MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

COMPLEX NUMBERS

Example

1. let z_1=1-2i, z_2=1 and z_3=3+4i. If $z = \left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2}$: then express z in the form A + Ib.

2. let z_1=1-2i, z_2=1 and z_3=3+4i. If $z = \left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2}$: find |z| , $\bar{z}$ and amp z.

3. let z_1=1-2i, z_2=1 and z_3=3+4i. If $z = \left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2}$:express z in the trignometrical form.

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4. Find real $\theta \in$ (-pi,pi) so that $\frac{3 + 2i \sin\theta}{1 - 2i \sin\theta}$ is purely real .

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5. Find real $\theta\varepsilon$ (-pi,pi) so that $\frac{3 + 2i \sin\theta}{1 - 2i \sin\theta}$ is purely imaginary .

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6. Solve for x,y $\in$ R: $\left(x^4 + 2xi\right) - \left(3x^2 + yi\right) = (1 + 2yi) + \frac{34}{3 + 5i}$.

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7. Prove that $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^{3n}$ +1=0 for all odd integral values of n.

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8. Let $f(x)=x^4+9x^3+35x^2-x+4$. Find f(-5+2$\sqrt{-4}$).

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9. If $\sqrt{a+ib}=x+iy,$ then possible value of $\sqrt{a-ib}$ is x^2+y^2 b. $\sqrt{x^2+y^2}$ c. $x+iy$ d. $x-iy$ e. $\sqrt{x^2-y^2}$

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10. Express $(1+a^2)(1+b^2)(1+c^2)$ as the sum of two squares.

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11. Evaluate: $\sqrt{-4-3i}$

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12. If z=a+ib, |z|=1 and b $\neq$ 0, show that z can be represented as $z=\frac{c+i}{c-i}$ where c is a real number.

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13. If a, b, c are real numbers and z is a complex number such that, $a^2+b^2+c^2=1$ and $b+ic=(1+a)z$ then $\frac{1+iz}{1-iz}$ equals. (a) $\frac{b-ic}{1-ia}$ (b) $\frac{a+ib}{1+c}$ (c) $\frac{1-c}{a-ib}$ (d) $\frac{1+a}{b+ic}$

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14. Find all non zero complex numbers z satisfying $\bar{z}=iz^2$

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15. Find the complex number z satisfying the equations $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}, \left|\frac{z-4}{z-8}\right| = 1$

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16. For every real number $a \geq 0$, find all the complex numbers z that satisfy the equation $2|z| - 4az + 1 + ia = 0$

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17. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is a complex number such that the argument of $\frac{z - z_1}{z - z_2} is \frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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18. Let z_1 , z_2 be two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular . If z_2 is not unimodular then find $|z_1|$.

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19. If u=$\sqrt{z_1 z_2}$, prove that $|z_1| + |z_2| = \left|\frac{z_1 + z_2}{2} + u\right| + \left|\frac{z_1 + z_2}{2} - u\right|$.

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20. If $|z - 3i| < \sqrt{5}$ then prove that the complex number z also satisfies the inequality $|i(z + 1) + 1| < 2\sqrt{5}$.

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21. Statement-1: for any non-zero complex number $|z - 1| \leq ||z| - 1| + |z|$ arg (z)

Statement-2 : For any non-zero complex number z

$$\left|\frac{z}{|z|}-1\right| \leq \arg(z)$$

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22. Show that for any two non zero complex numbers $z_1, z_2(|z_1|+|z_2|)|z_1|z_1|+z_2|z_2|| \leq 2|z_1+z_2|$

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23. $|z| \leq 1, |w| \leq 1,$ then show that $|z-w|^2 \leq (|z|-|w|)^2+(argz-argw)^2$

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24. z0 is a root of the equation $z^n \cos[\theta o]+z^{n-1}\cos[\theta 1]+z^{n-2}\cos[\theta 2]$ +.......+$z^n \cos[\theta[n-1]]+\cos\theta[n]$=2 where $\Theta[i] \in R$ then

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25. If $(\log)_{\sqrt{3}}\left(\frac{|z|^2-|z|+1}{2+|z|}\right)>2,$ then locate the region in the Argand plane which represents z.

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26. Locate the region in the argand plane for z satisfying |z+i|=|z-2|.

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27. Locate the complex number z such that $\log_{\frac{\cos\pi}{6}}\left[\frac{|z-2|+5}{4|z-2|-4}\right]<2$

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28. Find the fourth roots of -16i.

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29. Find the roots common to the equations $x^5 - x^3 + x^2 - 1 = 0$ and $x^4 = 1$.

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30. Find non zero integral solutions of $|1 - i|^x = 2^x$.

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31. If $z + 1/z = 2\cos\theta$, prove that $\left|\left(z^{2n} - 1\right)/\left(z^{2n} + 1\right)\right| = |\tan n\theta|$

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32. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and also $\sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that

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33. If $\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0,$ then (A) cos3alpha+cos3beta+cos3gamma=cos3gamma=3cos(alpha+beta+gamma) (B)cos 3alpha+cos3beta+cos3gamma=0(C) sin3theta+sin3beta+sin3gamma=0(D)sin3alpha+sin3beta+sin3gamma= 3sin(alpha+beta+gamma)`

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34. Solve the equation $z^8+1=0$ and deduce that

$\cos 4\theta=8\left(\cos\theta-\cos\left(\frac{\pi}{8}\right)\right)\left(\cos\theta-\cos\left(\frac{3\theta}{3}\right)\right)\left(\cos\theta-\cos\left(\frac{5\pi}{8}\right)\right)\left(\cos\right.$

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35. $\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)\cos\left(\frac{5\pi}{8}\right)\cos\left(\frac{7\pi}{8}\right)=$

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36. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$, find the sum $a_0 + a_4 + a_8 + a_{12} + \ldots\ldots$.

