n} C_{1}+(a+2 d) \cdot{ }^{n} C_{2}+\ldots+(a+n d) \cdot{ }^{n} C_{n} \quad$ then prove that $(n+1) S=2^{n} \cdot s$.

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18. 

Prove
that
${ }^{\wedge}{ }^{n} C_{0}-{ }^{n} C_{1}+{ }^{n} C_{2}-{ }^{n} C_{3}+\ldots .+(-1)^{r} \wedge C_{r}+\ldots .=(-1)^{r-1}{ }^{\wedge}(n$

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19. Evaluate : $\sum_{r=1}^{n}(r+1)(r+3)$

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20. Show that the HM of $(2 n+1) C_{-} r$ and ( $\left.2 n+1\right) C_{-}(r+1) i s \frac{2 n+1}{n+1}$ times of
$(2 n) C_{r}$ Also show that $\sum_{r=1}^{2 n-1}(-1)^{r-1} \cdot \frac{r}{2 n C_{r}}=\frac{n}{n+1}$.

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21. If $n$ is a positive integer and $C_{k}={ }^{n} C_{k}$ then find the value of $\sum_{k=1}^{n} k^{3} \cdot\left(\frac{C_{k}}{C_{k-1}}\right)^{2}$.

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## Exercise

1. Determine the constant term in the expansion of $\left(1+x+x^{2}+x^{3}\right)^{10}$

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2. If the fourth term in the expansion of $\left(p x+\frac{1}{x}\right)^{n}$ is $\frac{5}{2}$, then $(n, p)=$

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3. Show that there wil be a term independent of $x$ in the expansion of $\left(x^{a}+x^{-b}\right)^{n}$ only if an is a multiple of $(a+b)$.

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4. Find the term which does not contain irrational expression in the expansion of $(\sqrt[5]{3}+\sqrt[7]{5})^{24}$
5. If in any binomial expansion $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d be the 6th, 7th, 8th and 9th terms respectively, prove that $\frac{b^{2}-a c}{c^{2}-b d}=\frac{4 a}{3 c}$

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6. The value of $x$ in the expression $\left(x+x^{(\log )_{10}}\right)^{5}$ if third term in the expansion is $10,00,000$ is/are a. $10 \mathrm{~b} .100 \mathrm{c} .10^{-5 / 2}$ d. $10^{-3 / 2}$

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7. Prove that the coefficient of the middle term in the expansion of $(1+x)^{2 n}$ is equal to the sum of the coefficients of middle terms in the expansion of $(1+x)^{2 n-1}$

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8. In the expansion of $(1+x)^{43}$,the co-efficients of $(2 r+1) t h$ and $(r+2) t h$ terms are equal. Find $r$.

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9. Prove that in the expansion of $(1+x)^{2 n}$, the coefficient of $x^{n}$ is double the coefficient of $x^{n}$ in the expansion of $\frac{\left(1+2 x+x^{2}\right)^{n}}{1+x}$

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10. The coefficient of 5th, 6th and 7th terms in the expansion of $(1+x)^{n}$ are in A.P. Find the value of $n$.

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11. Given positive integers $r>1, n>2, n$ being even and the coefficient of $(3 r) t h$ term and $(r+2) t h$ term in the expansion of $(1+x)^{2 n}$ are

## equal; find $r$

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12. If the coefficients of three consecutive terms in the expansion of $(1+x)^{n}$ are 165,330 and 462 respectively, the value of n is is

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13. If $a, b, c$ be the three consecutive coefficients in the expansion of $a$ power oif $(1+x)$, prove that the index power is $\left(2 a c+b \frac{a+c}{b^{2}-a c}\right.$

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14. If $a, b, c$ and $d$ are any four consecutive coefficients in the expansion of $(1+x)^{n}$, then prove that
(i) $\frac{a}{a+b}+\frac{c}{b+c}=\frac{2 b}{b+c}$
(ii) $\left(\frac{b}{b+c}\right)^{2}>\frac{a c}{(a+b)(c+d)}$, if $x>0$.

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15. If a,b,c and d are any four consecutive coefficients in the expansion of $(1+x)^{n}$, then prove that
(i) $\frac{a}{a+b}+\frac{c}{b+c}=\frac{2 b}{b+c}$
(ii) $\left(\frac{b}{b+c}\right)^{2}>\frac{a c}{(a+b)(c+d)}$, if $x>0$.

