

MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Binomial Theorem for Positive Integrel Index

Example

1. Find the number of terms in the expansion of $\left(x+\sqrt{x^2-1}
ight)^6+\left(x-\sqrt{x^2-1}
ight)^6$

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2. Prove that $13^{99}-19^{93}$ is divisible by 162.

3. In $\left(33 + \frac{1}{33}\right)^n$ if the ratio of 7th term from the beginning to the 7th

term from the end is 1/6, then find the value of n.

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4. The sixth term in the expansion of $\left(\sqrt{2^{\log(10-3^x)}} + \left(2^{(x-2)\log 3}\right)^{\frac{1}{5}}\right)^m$ is equal to 21, if it is known that the binomial coefficient of the 2nd 3rd and 4th terms in the expansion represent, respectively, the first, third and fifth terms of an A.P. (the symbol log stands for logarithm to the base 10) The value of m is

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5. Find the coefficient of
$$x^3$$
 in the expansion of $(1+x+2x^2)\left(2x^2-\frac{1}{3x}
ight)^9$

6. The coefficient of $x^r [0 \le r \le (n-1)]$ in Ithe expansion of $(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + + (x+2)^{n-1}$ is $\ nC_r(3^r-2^n)$ b. $\ nC_r(3^{n-r}-2^{n-r})$ c. $\ nC_r(3^r+2^{n-r})$ d. none of these

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7. Determine the term independent of a in the expansion of $\left(\begin{array}{cc} a+1 & a-1 \end{array}\right)^{10}$

$$\left(rac{a^{2}}{a^{2} - a^{1} - a^{1}} - rac{a^{2}}{a - a^{1}}
ight)$$

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8. If a,b,c and d are any four consecutive

coefficients in the expansion of $\left(1+x
ight)^n$, then prove that

(i)
$$\frac{a}{a+b} + \frac{c}{b+c} = \frac{2b}{b+c}$$

(ii) $\left(\frac{b}{b+c}\right)^2 > \frac{ac}{(a+b)(c+d)}$, if $x > 0$.





11. Given that the 4th term in the expansion of $\left[2+\left(3/8x
ight)
ight]^{10}$ has the

maximum numerical value. Then find the range of value of x_{\cdot}



12. Find the coefficient of
$$x^n$$
 in the expansion of $\left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}\right)^2$.
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13. If
$$\sum_{r=0}^{2n}a_r(x-2)^r=\sum_{r=0}^{2n}b_r(x-3)^randa_k=1$$
 for all $k\geq n,$ then

show that $b_n =^{2n+1} C_{n+1}$.

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14. Sum the series : ${}^{1000}C_{50}+2\cdot {}^{999}C_{49}+3\cdot {}^{998}C_{48}+...+51\cdot {}^{950}C_{0}$

15.

Prove

that

 $nC_1\sin x.\cos(n-1)x+nC_2\sin 2x.\cos(n-2)x+nC_3\sin 3x.\cos(n-3)x$

16. find the sum of the series
$$\sum_{r=0}^{n} (-1)^{r} \cdot {}^{n}C_{r}$$
$$\left[\frac{1}{2^{r}} + \frac{3^{r}}{2^{2r}} + \frac{7^{r}}{2^{3r}} + \frac{15^{r}}{2^{4r}} \dots \text{ up to m terms}\right]$$
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17. If
$$s = a + (a + d) + (a + 2d) + ... + (a + nd)$$
 and

$$S=a+(a+d)\cdot {}^nC_1+(a+2d)\cdot {}^nC_2+...+(a+nd)\cdot {}^nC_n$$
 then

prove that $(n+1)S = 2^n \cdot s.$



19. Evaluate :
$$\sum_{r=1}^n {(r+1)(r+3)}$$

20. Show that the HM of (2n+1)C_r and (2n+1)C_(r+1) $is\frac{2n+1}{n+1}$ times of

$$(2n)C_r$$
 Also show that $\sum_{r=1}^{2n-1}{(-1)^{r-1}}\cdot rac{r}{2nC_r}=rac{n}{n+1}.$

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21. If n is a positive integer and $C_k = \cdot^n C_k$ then find the value of $\sum_{k=1}^n k^3 \cdot \left(\frac{C_k}{C_{k-1}}\right)^2$.