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37. If $1, z_1, z_2, z_3 \ldots . z_{n-1}$ be n^{th} roots if unity and w be a non real complex cube root of unity, then $\prod_{r=1}^{n-1} (w - z_r)$ can be equal to

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38. If $1, z_1, z_2, z_3 \ldots . z_{n-1}$ be n^{th} roots if unity and w be a non real complex cube root of unity, then $\prod_{r=1}^{n-1} (w - z_r)$ can be equal to

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39. If $1, z_1, z_2, z_3.... z_{n-1}$ be n^{th} roots if unity and w be a non real complex cube root of unity, then $\prod_{r=1}^{n-1} (w - z_r)$ can be equal to

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40. If n is an odd integer but not a multiple of 3, then prove that $xy(+ y)\left(x^2 + y^2 + xy\right)$ is a factor of $(x + y)^n - x^n - y^2$.

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41. Find the complex number z such that |z-2+2i| $\leq$ 1 and z has the : least absoluate value.

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42. Find the complex number z such that |z-2+2i| $\leq$ 1 and z has the : numerically least amplitude.

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43. Let $A(z_1)$ and (z_2) represent two complex numbers on the complex plane. Suppose the complex slope of the line joining A and B is defined as $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$.If the line l_1, with complex slope $\omega_1, \text{ and } l_2$, with complex slope $omeg_2$, on the complex plane are perpendicular then prove that $\omega_1 + \omega_2 = 0.$

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44. Two different non-parallel lines cut the circle $|z| = r$ at points $a, b, c \text{ and } d$, respectively. Prove that these lines meet at the point given by $\frac{a^{-1} + b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}$

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45. If $z_1 + z_2 + z_3 + z_4 = 0$ where $b_1 \in R$ such that the sum of no two values being zero and $b_1z_1 + b_2z_2 + b_3z_3 + b_4z_4 = 0$ where z_1, z_2, z_3, z_4

are arbitrary complex numbers such that no three of them are collinear, prove that the four complex numbers would be concyclic if $|b_1b_2||z_1 - z_2|^2 = |b_3b_4||z_3 - z_4|^2$.

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Exercise

1. Express $\frac{(2+i)^2}{2+3i}$ in the form x+iy.

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2. if z=$\frac{3-4i}{(4-2i)(1+i)}$ then find Re(z).

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3. IF $\frac{\sin\frac{x}{2} + \cos\frac{x}{2} + i\tan\frac{x}{2}}{1 + 2i\sin\frac{x}{2}}$ is purely real, find the set of value of x.

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4. Simplify $\frac{20}{\sqrt{3}-\sqrt{-2}}+\frac{30}{3\sqrt{-2}-2\sqrt{3}}-\frac{14}{2\sqrt{3}-\sqrt{-2}}$ in the form A+iB.

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5. Find all values of θ for which $\frac{1-i\sin\theta}{1+i\cos\theta}$ is purely real.

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6. Find the real values of $x\ and\ y$, if $:(3m-2i)(2+i)^2=10(1+i)$

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7. $(x+iy)(2-3i)=4+i$

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8. Find all complex numbers z for which $\frac{z-2}{z+2}$ is purely imaginary.

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9. Find the real value of x and y if $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$

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10. If z_1=-3+ixy,z_2=x+y+4i and z_1=$\bar{z}_2$ then find x and y .

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11. Evaluate: $\left[i^{19}+\left(\frac{1}{i}\right)^{25}\right]^2$

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12. Evaluate $\left(i^{17} - \frac{1}{i^{34}}\right)^3$.

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13. If $\left(\frac{1+i}{1-i}\right)^m = 1,$ then find the least positive integral value of $m\cdot$

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14. Prove that $(1+i)^{2n} + (1-i)^{2n}$ = 0 if n is an odd integer and, equal to $\frac{2^{n+1}}{(-1)^{n/2}}$ if n is an even integer.

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15. If $x = \sqrt{-2} - 1$ find he value of $x^4 + 4x^3 + 6x^2 + 4x + 9$

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16. If $a = \frac{1+i}{\sqrt{2}}$ find the value of $a^6 + a^4 + a^2 + 1$

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17. If $\sqrt{a - ib}$= x-iy , prove that $\sqrt{a + ib}$=x+iy.

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18. If $\sqrt[3]{a + ib}$ = x + iy, prove that : $\sqrt[3]{a - ib}$ = x- iy .

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19. $(x + iy)^{\frac{1}{3}} = (a + ib)$ then prove that $\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)$

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20. Express $(1 + a^2)(1 + b^2)$ as the sum of two squares.

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21. If $\sqrt{7} - 24i = x + iy$ and $x = \pm 4, y = \pm 3$ then $\sqrt{7} - 24i$=?

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22. Find the square root of $8 - 15i$.

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23. If z^2 = 12$\sqrt{-1}$, find the complex number z .

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24. Find the square root of $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{31}{16}$.

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25. Find the conjugate, modulus and argumetn of $\sqrt{2-\sqrt{2i}}$

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26. If z = $\dfrac{(1+i)\left(1+\sqrt{3}i\right)^2}{1-i}$, find |z| , arg z and $\bar{z}$.