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16. If the four consecutive coefficients in any binomial expansion be a, b, c, d, then prove that (i) $\frac{a+b}{a}, \frac{b+c}{b}, \frac{c+d}{c}$ are in H.P.
$(b c+a d)(b-c)=2\left(a c^{2}-b^{2} d\right)$

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17. Let $\left(1+x^{2}\right)^{2} \cdot(1+x)^{n}=\sum_{k=0}^{n+4} a_{k} \cdot x^{k}$ If $a_{1}, a_{2}$ and $a_{3}$ are iun $A P$, find $n$.
18. If $n$ be a positive integer then prove that the integral part $P$ of $(5+2 \sqrt{6})^{n}$ is an odd integer. If $f$ be the fractional part of $(5+2 \sqrt{6})^{n}$, prove that $P=\frac{1}{1-f}-f$

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19. If $(9+4 \sqrt{5})^{n}=p+\beta$ where n and p are positive integers and $\beta$ is a positive proper fraction, prove that $(1-\beta)(p+\beta)=1$.

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20. Integer just greater tehn $(\sqrt{3}+1)^{2 n}$ is necessarily divisible by (A)
$n+2$ (B) $2^{n+3}$
(C) $2^{n}$
(D) $2^{n+1}$

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21. The greatest coefficient in the expansion of $(1+x)^{2 n}$ is

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22. If $x=1 / 3$, find the greatest tem in the expansion of $(1+4 x)^{8}$.

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23. Find the numerically greatest term in the expansion of $(3-2 x)^{9}$ when $\mathrm{x}=1$

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24. Find the value of the greatest term in the expansion of $\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$.

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25. In the expansion of $(x+a)^{15}$, if the eleventh term is the geometric mean of the eighth and the twelfth terms, which term in the expansion is the greatest?

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26. In the expansion of $\left(\frac{3}{2}+\frac{x}{3}\right)^{n}$ when $x=\frac{1}{2}$, it is know that the 6th term is the greatest term. Find the possible positive integral values of $n$.

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27. Prove that the greatest coefficient in the expansion of $(1+x)^{2 n}$ is double the greatest coefficient in expansion $(1+x)^{2 n-1}$.

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28. Find the sum : ${ }^{2 n+1} C_{0}+{ }^{2 n+1} C_{1}+{ }^{2 n+1} C_{2}+\ldots+{ }^{2 n+1} C_{n}$.
29. 

The
sum
of
the
series
$\frac{1}{1!(n-1)!}+\frac{1}{3!(n-3)!}+\frac{1}{5!(n-5)!}+\ldots .+\frac{1}{(n-1)!1!}$ is $=(\mathrm{A})$
$\frac{1}{n!2^{n}}$ (B) $\frac{2^{n}}{n}$ ! (C) $\frac{2^{n-1}}{n}$ ! (D) $\frac{1}{n!2^{n-1}}$

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30. 

Find
the
sum
$\frac{1}{2} \cdot{ }^{n} C_{0}+{ }^{n} C_{1}+2 \cdot{ }^{n} C_{2}+2^{2} \cdot{ }^{n} C_{3}+\ldots+2^{n-1} \cdot{ }^{n} C_{n}$.

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31. 

Prove
that
$\left(1+{ }^{n} C_{1}+{ }^{n} C_{2}+{ }^{n} C_{3}+\ldots+{ }^{n} C_{n}\right)^{2}=1+{ }^{2 n} C_{1}+{ }^{2 n} C_{2}+{ }^{2 n} C_{3}+\ldots+{ }^{2 n}$

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32. If $t_{0}, t_{1}, t_{2}, \ldots, t_{n}$ are the terms in the expansion of $(x+a)^{n}$ then prove that $\left(t_{0}-t_{2}+t_{4}-\ldots\right)^{2}+\left(t_{1}-t_{3}+t_{5}-\ldots\right)^{2}=\left(x^{2}+a^{2}\right)^{n}$.

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33. If $\frac{\left(1+x-x^{2}\right)^{10}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{n} x^{n}+\ldots$ then find $a_{0}+a_{1}+a_{2}+\ldots$.

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34. If $\frac{\left(1+x-x^{2}\right)^{10}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{n} x^{n}+\ldots$ then find $a_{0}-a_{1}+a_{2}-\cdots$.