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Exercise

1. Determine the constant term in the expansion of $ig(1+x+x^2+x^3ig)^{10}$



4. Find the term which does not contain irrational expression in the expansion of $\left(\sqrt[5]{3} + \sqrt[7]{5}\right)^{24}$

5. If in any binomial expansion a, b, c and d be the 6th, 7th, 8th and 9th

terms respectively, prove that $\displaystyle rac{b^2-ac}{c^2-bd}=rac{4a}{3c}$

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6. The value of x in the expression $\left(x + x^{(\log)_{10}}\right)^5$ if third term in the expansion is 10,00,000 is/are a. 10 b. 100 c. $10^{-5/2}$ d. $10^{-3/2}$

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7. Prove that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is equal to the sum of the coefficients of middle terms in the expansion of $(1+x)^{2n-1}$

8. In the expansion of $(1+x)^{43}$,the co-efficients of (2r+1)th and (r+2)th terms are equal. Find r.

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9. Prove that in the expansion of $(1+x)^{2n}$, the coefficient of x^n is double

the coefficient of x^n in the expansion of $rac{ig(1+2x+x^2ig)^n}{1+x}$

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10. The coefficient of 5th, 6th and 7th terms in the expansion of $\left(1+x
ight)^n$

are in A.P. Find the value of n.



11. Given positive integers r>1, n>2, n being even and the coefficient

of (3r)th term and (r+2)th term in the expansion of $(1+x)^{2n}$ are



13. If a,b,c be the three consecutive coefficients in the expansion of a power oif (1 + x), prove that the index power is $\left(2ac + b\frac{a+c}{b^2 - ac}\right)$

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14. If a,b,c and d are any four consecutive

coefficients in the expansion of $\left(1+x
ight)^n$, then prove that

$$\begin{aligned} \text{(i)} & \frac{a}{a+b} + \frac{c}{b+c} = \frac{2b}{b+c} \\ \text{(ii)} & \left(\frac{b}{b+c}\right)^2 > \frac{ac}{(a+b)(c+d)}, \text{ if } \ x > 0 \,. \end{aligned}$$

15. If a,b,c and d are any four consecutive

coefficients in the expansion of $\left(1+x
ight)^n$, then prove that

(i)
$$rac{a}{a+b} + rac{c}{b+c} = rac{2b}{b+c}$$

(ii) $\left(rac{b}{b+c}
ight)^2 > rac{ac}{(a+b)(c+d)}$, if $x > 0$.

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16. If the four consecutive coefficients in any binomial expansion be a, b, c, d, then prove that (i) $\frac{a+b}{a}$, $\frac{b+c}{b}$, $\frac{c+d}{c}$ are in H.P. (ii) $(bc+ad)(b-c) = 2(ac^2 - b^2d)$

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17. Let
$$\left(1+x^2
ight)^2\cdot\left(1+x
ight)^n=\sum_{k=0}^{n+4}a_k\cdot x^k$$
 If $a_1,a_2\,\, ext{and}\,\,a_3$ are iun AP ,

find n.

18. If n be a positive integer then prove that the integral part P of $(5+2\sqrt{6})^n$ is an odd integer. If f be the fractional part of $(5+2\sqrt{6})^n$, prove that $P=\frac{1}{1-f}-f$

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19. If $\left(9+4\sqrt{5}
ight)^n=p+eta$ where n and p are positive integers and eta is a

positive proper fraction, prove that $(1 - \beta)(p + \beta) = 1$.

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20. Integer just greater tehn $\left(\sqrt{3}+1
ight)^{2n}$ is necessarily divisible by (A) n+2 (B) 2^{n+3} (C) 2^n (D) 2^{n+1}

21. The greatest coefficient in the expansion of $(1+x)^{2n}$ is



when x=1

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24. Find the value of the greatest term in the expansion of
$$\sqrt{3}\left(1+\frac{1}{\sqrt{3}}\right)^{20}$$
.

25. In the expansion of $(x + a)^{15}$, if the eleventh term is the geometric mean of the eighth and the twelfth terms, which term in the expansion is the greatest?