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27. Write $1+\dfrac{i}{2}-i$ in the trigonometrical form.

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28. If z_1= 3i and z_2 = -1 -i , find the value of arg $\dfrac{z_1}{z_2}$.

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29. Find the modulus and amplitude of $\frac{1}{(1-i)^2} - \frac{1}{(1+i)^2}$. Also write the complex number in its polar form.

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30. Solve the equation $z^2 + |z| = 0$, where z is a complex number.

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31. Find the complex number z if $z^2 + \bar{z}$=0

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32. Solve for z: z^2 =$(i\bar{z})^2$.

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33. which of the foolowing satisfies $|z+1| = z+2+2i$

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34. Determine the complex numbers which satisfy the equation $-i\bar{z}$=z^2.

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35. Find all complex numbers satisfying $|z|^2 + z^2 - 5 + i\sqrt{3}$ = 0.

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36. Is there any z such that $\left|\frac{z+2}{z+4i}\right| = 1$? If so, find z.

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37. IF 1age1, $f \in dallcomp \leq x\nu mberszsatiy \in gtheequatio \in$

z+a|z+|+i=0`

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38. Find all complex numbers satisfying $\bar{z}$= z^2 .

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39. For every real number c $\geq$ 0, find all complex numbers z which satisfy the equation $|z|^2 - 2iz + 2c(1+i) = 0$, where $i = \sqrt{-1}$ and passing through (-1,4).

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40. Let the complex numbers z of the form $x + iy$ satisfy arg $\left(\frac{3z-6-3i}{2z-8-6i}\right) = \frac{\pi}{4}$ and $|z-3+i| = 3$. Then the ordered pairs

(x, y) are (A) $\left(4 - \frac{4}{\sqrt{5}}, 1 + \frac{2}{\sqrt{5}}\right)$ (B) $\left(4 + \frac{5}{\sqrt{5}}, 1 - \frac{2}{\sqrt{5}}\right)$ (C) $(6 - 1)$ (D) $(0, 1)$

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41. If z = x + iy and |z+6| = |2z+3| , prove that $x^2 + y^2$=9.

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42. if argument of $\frac{z-1}{z+1} = \frac{\pi}{4}$ then prove that $x^2 + y^2 - 2y = 1$

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43. If z is a normal complex for which |z| = 1 , prove that $\frac{z-1}{z+1}$ is a purely imaginary number.

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44. Show that a real value of x will satisfy hte equation $(1 - ix)/(1 + ix) = a - ib$ if $a^2 + b^2 = 1, wherea, b$ real.

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45. If $(a_1 + ib_1)(a_2 + ib_2)...(a_n + ib_n)$ = x+iy , prove that : $\left(a_1^2 + b_1^2\right)\left(a_2^2 + b_2^2\right)...\left(a_n^2 + b_n^2\right) = x^2 + y^2$.

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46. If $(a_1 + ib_1)(a_2 + ib_2)...(a_n + ib_n)$ = x+iy , prove that : $\tan^{-1}\frac{b_1}{a_1} + \tan^{-1}\frac{b_2}{a_2} + ... + \tan^{-1}\frac{b_n}{a_n} = \tan^{-1}\frac{y}{x}$.

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47. If the complex numbers $z_1, z_2, \ldots\ldots. z_n$ lie on te unit circle |z| = 1 then show that $|z_1 + z_2 + \ldots. + z_n| = \left|z_1^{-1}1 + z_2^{-1} + \ldots\ldots. + z_n^{-1}\right|$.

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48. Show that if $iz^3 + z^2 - z + i = 0$, then $|z| = 1$

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49. If f(z) = $a_0 z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_n$ where z is a complex number and a_1, a_2 , etc , are real , prove that $\overline{f(z)} = f(\bar{z})$.

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50. If $|z_1 + z_2|$ = $|z_1 - z_2|$, prove that ampz_1 - ampz_2 = $\frac{\pi}{2}$.

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51. Prove that $|z_1 + z_2|^2$ = $|z_1|^2$+$|z_2|^2$ if $\frac{z_1}{z_2}$ is purely imaginary.

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52. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$.

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53. Prove that $\left|\alpha + \sqrt{\alpha^2 - \beta^2}\right| + \left|\alpha - \sqrt{\alpha^2 - \beta^2}\right| = |\alpha + \beta| + |\alpha - \beta|$

where α, β are complex numbers.

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54. Prove that $|1 - \bar{z}_1 z_2|^2 - |z_1 - z_2|^2 = \left(1 - |z_1|^2\right)\left(1 - |z_2|^2\right)$.

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55. If $|z - 1| < 3$, prove that $|iz + 3 - 5i| < 8$.

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56. Prove that for nonzero complex numbers $|z_1 + z_2| \left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2(|z_1| + |z_2|)$.

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57. Prove that following inequalities:

(i) $\left|\frac{z}{|z|} - 1\right| \leq |argz|$ (ii) $|z - 1| \leq |z||argz| + |z| - 1|$

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58. Find the value of $w^4 + w^6 + w^8$, if w is a complex cube root of unity.

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59. Find the range of real number α for which the equation $z + \alpha|z - 1| + 2i = 0$ has a solution.

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60. If a, b are real, prove that the equation z^2+az+b=0 will not have any purely real solution if a^2< 4b.

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61. Find the maximum and minimum values of $|z|$ satisfying $\left|z+\frac{1}{z}\right|=2$

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62. If $|z|\geq 3,$ then determine the least value of $\left|z+\frac{1}{z}\right|$.