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35. If $\frac{\left(1+x-x^{2}\right)^{10}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{n} x^{n}+\ldots$ then find $a_{0}+a_{2}+a_{4}+\ldots$
36. If $\frac{\left(1+x-x^{2}\right)^{10}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{n} x^{n}+\ldots$ then find $a_{1}+a_{3}+a_{5}+\ldots$

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37. If $\frac{\left(1+x-x^{2}\right)^{n}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{2 n} x^{2 n}$ then find $a_{0}+a_{1}+a_{2}+\ldots+a_{2 n}$

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38. If $\frac{\left(1+x-x^{2}\right)^{n}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{2 n} x^{2 n}$ then find $a_{0}-a_{1}+a_{2}-\ldots+a_{2 n}$

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39. If $\frac{\left(1+2 x-x^{2}\right)^{n}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{2 n} x^{2 n}$ then find $a_{0}+a_{2}+a_{4}+\ldots+a_{2 n}$

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40. If $\frac{\left(1+x-x^{2}\right)^{n}}{1+x^{2}}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots+a_{2 n} x^{2 n}$ then find $a_{1}+a_{3}+a_{5}+\ldots+a_{2 n-1}$

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41. The sum of the binomial coefficients in the expansion of $\left(x^{2}+\frac{1}{x}\right)^{n}$ is 1024 . find the coefficient of $x^{11}$ in the binomial expansion.

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42. The exponent of a binomial exceeds that of another by 3 . the sum of the binomial coefficients in expansions of both binomial taken together is
43. find the smaller of the two exponents.

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43. Find the coefficient of $x^{3}$ in the expansion of $1+(1+x)+(1+x)^{2}+(1+x)^{3}+\ldots+(1+x)^{n}$

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44. Find the coefficients of $x^{50}$ in the expression $(1+x)^{1000}+2 x(1+x)^{999}+3 x^{2}(1+x)^{998}++1001 x^{1000}$.

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45. (b) Find the value of $\sum_{r=m}^{n} .{ }^{r} C m, n>m$

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$$
\left({ }^{3} C_{3}+{ }^{4} C_{3}+{ }^{5} C_{3}+\ldots+{ }^{n} C_{3}\right) \times\left({ }^{n} C_{3}+{ }^{n} C_{4}+{ }^{n} C_{5}+\ldots+{ }^{n} C_{n}\right)
$$

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47. The value of ${ }^{\wedge} n C_{1}+{ }^{n+1} C_{2}+{ }^{n+2} C_{3}++{ }^{n+m-1} C_{m}$ is equal to
${ }^{\wedge} m+n C_{n-1}{ }^{\wedge} m+n C_{n-1}{ }^{\wedge} m C_{1}+{ }^{m+1} C_{2}+{ }^{m+2} C_{3}++{ }^{m+n-1}$
${ }^{\wedge} m+1 C_{m-1}$

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48. Prove that ${ }^{n+1} C_{2}+2 \cdot \sum_{k=2}^{n}{ }^{k} C_{2}=\sum_{k=1}^{n} k^{2}$

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49. If $(1+x)^{n}=C_{0}+C_{1} x+C_{2} x^{2}+\ldots+C_{n} x^{n}$, find the sum of the following series
$a C_{1}+(a+d) C_{2}+(a+2 d) C_{3}+\ldots+(a+\overline{n-1} d) C_{n}$

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50. 

Prove
${ }^{n} C_{0}+2 \cdot{ }^{n} C_{1}+3 \cdot{ }^{n} C_{2}+\ldots+(n+1){ }^{n} C_{n}=(n+2) 2^{n-1}$

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51. Prove that ${ }^{n} C_{0}+3 \cdot{ }^{n} C_{1}+5 \cdot{ }^{n} C_{2}+\ldots+(2 n+1)^{n} C_{n}=(n+1) 2^{n}$

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52. Prove that ${ }^{n} C_{0}-2 \cdot{ }^{n} C_{1}+3 \cdot{ }^{n} C_{2}-\ldots+(-1)^{n}(n+1)^{n} C_{n}=0$
53. If $s_{n}={ }^{n} C_{0}+2 \cdot{ }^{n} C_{1}+3 \cdot{ }^{n} C_{2}+\ldots+(n+1) \cdot{ }^{n} C_{n}$ then find $\sum_{n=1}^{\infty} s_{n}$.