26. In the expansion of
$$\left(rac{3}{2}+rac{x}{3}
ight)^n$$
 when $x=rac{1}{2}$, it is know that the 6th

term is the greatest term. Find the possible positive integral values of n.

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27. Prove that the greatest coefficient in the expansion of $\left(1+x
ight)^{2n}$ is

double the greatest coefficient in expansion $(1 + x)^{2n-1}$.

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28. Find the sum : ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n$.





32. If $t_0, t_1, t_2, ..., t_n$ are the terms in the expansion of $(x + a)^n$ then prove that $(t_0 - t_2 + t_4 - ...)^2 + (t_1 - t_3 + t_5 - ...)^2 = (x^2 + a^2)^n$.

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33. If
$$\frac{\left(1+x-x^2\right)^{10}}{1+x^2} = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n + \ldots$$
 then find $a_0 + a_1 + a_2 + \ldots$

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34. If
$$\frac{\left(1+x-x^2
ight)^{10}}{1+x^2}=a_0+a_1x+a_2x^2+...+a_nx^n+\ldots$$
 then find $a_0-a_1+a_2-...$

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35. If
$$\frac{\left(1+x-x^2
ight)^{10}}{1+x^2}=a_0+a_1x+a_2x^2+...+a_nx^n+\ldots$$
 then find $a_0+a_2+a_4+\ldots$

36. If
$$\frac{\left(1+x-x^2\right)^{10}}{1+x^2}=a_0+a_1x+a_2x^2+\ldots+a_nx^n+\ldots$$
 then find $a_1+a_3+a_5+\ldots$

37. If
$$rac{ig(1+x-x^2ig)^n}{1+x^2}=a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$$
 then find

$$a_0 + a_1 + a_2 + \ldots + a_{2n}$$

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38. If
$$rac{ig(1+x-x^2ig)^n}{1+x^2}=a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$$
 then find

$$a_0 - a_1 + a_2 _ ... + a_{2n}$$

39. If $\frac{\left(1+2x-x^2
ight)^n}{1+x^2}=a_0+a_1x+a_2x^2+...+a_{2n}x^{2n}$ then find $a_0+a_2+a_4+...+a_{2n}$

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40. If
$$\frac{\left(1+x-x^2\right)^n}{1+x^2} = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$$
 then find

$$a_1 + a_3 + a_5 + ... + a_{2n-1}$$

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41. The sum of the binomial coefficients in the expansion of $\left(x^2+rac{1}{x}
ight)^n$

is 1024. find the coefficient of x^{11} in the binomial expansion.



42. The exponent of a binomial exceeds that of another by 3. the sum of

the binomial coefficients in expansions of both binomial taken together is

144. find the smaller of the two exponents.



:

$$\left({}^{3}C_{3} + {}^{4}C_{3} + {}^{5}C_{3} + ... + {}^{n}C_{3}\right) imes \left({}^{n}C_{3} + {}^{n}C_{4} + {}^{n}C_{5} + ... + {}^{n}C_{n}\right)$$

47. The value of
$$nC_1 + n+1 C_2 + n+2 C_3 + n+n+1 C_m$$
 is equal to

$$\hat{\ }m + nC_{n-1} \quad \hat{\ }m + nC_{n-1} \quad \hat{\ }mC_1 + ^{m+1}C_2 + ^{m+2}C_3 + \ + ^{m+n-1}$$

$$\hat{} m + 1C_{m-1}$$

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48. Prove that
$${}^{n+1}C_2 + 2 \cdot \sum_{k=2}^n {}^kC_2 = \sum_{k=1}^n k^2$$

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49. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, find the sum of the

following

series







56. Evaluate $\sum_{r=1}^{n} \frac{p_r}{r} \cdot {}^nC_r$ where p_r denotes the sum of the first r natural

numbers.