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63. If z is a complex number such that $\left|z+\frac{1}{z}\right|=1$ show that $Re(z)=0$ when $|z|$ is the maximum.

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64. Locate the region in the Argand plane for the complex number z satisfying |z-4| < |z-2|.

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65. Indicate the region represented by $\frac{\pi}{6} \leq argz \leq \frac{\pi}{4}$.

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66. Indicate the region in the Argand plane respresented by $|z+1|^2 + |z-1|^2$=4 .

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67. Prove that the product of any number of unimodular complex numbers is also a unimodular complex number whose amplitude is equal to the sum of their amplitudes.

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68. Separate the real and imaginary parts of $\frac{(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)}{(\cot\gamma + i)(1 + i\tan\gamma)}$

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69. Prove that $\left(\frac{1 + \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}\right)^n = \cos n\theta + i\sin n\theta$.

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70. If $\alpha = \frac{\pi}{12}$,prove that $\frac{(\cos\alpha + i\sin\alpha)(\cos 2\alpha + i\sin 2\alpha)}{\cos 3\alpha - i\sin 3\alpha} = i$.

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71. Prove that

$$\left(\frac{\cos x + i\sin x}{\sin x + i\cos x}\right)^5 = \cos\left(10x - \frac{5\pi}{2}\right) + i\sin\left(10x - \frac{5\pi}{2}\right)$$

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72. Find the value of : $\sqrt[4]{1}$.

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73. Find the value of : $1^{1/5}$.

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74. Find the value of : $(1-i)^{1/3}$.

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75. Find the value of : $(1-\sqrt{3}i)^{2/5}$.

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76. Find the value of : $\sqrt[4]{-64}$.

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77. Find the value of : $(1+i)^{n}, n \in N$

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78. Solve for x : $x^{5}-1$=0.

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79. Solve for x : $x^{6}-1$=0.

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80. Solve for x : $x^5 + x^3 - x^2 - 1$ =0.

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81. Solve for x : $x^4 + x^3 + x^2 + x + 1$ =0.

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82. Sum of common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ is

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83. Solve: ${}^8C_0x^8 +{}^8 C_2x^6 +{}^8 C_4x^4 +{}^8 C_6x^2 + 1$=0.

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84. Find the integral solutions of the following equation: $(1-i)^x = (1+i)^x$

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85. Solve $(1+i)^x = 2^x$ where x is an integer $\neq$ 0.

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86. If $x^2 - 2x\cos\theta + 1 = 0,$ then the value of $x^{2n} - 2x^n\cos n\theta + 1, n \in N$ is equal to

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87. If $2\cos A = x + \frac{1}{x}$, $2\cos\beta = y + \frac{1}{y}$ then show that $2\cos(A-B) = \frac{x}{y} + \frac{y}{x}.$

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88. If $x_n = \cos\left(\frac{\pi}{2^n}\right) + i\sin\left(\frac{\pi}{2^n}\right)$ Prove that $x_1 x_2 \ldots \ldots x_\infty = -1$

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89. If z_1, z_2 be the complex numbers satisfying x^2+4=2x, prove that $z_1^n + z_2^n = 2^{n+1}\cos\frac{n\pi}{3}$.

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90. If $z = \cos\theta + i\sin\theta$ is a root of the equation $a_0 z^n + a_2 z^{n-2} + \ + a_{n-1} z^+ a_n = 0,$ then prove that

$a_0 + a_1\cos\theta + a_2^{\cos 2}\theta + \ + a_n\cos n\theta = 0$

$a_1 \sin\theta + a_2^{\sin 2}\theta + \ + a_n\sin n\theta = 0$

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91. If $z = \cos\theta + i\sin\theta$ is a root of the equation $a_0 z^n + a_2 z^{n-2} + \quad + a_{n-1} z^{+} a_n = 0,$ then prove that

$a_0 + a_1 \cos\theta + a_2^{\cos 2}\theta + \quad + a_n \cos n\theta = 0$

$a_1 \sin\theta + a_2^{\sin 2}\theta + \quad + a_n \sin n\theta = 0$

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92. If $z^7 + 1 = 0$ then $\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{5\pi}{7}\right)$ is (A) $\frac{1}{8}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{1}{2}$

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93. Hence deduce that $\frac{\cos(\pi)}{9}\frac{\cos(2\pi)}{9}\frac{\cos(3\pi)}{9}\frac{\cos(4\pi)}{9} = \frac{1}{16}$

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94. $(2 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^3 = (1 - 3\omega + \omega^2)^4 = 1$

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95. (i) If α, β be the imaginary cube root of unity, then show that $\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$

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96. If $x = a + b, y = a\omega + b\omega^2 ndz = \omega^2 + b\omega,$ prove that $x^3 + y^3 + z^3 = 3(a^3 + b^3)$

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97. If $x = a + b, y = aw + bw^2$and $z = aw^2 + bw,$ where w is an imaginary cube. root of unity, prove that $x^2 + y^2 + z^2 = 6ab.$

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98. Prove that $\left(\frac{i+\sqrt{3}}{2}\right)^{100}+\left(\frac{i-\sqrt{3}}{2}\right)^{100}$ +1=0.

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99. What is the value of $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{200}+\left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200}+1?$

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100. Prove that $\sqrt{-1-\sqrt{-1-\sqrt{1}-\ldots}}\ \propto\ =w \text{ or } w^2$.