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54. Find the sum :1 $\cdot{ }^{n} C_{0}+2 \cdot{ }^{n} C_{1}+3 \cdot{ }^{n} C_{2}+4 \cdot{ }^{n} C_{3}+\ldots$, where n is an odd integer

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$$
\begin{aligned}
& \text { 55. } \begin{array}{c}
\text { Show } \\
{ }^{n} C_{0} \cdot m-{ }^{n} C_{1} \cdot(m-1)+{ }^{n} C_{2} \cdot(m-2)-\ldots+(-1)^{n} \cdot{ }^{n} C_{n} \cdot(m-n)
\end{array}
\end{aligned}
$$

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56. Evaluate $\sum_{r=1}^{n} \frac{p_{r}}{r} \cdot{ }^{n} C_{r}$ where $p_{r}$ denotes the sum of the first $r$ natural numbers.

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57. Prove by binomial expansion that $\sum_{k=1}^{n} k^{2} \cdot{ }^{n} C_{k}=n(n+1) 2^{n-2}$

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58. Evaluate $\sum_{r=0}^{n}(r+1)^{2} \cdot{ }^{n} C_{r}$

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59. If $(1+x)^{n}=C_{0}+C_{1} x+C_{2} x^{2}+C_{3} x^{3}+\ldots+C_{n} x^{n}$ then prove that 2. $C_{0}+2^{2} \frac{C_{1}}{2}+2^{3} \frac{C_{2}}{3}+2^{4} \frac{C_{3}}{4}+\ldots+2^{n+1} \frac{C_{n}}{n+1}=\frac{3^{n+1}-1}{n+1}$

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60. If $(1+x)^{n}=C_{0}+C_{1} x+C_{2} x^{2}+C_{3} x^{3}+\ldots+C_{n} x^{n}$ then prove that $C_{0}-\frac{1}{2} C_{1}+\frac{1}{3} C_{2}-\frac{1}{4} C_{3}+\ldots+(-1)^{n} \cdot \frac{C_{n}}{n+1}=\frac{1}{n+1}$

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61. 

## Prove

$3 \cdot{ }^{10} C_{0}+3^{2} \cdot \frac{{ }^{10} C_{1}}{2}+3^{3} \cdot \frac{{ }^{10} C_{2}}{3}+\ldots 3^{11} \cdot \frac{{ }^{10} C_{10}}{11}=\frac{4^{11}-1}{11}$

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62. 

Prove
that
$2 \cdot{ }^{n} C_{0}+2^{2} \cdot \frac{{ }^{n} C_{1}}{2}+2^{3} \cdot \frac{{ }^{n} C_{2}}{3}+\ldots 2^{n+1} \cdot \frac{{ }^{n} C_{n}}{n+1}=\frac{3^{n+1}-1}{n+1}$

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63. Find the sum $\sum_{k=0}^{n} \frac{{ }^{n} C_{k}}{(k+1)(k+2)}$

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64. Find the sum $\sum_{r=0}^{n}(-1)^{r} \cdot \frac{{ }^{n} C_{r}}{{ }^{r+3} C_{r}}$

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65. Find the sum $\sum_{k=1} \frac{{ }^{n} C_{2 k-1}}{2 k}$

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66. Find the sum $\sum_{k=0}^{n} \frac{{ }^{n} C_{k}}{k+1}$

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$$
\begin{aligned}
& \text { 67. If }(1+x)^{n}=\sum_{r=0}^{n} C_{r} x^{r} \quad \text { then prove that } \\
& \sum_{r=0}^{n} \frac{C_{r}}{(r+1) 2^{r+1}}=\frac{3^{n+1}-2^{n+1}}{(n+1) 2^{n+1}}
\end{aligned}
$$

68. Find $\sum_{r=0}^{n}(r+1) \cdot{ }^{n} C_{r} x^{r}$

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69. Show that $C_{0}^{2}-C_{1}^{2}+C_{2}^{2}-C_{3}^{2}+\ldots \ldots \ldots \ldots+(-1)^{n} C_{n}^{2}=0$ or $(-1)^{\frac{n}{2}} C_{\frac{n}{2}}$ according as $n$ is odd or even.

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70. 

Prove
that
$\left({ }^{2 n} C_{0}\right)^{2}-\left({ }^{2 n} C_{1}\right)^{2}+\left({ }^{2 n} C_{2}\right)^{2}-\ldots+\left({ }^{2 n} C_{2 n}\right)^{2}=(-1)^{n} \cdot{ }^{2 n} C_{n}$.

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71. Sum of the products of the binomial coefficients $C_{0}, C_{1}, C_{2}, \ldots \ldots C_{n}$ taken two at a time is:
72. 