61. Prove that

$$3 \cdot {}^{10}C_0 + 3^2 \cdot \frac{{}^{10}C_1}{2} + 3^3 \cdot \frac{{}^{10}C_2}{3} + \dots 3^{11} \cdot \frac{{}^{10}C_{10}}{11} = \frac{4^{11} - 1}{11}$$

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62. Prove that
 $2 \cdot {}^nC_0 + 2^2 \cdot \frac{{}^nC_1}{2} + 2^3 \cdot \frac{{}^nC_2}{3} + \dots 2^{n+1} \cdot \frac{{}^nC_n}{n+1} = \frac{3^{n+1} - 1}{n+1}$
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63. Find the sum
$$\sum_{k=0}^n rac{{}^nC_k}{(k+1)(k+2)}$$

64. Find the sum
$$\sum_{r=0}^n (-1)^r \cdot rac{{}^n C_r}{{}^{r+3}C_r}$$

65. Find the sum
$$\sum_{k=1}^{n} rac{{}^nC_{2k-1}}{2k}$$

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66. Find the sum
$$\sum_{k=0}^n rac{{}^n C_k}{k+1}$$

67. If
$$(1+x)^n = \sum_{\substack{r=0\\r=0}}^n C_r x^r$$
 then prove that $\sum_{r=0}^n \frac{C_r}{(r+1)2^{r+1}} = \frac{3^{n+1}-2^{n+1}}{(n+1)2^{n+1}}$ Watch Video Solution

68. Find
$$\sum_{r=0}^n \left(r+1
ight) \cdot {}^n C_r x^r$$



69. Show that
$$C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = 0$$
 or $(-1)^{\frac{n}{2}} C_{\frac{n}{2}}$ according as n is odd or even.

70.
 Prove
 that

$$({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - \ldots + ({}^{2n}C_{2n})^2 = (-1)^n \cdot {}^{2n}C_n.$$
 that

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71. Sum of the products of the binomial coefficients $C_0, C_1, C_2, \dots, C_n$

taken two at a time is:

72. Find the sum

$$^{2} 20C_{10}.^{15}C_{0}+^{20}C_{9}.^{15}C_{1}+^{20}C_{8}.^{15}C_{2}+....+^{20}C_{0}.^{15}C_{10}$$

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73. Prove that $\sum_{r=1}^{k} (-3)^{r-1} {}^{3n}C_{2r-1} = 0$, where k=(3n)/2 and n is an even integer
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74. If $p + q = 1$, then show that $\sum_{r=0}^{n} r^{2} {}^{n}C_{r}p^{r}q^{n-r} = npq + n^{2}p^{2}$.
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75. Use a combinatorial argument to prove that $(C(n, 1))^{2} + 2(C(n, 2))^{2} + 3(C(n, 3))^{2} + \dots + n(C(n, n))^{2} = \frac{(2n)}{(n-1)}$

76. Prove that

$$\frac{{}^{n}C_{1}}{{}^{n}C_{0}} + 2 \cdot \frac{{}^{n}C_{2}}{{}^{n}C_{1}} + 3 \cdot \frac{{}^{n}C_{3}}{{}^{n}C_{2}} + \dots + n \cdot \frac{{}^{n}C_{n}}{{}^{n}C_{n-1}} = \frac{n(n+1)}{2}$$
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77. Given,

$$s_{n} = 1 + q + q^{2} + + q^{n}, S_{n} = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^{2} + + \left(\frac{q+1}{2}\right)^{n}, q$$
prove that $\hat{n} + 1C_{1} + {}^{n+1}C_{2}s_{1} + {}^{n+1}C_{3}s_{2} + + {}^{n+1}C_{n+1}s_{n}2^{n}S_{n}$
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78. Find the value of $\sum_{n}^{n} \left(\sum_{n=1}^{n} nC_{m}^{m}C_{p}\right)$ And hence, find the value of

(
$$\lim_{n \to \infty} \frac{1}{3^n} \sum_{p=1}^n \left(\sum_{m=p}^n n C_m C_p \right)$$
. And hence, and the value of

79. The value of
$$(2n + 1)C_0^2 + 2n+1 C_1^2 + 2n+1 C_2^2 + \dots + 2n+1 C_n^2$$
 is equal to
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80. Find the sum 'sumsum_(Olt=i
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81. If $(1 + x + x^2 + x^3)^n = a_0 + a_1x + a_2x^2 + \dots + a_{3n}x^{3n}$ then which of following are correct
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82. The coefficient of a^4b^5 in the expansion of $(a + b)^9$ is _____.