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101. Resolve into the linear factor: a^3+b^3

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102. Resolve into the linear factor: $a^3 + b^3 + c^3 - 3abc$

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103. value of the series $a_0 - a_2 + a_4 - a_6 + \ldots\ldots$ is

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104. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ then find : $a_1 - a_3 + a_5 - a_7 + \ldots.$

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105. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ then find : $a_0 + a_3 + a_6 + a_9 + \ldots.$

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106. If $\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + \quad + a_{2n}x_{2n},$ find the value of $a_0 + a_6 + \quad + \ , n \in N.$

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107. If 1,a_1, a_2, a_3,...,a_{n-1} are the nth roots of unity then prove that : $(1-a_1)(1-a_2)(1-a_3)...(1-a_{n-1})$=n.

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108. If 1,a_1, a_2, a_3,...,a_{n-1} are the nth roots of unity then prove that : $1 + a_1 + a_2 + \ldots + a_{n-1}$=0.

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109. If α is an n^{th} roots of unity, then $1 + 2\alpha + 3\alpha^2 + \ldots\ldots\ldots + n\alpha^{n-1}$ equals

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110. If $\alpha(\neq 1)$ is a nth root of unity then $S = 1 + 3\alpha + 5\alpha^2 + \ldots\ldots\ldots$ upto n terms is equal to

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111. $\sum_{p=1}^{\pi} (3p+2)\left[\sum_{q=1}^{10} \frac{\sin(2q\pi)}{11} - i\frac{\cos(2q\pi)}{11}\right]^p$

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112. Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1, wherep, q, r, s \in n.$

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113. If α ia ROOT OF x^2+x+1= 0 then show that α is also a root of $x^{3p}+x^{3q+1}+x^{3r+2}$ = 0. where p,q,r are integers.

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114. If p,q,r are three consecutive integers $\geq$ 3, prove that $x^p+x^q+x^r$ is divisible by x^3+x^2+x.

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115. If n is n odd integer that is greater than or equal to 3 but not la ultiple of 3, then prove that $(x+1)^n = x^n - 1$ is divisible by $x^3+x^2+x\cdot$

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116. If $z^2(\overline{\omega})^4 = 1$ where ω is a nonreal complex cube root of 1 then find z.

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117. Find z satisfying $|z-5i| \leq 3$ such that amp z is the minimum.

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118. If $|z-25i| \leq 15.$ then $|\text{maximum } arg(z) - \text{minimum } arg(z)|$ equals

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119. Find the complex number Z, the greatest in absolute value which satisfies $|z-2+2i| = 1$

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120. Complex number z_1, z_2 and z_3 in AP

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121. if $\left|\frac{1-iz}{z-i}\right| = 1$ prove that the locus of the variable point z in the Argand plane is the real axis.

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122. The equation $\bar{b}z + \mathbf{a}rz = c$, where b is a non-zero complex constant and c is a real number, represents

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123. Let $A(z_1)$ and (z_2) represent two complex numbers on the complex plane. Suppose the complex slope of the line joining A and B is defined as $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$.If the line l_1, with complex slope ω_1, and l_2, with complex slope $omeg_2$, on the complex plane are perpendicular then prove that $\omega_1 + \omega_2 = 0$.

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124. Find the area of the triangle whose vertices in the Argand plane are $i, \ -i, 1+i.$

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125. Show that the area of the triangle on the Argand diagram formed by the complex number $z, izandz + iz$ is $\frac{1}{2}|z|^2$

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126. Find the area of the triangle whose vertices represent the three roots of the complex equation $z^4 = (z+1)^4$.

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127. The complex numbers z_1, z_2 and the origin form an equilateral triangle only if (A) $z_1^2 + z_2^2 - z_1 z_2 = 0$ (B) $z_1 + z_2 = z_1 z_2$ (C) $z_1^2 - z_2^2 = z_1 z_2$ (D) none of these

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128. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C. Show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

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129. If z_1, z_2, z_3 be the vertices of an equilateral triangle, show that $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0 \text{ or } z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

.

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130. If z_1, z_2, z_3be the vertices of an equilateral triangle, show that $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$ or $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$.

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131. Complex numbers z_1, z_2, z_3 are the vertices A, B, C respectively of an isosceles right angled triangle with right angle at C. Show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

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132. The points representing cube roots of unity

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133. z, w, w^2 are the vertices of an equilateral triangle in the Argand plane, w and w^2 being complex cube roots of 1. If $Im(z) = 0$, then possible values of z is/are:

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134. If a, b, c and u, v, w are the complex numbers representing the vertices of two triangles such that $(c = (1 - r)a + rb$ and $w = (1 - r)u + rv,$ where r is a complex number, then the two triangles have the same area (b) are similar are congruent (d) None of these

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135. If two consecutive vertices of a regular hexagon be z and $\bar{z}$, find the other vertices if the centre is z = 0.

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136. If z_1, z_2 are two complex numbers representing two consecutive vertices of a square then what is the complex number represented by the centre of the square ?

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137. If two vertices of an equilateral triangle are z_1, z_2 then find the complex number z_3 represented by the third vertex where z_1, z_2, z_3 are anticlockwise.

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138. The equation $z\bar{z} + a\bar{z} + \bar{a}z + b = 0, b \in R$ represents circle, if

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139. Prove that $|z - z_1|^2 + |z - z_2|^2$ = k will represent a circle if $|z_1 - z_2|^2 \leq$ 2k.

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140. Find all circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4.$

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141. If z=4i-5 then |z| = _______ and $\bar{z}$ = ______.