Find
the
sum
${ }^{\wedge} 20 C_{10} \cdot{ }^{15} C_{0}+{ }^{20} C_{9} \cdot{ }^{15} C_{1}+{ }^{20} C_{8} \cdot{ }^{15} C_{2}+\ldots .+{ }^{20} C_{0} \cdot{ }^{15} C_{10}$

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73. Prove that $\sum_{r=1}^{k}(-3)^{r-1}{ }^{3 n} C_{2 r-1}=0$, where $\mathrm{k}=(3 \mathrm{n}) / 2$ and n is an even integer

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74. If $p+q=1$, then show that $\sum_{r=0}^{n} r^{2}{ }^{\wedge} n C_{r} p^{r} q^{n-r}=n p q+n^{2} p^{2}$.

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75. Use a combinatorial argument to prove that
$(C(n, 1))^{2}+2(C(n, 2))^{2}+3(C(n, 3))^{2}+\ldots \ldots \ldots . .+n(C(n, n))^{2}=\frac{(2 n}{((n}$

## (D) Watch Video Solution

76. 

Prove
$\frac{{ }^{n} C_{1}}{{ }^{n} C_{0}}+2 \cdot \frac{{ }^{n} C_{2}}{{ }^{n} C_{1}}+3 \cdot \frac{{ }^{n} C_{3}}{{ }^{n} C_{2}}+\ldots+n \cdot \frac{{ }^{n} C_{n}}{{ }^{n} C_{n-1}}=\frac{n(n+1)}{2}$

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77. 

Given,
$s_{n}=1+q+q^{2}++q^{n}, S_{n}=1+\frac{q+1}{2}+\left(\frac{q+1}{2}\right)^{2}++\left(\frac{q+1}{2}\right)^{n}, \varrho$ prove that ${ }^{\wedge} n+1 C_{1}+{ }^{n+1} C_{2} s_{1}+{ }^{n+1} C_{3} s_{2}++{ }^{n+1} C_{n+1} s_{n} 2^{n} S_{n}$.

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78. Find the value of $\sum_{p=1}^{n}\left(\sum_{m=p}^{n}{ }^{\wedge} n C_{m}^{m} C_{p}\right)$. And hence, find the value of $(\lim )_{n} \vec{\infty} \frac{1}{3^{n}} \sum_{p=1}^{n}\left(\sum_{m=p}^{n}{ }^{\wedge} n C_{m}^{m} C_{p}\right)$.
79. The value of ${ }^{\wedge}(2 n+1) C_{0}^{2}+{ }^{2 n+1} C_{1}^{2}+{ }^{2 n+1} C_{2}^{2}+\ldots .+{ }^{2 n+1} C_{n}^{2}$ is equal to

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80. Find the sum 'sumsum_(Olt=i

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81. If $\left(1+x+x^{2}+x^{3}\right)^{n}=a_{0}+a_{1} x+a_{2} x^{2}+\ldots \ldots \ldots \ldots a_{3 n} x^{3 n}$ then which of following are correct

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82. The coefficient of $a^{4} b^{5}$ in the expansion of $(a+b)^{9}$ is $\qquad$ .
83. The coefficient in the third term of the expansion of $\left(x^{2}-\frac{1}{4}\right)^{n}$ when expanded in descending powers of $x$ is 31 . then $n$ is equal to $\qquad$ .

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84. Which is larger : $\left(99^{50}+100^{50}\right)$ or $(101)^{50}$.

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85. The sum of the coefficeints of the polynominal $\left(1+x-3 x^{2}\right)^{2163}$ is

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86. The sum of the numerical coefficients in the expansion of $(2 x+3 y)^{10}$ is $\qquad$ .
87. If the fifth term of the expansion $\left(a^{2 / 3}+a^{-1}\right)^{n}$ does not contain ' $a^{\prime}$. Then $n$ is equal to 2 b .5 c .10 d . none of these

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88. The sum of the rational terms in the expansion of
$(\sqrt{2}+\sqrt[5]{3})^{10}$ is

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89. Write the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{10}$.

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90. If the coefficients of 2 nd , 3 rd and 4 th terms in the expansion of $(1+x)^{n}$ are in A.P., then find the value of n .

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91. If $x^{r}$ occurs in the expansion of $\left(x+\frac{1}{x}\right)^{n}$ then its coefficient is $\qquad$ .

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92. If the sum of the coefficients in the expansion of $(a+b)^{n}$ is 4096, then the greatest coefficient in the expansion is 924 b .792 c .1594 d . none of these

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93. The number of terms in the expansion of $\left(1+x^{\frac{1}{5}}\right)^{55}$ which are free from radicals is $\qquad$ .