87. If the fifth term of the expansion $\left(a^{2/3}+a^{-1}
ight)^n$ does not contain

'a' Then n is equal to 2 b. 5 c. 10 d. none of these



 $\left(1+x
ight)^n$ are in A.P., then find the value of n.

91. If x^r occurs in the expansion of $\left(x+rac{1}{x}
ight)^n$ then its coefficient is _____.

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92. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is 924 b. 792 c. 1594 d. none of these

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93. The number of terms in the expansion of $\left(1+x^{rac{1}{5}}
ight)^{55}$ which are free

from radicals is _____.

94. If n is even then the coefficient of x in the expansion of $(1+x)^n \cdot \left(1-\frac{1}{x}\right)^n$ is _____.

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95. The sum of ${}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}$ is equal to _____.

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96. The coefficient of $x^n y^n$ in the expansion of

$$\left[(1+x)(1+y)(x+y)
ight]^n$$
 , is

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97. The number of terms in the expansion of $\left(1+2x+x^2
ight)^n$ is :

98. The number of terms in the expansion of
$$(1 + 7\sqrt{2x})^9 + (1 - 7\sqrt{2x})^9$$
 is
A. 5
B. 7
C. 9
D. 10

Answer: A

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99. In the expansion of
$$\left(x^3-rac{1}{x^2}
ight)^{15}$$
 , the constant term,is A. $^{15}C_6$

B. 0

 $\mathsf{C.}-^{15}C_6$

D. none of these

Answer:



100. The largest coefficient in the expansion of $\left(1+x
ight)^{24}$ is

A. $^{24}C_{24}$

- B. ${}^{24}C_{13}$
- C. ${}^{24}C_{12}$
- D. $^{24}C_{11}$

Answer: B



101. 3^{51} when divided by 8 leaves the remainder 2 2. 6 3. 3 4. 5 5. 1

A. 1	
B. 6	
C. 5	
D. 3	

Answer:

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102. The sum of the series ${}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + ... + {}^{20}C_9$ is =

A. 2^{20}

 $\mathsf{B.}\,2^{19}$

C.
$$2^{19} + rac{1}{2} \cdot {}^{20}C_{10}$$

D. $2^{19} - rac{1}{2} \cdot {}^{20}C_{10}$

Answer: D

103. The sum of the last eight coefficients in the expansion of $\left(1+x
ight)^{16}$ is

equal to

A.
$$2^{15}$$

B. 2^{14}
C. $2^{15} - \frac{1}{2} \cdot \frac{16!}{(8!)^2}$

D. none of these

Answer:



104. If C_r stands for nC_r , then the sum of the series $\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \dots + (-1)^n(n+1)C_n^2\right],$ where n is

an even positive integers, is:

B.
$$(-1)^{rac{n}{2}} \cdot (n+1)$$

C. $(-1)^n \cdot (n+1)$
D. $(-1)^n \cdot n$

Answer:

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105. If pandq are ositive, then prove that the coefficients of x^pandx^q in the expansion of $(1 + x)^{p+q}$ will be equal.

A. equal

B. equal but opposite in sgin

C. reciprocal to each other

D. none of these

Answer:

106. The number of dissimilar terms in the expansion of $\left(a+2b+3c
ight)^8$ is

A. 9	
B. 24	
C. 45	
D. 10	

Answer: C

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107. In the expansion of
$$(1+x)^{2m} \left(rac{x}{1-x}
ight)^{-2m}$$
 the term independent

of x is

A. ${}^{2m}C_m$

B. ${}^{2m}C_0$

 $\mathsf{C.} \left(\, -1 \right)^m \cdot \, {}^{2m} C_m$

D. none of these

Answer: C



110. State true or false: In the expansion of $(1 + 2x + x^2)^9$ there is exactly one term whose coefficient is not equal to coefficient of any other

term.



112. State true or false : If
$$f(x) = \left(x+rac{1}{x}
ight)^{2n} + \left(x-rac{1}{x}
ight)^{2n}$$
 the $f(x)$ is

a polynomial function which is an even function.

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113. State whether the statements are true or false :
$${}^{16}C_0 - {}^{16}C_1 + {}^{16}C_2 - ... + {}^{16}C_{16} = 0$$