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142. If the conjugate of $(x + iy)(1 - 2i)$ be $1 + i$, then:

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143. If $|z| = 2$ and arg $(z) = \frac{\pi}{4}$, find z.

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144. In the Argand plane, the vector z = 4 - 3i is turned in the clockwise sense through 180^@` and stretched three times. The complex number represented ny the new vector is

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145. The value of $i^{103} + i^{-99}$ = ______ where $i = \sqrt{-1}$.

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146. The greatest and the least absolute value of z+1 where |z+4| $\leq$ 3 are ______ and ______ .

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147. If a+ib = x-iy then in terms of x,y we have a-ib = _____ .

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148. If n is of the form 3m+2, $1+\omega^{-11}+\omega^{-2n}$ = ______ where ω is a nonreal cube root of 1.

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149. If α, β, γ are the cube roots of p then for any x, y and z $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is

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150. The value of $\left|\sqrt{\alpha}\right|$, where α is a nonreal cube root of unity , is ______.

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151. If ω is a cube root of unity, then find the value of the following: $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+c\omega^2}{c+c\omega+a\omega^2}$

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152. Let the point P represent the complex number z. If OP is turned round O by an angle $\dfrac{\pi}{2}$ in the clockwise sense where O is the origin then in the new position P represents the complex number ______.

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153. $\left|\sqrt{z}\right|^4$ = ______ where z is a unimodular complex number.

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154. $\dfrac{z\left|\bar{z}\right|^2}{\bar{z}\left|z\right|^2}$ = ______ where z is a complex number of amplitude $\dfrac{\pi}{3}$.

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155. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$.If he point D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number......or......

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156. The angle that the vector representing the complex number $\frac{1}{\left(\sqrt{3}+i\right)^{100}}$ makes with the positive direction of real axis is _____.

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157. If |z| =2 then then the complex number 1/z is represented on a fixed _______.

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158. If z_1 and z_2 are two complex number and a, b, are two real number then $|az_1 - bz_2|^2 + |bz_1 + az_2|^2$ equals

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159. a and b are real numbers between 0 and 1 such that the points $z_1 = a + i, z_2 = 1 + bi, z3 = 0$ form an equilateral triangle, then a and b are equal to

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160. If the complex number z_1, z_2 and z_3 represent the vertices of an equilateral triangle inscribed in te circel $|z| = 2$ and $z_1 = 1 + i\sqrt{3}$ then (A) $z_2 = 1, z_3 = 1 - i\sqrt{3}$ (B) $z_2 = 1 - i\sqrt{3}, z_3 = - - i\sqrt{3}$ (C) $z_2 = 1 - i\sqrt{3}, z_3 = -1 + i\sqrt{3}$ (D) $z_2 = , z_3 = 1 - i\sqrt{3}$

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161. If the real part of $\frac{z+4}{2z+i}$ is $\frac{1}{2}$ then the point z lies on ______.

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162. If $a > 0, b > 0$ then $\sqrt{-a} \cdot \sqrt{b}$ is equal to

A. $-\sqrt{ab}$

B. $\sqrt{ab}i$

C. $\sqrt{ab}$

D. none of these

Answer: B

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163. If $i^2 = -1$ then the value of $\sum_{n=1}^{200} i^n$ is

A. 0

B. 50

C. -50

D. none of these

Answer:

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164. Find the smallest positive integer value of n for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number.

A. 3

B. 2

C. 1

D. 5

Answer:

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165. The digit in the units place in the value of $(727)^{39}$ is

A. 3

B. 7

C. 1

D. 9

Answer: A

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166. If n is an integer, not a multiple of 3, the sum of $w^n + w^{2n}$, w being a nonreal complex cube root of unity , is

A. 2

B. -1

C. 2 or -1

D. none of these

Answer: B

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167. If $\alpha = \sqrt[3]{1}$ and α is not real then $\alpha^{3n+1} + \alpha^{3n+3} + \alpha^{3n+5}$ has the value

A. -1

B. 0

C. 1

D. 3

Answer: B

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168. If $1, \omega, \omega^2$ are the cube roots of unity, then the roots of the equation $(x-1)^3 + 8 = 0$ are

A. $-1, 1+2w, 1+2w^2$

B. $-1, 1-2w, 1-2w^2$

C. $-1 -1 -1$

D. none of these

Answer:

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169. If z is a complex number then

A. $z+\bar{z}$ and $z\bar{z}$ are both purely real

B. $z+\bar{z}$ is purely real but $z\bar{z}$ is not

C. $z\bar{z}$ is purely real nut $z+\bar{z}$ is not

D. neither $z+\bar{z}$ nor $z\bar{z}$ need be purely real

Answer: A

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170. If $z = i\bar{z}$ then

A. z is purely real

B. z is purely imaginary

C. $z = x(1 + i), x \in R$

D. $z = 0$

Answer: C

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171. The value of $\sqrt{i}$ is

A. 1-i

B. 1+i

C. $\pm(1+i)$

D. $\frac{\pm 1}{\sqrt{2}}(1+i)$

Answer: D

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172. Prove that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ is purely real.