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94. If $n$ is even then the coefficient of $x$ in the expansion of $(1+x)^{n} \cdot\left(1-\frac{1}{x}\right)^{n}$ is $\qquad$ .

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95. The sum of ${ }^{21} C_{0}+{ }^{21} C_{1}+{ }^{21} C_{2}+\ldots+{ }^{21} C_{10}$ is equal to $\qquad$ .

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96. The coefficient of $x^{n} y^{n}$ in the expansion of
$[(1+x)(1+y)(x+y)]^{n}$, is

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97. The number of terms in the expansion of $\left(1+2 x+x^{2}\right)^{n}$ is:
98. The number of terms in the expansion of $(1+7 \sqrt{2 x})^{9}+(1-7 \sqrt{2 x})^{9}$ is
A. 5
B. 7
C. 9
D. 10

## Answer: A

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99. In the expansion of $\left(x^{3}-\frac{1}{x^{2}}\right)^{15}$, the constant term,is
A. ${ }^{15} C_{6}$
B. 0
C. $-{ }^{15} C_{6}$
D. none of these

Answer:

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100. The largest coefficient in the expansion of $(1+x)^{24}$ is
A. ${ }^{24} C_{24}$
B. ${ }^{24} C_{13}$
C. ${ }^{24} C_{12}$
D. ${ }^{24} C_{11}$

## Answer: B

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101. $3^{51}$ when divided by 8 leaves the remainder 22.63 .34 .55 .1
A. 1
B. 6
C. 5
D. 3

## Answer:

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102. The sum of the series ${ }^{20} C_{0}+{ }^{20} C_{1}+{ }^{20} C_{2}+\ldots+{ }^{20} C_{9}$ is $=$
A. $2^{20}$
B. $2^{19}$
C. $2^{19}+\frac{1}{2} \cdot{ }^{20} C_{10}$
D. $2^{19}-\frac{1}{2} \cdot{ }^{20} C_{10}$

## Answer: D

103. The sum of the last eight coefficients in the expansion of $(1+x)^{16}$ is equal to
A. $2^{15}$
B. $2^{14}$
C. $2^{15}-\frac{1}{2} \cdot \frac{16!}{(8!)^{2}}$
D. none of these

## Answer:

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104. If $C_{r}$ stands for ${ }^{n} C_{r}$, then the sum of the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_{0}^{2}-2 C_{1}^{2}+3 C_{2}^{2}-\ldots+(-1)^{n}(n+1) C_{n}^{2}\right]$, where n is an even positive integers, is:
A. 0
B. $(-1)^{\frac{n}{2}} \cdot(n+1)$
C. $(-1)^{n} \cdot(n+1)$
D. $(-1)^{n} \cdot n$

## Answer:

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105. If pandq are ositive, then prove that the coefficients of $x^{p} a n d x^{q}$ in the expansion of $(1+x)^{p+q}$ will be equal.
A. equal
B. equal but opposite in sgin
C. reciprocal to each other
D. none of these

## Answer:

106. The number of dissimilar terms in the expansion of $(a+2 b+3 c)^{8}$ is
A. 9
B. 24
C. 45
D. 10

## Answer: C

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107. In the expansion of $(1+x)^{2 m}\left(\frac{x}{1-x}\right)^{-2 m}$ the term independent of $x$ is
A. ${ }^{2 m} C_{m}$
B. ${ }^{2 m} C_{0}$
C. $(-1)^{m} \cdot{ }^{2 m} C_{m}$
D. none of these

## Answer: C

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108. State true or false : The integral part of $(8+3 \sqrt{7})^{20}$ is odd.

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109. State true or false : In the expansion of $\left(\frac{x^{2}}{y}+\frac{y^{2}}{x}\right)^{15}$ there is no term independent of x and y both.

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110. State true or false: In the expansion of $\left(1+2 x+x^{2}\right)^{9}$ there is exactly one term whose coefficient is not equal to coefficient of any other term.
111. State true or false : In the expansion of $\left(x+\frac{1}{x}\right)^{13}$ every term is a function of $x$.

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112. State true or false : If $f(x)=\left(x+\frac{1}{x}\right)^{2 n}+\left(x-\frac{1}{x}\right)^{2 n}$ the $f(x)$ is a polynomial function which is an even function.

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113. State whether the statements are true or false : ${ }^{16} C_{0}-{ }^{16} C_{1}+{ }^{16} C_{2}-\ldots+{ }^{16} C_{16}=0$

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