A. Re(z)=0

B. Im(z)=0

C. Re(z)gt0,Im(z)gt0

D. Re(z)gt0,Im(z)lt0

Answer:

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173. If $z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2r\pi}{11}$ then : the value of $\sum_{r=0}^{10} z_r$ is equal to

A. -1

B. 0

C. $-i$

D. i

Answer: B

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174. If $z_r = \sin\frac{2\pi r}{11} - i\cos\frac{2r\pi}{11}$ then : the value of $\sum_{r=1}^{10} z_r$ is equal to

A. -1

B. 0

C. $-i$

D. i

Answer: D

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175. The complex numbers $\sin x - i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for

A. $x = n\pi$

B. $x = 0$

C. $x = \left(n + \frac{1}{2}\right)\pi$

D. no value of x

Answer: D

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176. If $|z_1| = |z_2| = 1$ and amp z_1+ampz_2=0 then

A. $z_1 z_2 = 1$

B. $z_1 + z_2$=0

C. $z_1 = \bar{z}_2$

D. none of these

Answer: A,C

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177. If z_1 and z_2 are two non zero$comp \le xvmbersucht\hat{}$ |z_1+z_2|=|z_1|+|z_2|$then$arg z_1-argz_2$isequal \to (A)$- pi/2(B)0(C)-pi (D)pi/2`

A. $-\pi$

B. $-\frac{\pi}{2}$

C. 0

D. $\frac{\pi}{2}$

π

Answer:

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178. The inequality $|z+2| < |z-2|$ represents the region given by

A. $Re(z) > 0$

B. $Re(z) < 0$

C. $Re(z) > 2$

D. none of these

Answer: B

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179. Let z_1 and z_2 be complex numbers of such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. $If z_1$ has positive real part and z_2 has negative imginary part, then which of the following statemernts are correct for te vaue of $\frac{z_1 + z_2}{z_1 - z_2}$ (A) 0 (B) real and positive (C) real and negative (D) purely imaginary

A. zero

B. real and positive

C. real and negative

D. purely imaginary

Answer:

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180. If $z_1 = a_i b$ and $z_2 = c + id$ are complex numbes such that $|z_1| = |z_2| = 1$ and $Re(z_1 \bar{z}_2) = 0$ then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfy which of the following relations? (A) $|\omega_1| = 1$ (B) $|\omega_2| = 1$ (C) $Re(\omega_1 \overline{\omega}_2) = 0$ (D) $Im(\omega_1 \overline{\omega}_2) = 0$

A. $|w_1|$=1

B. $|w_2|$=1

C. Re$(w_1\overline{w}_2)$=0

D. none of these

Answer:

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181. if $|z_1|$= $|z_2| \neq 0$ and amp $\frac{z_1}{z_2}$=π then

A. $z_1 = z_2$

B. $z_1 + z_2$=0

C. $z_1 z_2$=1

D. none of these

Answer: B

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182. If z_1 and z_2 are two nonzero complex numbers such that $|z_1 - z_2| = |z_1| - |z_2|$ then arg z_1-argz_2 is equal to

A. 0

B. π

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

Answer: A

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183. If z_1,z_2 are nonreal complex and $\left|\frac{z_1 + z_2}{z_1 - z_2}\right|$=1 then $\frac{z_1}{z_2}$ is

A. real positive

B. purely imaginary

C. negative real

D. none of these

Answer: B

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184. If $z = 2 + 3i$, then $\left|z^2\right|^3$ is equal to

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185. The equation $z^5 + z^4 + z^3 + z^2 + z + 1$=0 is satisfied by

A. $z = \pm 1$

B. $z = -1$

C. $\pm \frac{1}{2} + i\frac{\sqrt{3}}{2}$

D. $\frac{1}{2} \pm i\sqrt{\frac{1}{2}}$

Answer: B , C

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186. The points, $z_1, z_2, z_3, z_4,$ in the complex plane are the vartices of a parallelogram taken in order, if and only if $z_1 + z_4 = z_2 + z_3$ $z_1 + z_3 = z_2 + z_4$ $z_1 + z_2 = z_3 + z_4$ (d) None of these

A. $z_1 + z_{14} = z_{12} + z_3$

B. $z_1 + z_{13} = z_2 + z_4$

C. $z_1 + z_2 = z_3 + z_4$

D. none of these

Answer:

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187. If $z^4 = (z-1)^4$ then the roots are represented in the Argand plane by the points that are

A. collinear

B. concyclic

C. vertices of a parallelogram

D. none of these

Answer: A

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188. If $|z_1| = |z_2| = |z_3|$ = 1 and $z_1 + z_2 + z_3$ =0 then the area of the triangle whose vertices are z_1, z_2, z_3 is

A. $\frac{3\sqrt{3}}{4}$

B. $\sqrt{3}$

C. 1

D. none of these

Answer: A

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189. In the Argand plane $\left|\frac{z-i}{z+i}\right|$ = 4 represents a

A. pair of distinct lines

B. circle

C. a pair of coincident

D. none of these

Answer: B

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190. Suppose

$z_1+z_2+z_3+z_4=0$ and $|z_1|=|z_2|=|z_3|=|z_4|=1. If z_1, z_2, z_3, z_4$ are the vertices of a quadrilateral, then the quadrilateral can be a (a) parallelogram (c) rectangle (b) rhombus (d) square

A. trapezium

B. rectangle

C. square

D. none of these

Answer:

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191. If arg $\frac{z - z_1}{z_2 - z_1}$ = 0 for three distinct complex numbers z, z_1, z_2 then the three points are

A. concyclin

B. vertices of an equilateral triangle

C. collinear

D. vertics of a right-angled triangle

Answer: C

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192. The complex numbers $z = x + iy$ which satisfy the equation $\left|\frac{z - 5i}{z + 5i}\right| = 1$ lie on (a) The x-axis (b) The straight line $y = 5$ (c) A circle passing through the origin (d) Non of these

A. the x-axis

B. the straight line y = 5

C. a circle passing through the origin

D. none of these

Answer:

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193. The locus of the points z satisfying the condition arg $\left(\frac{z - 1}{z + 1}\right) = \frac{\pi}{3}$ is, a

A. a circle

B. a straight line

C. a pair of straight lines

D. none of these

Answer:

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194. If $arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$ then the locus of z is

A. a straight line

B. a pair of straight lines

C. a circle

D. an ellipse

Answer:

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195. $z\bar{z} + a\bar{z} + \bar{a}z + b$=0 where $b \in R$ represents a real circle of nonzero radius if

A. $|\bar{a}|^2 > b$

B. $|a|^2 < b$

C. $|\bar{a}|^2 \geq b$

D. $|a|^2 \leq b$

Answer: A

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196. Find the value of $\sqrt{20 + 48i}$

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197. If $i^p = i^q$ where $i^2 = -1$ then p-q is divisible by 4.

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198. If $z=\dfrac{2+3i}{3+2i}$, then $|z|=$

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199. Find the solutions to the equation $(z+i)^2$ = 16.

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200. If z is a nonreal compex number and |z|=1 then $z^2+\dfrac{1}{z^2}=2$.

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201. State true or false: If z= $\dfrac{\cos 2\theta + i\sin 2\theta}{\cos\theta + i\sin\theta}$ then z is unimodular.

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202. If two nonzero complex numbers z_1, z_2 be such that $z_1 + z_2$ is real then they are necessarily conjugate to each other.

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203. For complex numbers$z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ we write $z_1 \cap z_2$ if $x_1 \leq X_2$ and $y_1 \leq y_2$ Then for all compelx nubers z with $1 \cap z$ we have $\frac{1-z}{1+z} \cap 0$

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204. If z is a nonzero complex number then $\left(\overline{z^{-1}}\right) = (\bar{z})^{-1}$.

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205. If the points P and Q represent the complex numbers z and iz then $\angle POQ$ is a right angle. State true or false

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206. If $z_1 \neq -z_2$ and $|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$ then :

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207. Find z, if $\left|\frac{z+1}{z+i}\right| = 1$

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208. If in the Argand plane z_1, z_2, z_3 and z_4 are four points such that $|z_1| = |z_2| = |z_3| = |z_4|$ then the four points are vertices of a square. True or False.

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209. State true or false: If $z \neq 0$ then $arg z + arg \bar{z}$=0.

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210. Let z_1 and z_2 be the roots of $z^2 + pz + q$=0. Then the points represented by z_1, z_2 the origin form an equilateral triangle if p^2=3q.

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211. If z_1, z_2 are nonzero complex numbers then $\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \leq 2$.

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212. The nth roots of -1 can be n terms of a GP.

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213. In the following, each question has one or more than one correct answers. Indicate the correct answer(s). : IF a, b, c are multiples of 3 then $x^a + x^{b+1} + x^{c+2}$ is divisible by

A. x+1

B. $x^2 + 1$

C. $x^2 + x + 1$

D. none of these

Answer: C

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214. If $|z - 3i| = 3$ and $ampz \in \left(0, \frac{\pi}{2}\right)$, then $\cot(ampz) - \frac{\theta}{z}$ is eqaul to

A. i

B. 1

C. -1

D. $-i$

$$\frac{-1}{i}$$

Answer:

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215. If z $\neq$ 0 then amp z +amp$\bar{z}$ = ______.

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216. x+iy=$\left(\frac{\sqrt{3}+i}{1-\sqrt{3}i}\right)^{25}$ then x=______, y=______.

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217. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of x^5-1=0 then prove that $\frac{w-\alpha_1}{w^2-\alpha_1}\cdot\frac{w-\alpha_2}{w^2-\alpha_2}\cdot\frac{w-\alpha_3}{w^2-\alpha_3}\cdot\frac{w-\alpha_4}{w^2-\alpha_4}=w$ where w is a nonreal complex cube root of unity.

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218. Consider the quadratic equation $az^2 + bz + c = 0$ where a,b,c and non-zero complex numbers. Now answer the following:

Q. The condition that the equation has one complex root α such that $|\alpha| = 1$, is

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219. If $\left|z^2 - 4\right| = 2|z|$ then find the greatest value of |z|.

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220. If $z = 2 + t + i\sqrt{3 - t^2}$, where t is real and $t^2 < 3$, show that the modulus of $\frac{z+1}{z-1}$ is independent of t. Also show that the locus of the points z for different values of t is a circle and finds its centre and radius.

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221. If $|z_1| = 1, |z_2| = 1$ then prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2$ =4.

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222. z_1, z_2 are represented by two consecutive vertices of a rhombus, the angle at z_1 being $\frac{\pi}{4}$. Find the complex numbers z_3, z_4 represented by the other vertices, the vertices z_1, z_2, z_3, z_4 being in the anticlockwise sense and origin being the center.

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223. Prove that $t^2 + 3t + 3$ is a factor of $(t+1)^{n+1}$ ^ $(n+1) + (t+2)^{2n-1}$ for all intergral values of n $\in$ N.

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224. If $x^2 - x + 1$ =0 then find the value of $\left(x + \frac{1}{x}\right)^2 + \left(x^2 + \frac{1}{x^2}\right)^2 + \left(x^3 + \frac{1}{x^3}\right)^2 + \ldots + \left(x^{24} + \frac{1}{x^{24}}\right)^2.$

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