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## MATHS

## NCERT - FULL MARKS MATHEMATICS(TAMIL)

## SEQUENCES AND SERIES

## Example

1. Write the first three terms in each of the following sequences defined by the following:
(i) $a_{n}=2 n+5$, (ii) $a_{n}=\frac{n-3}{4}$

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2. What is the $20^{\text {th }}$ term of the sequence defined by
$a_{n}=(n-1)(2-n)(3+n) ?$

## - Watch Video Solution

3. Let the sequence $a_{n}$ be defined as follows:
$a_{1}=1, a_{n}=a_{n-1}+2$ for $n \geq 2$.
Find first five terms and write corresponding series

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4. In an A.P. if $m^{\text {th }}$ term is n and the $n^{\text {th }}$ term is m , where $m \neq n$, find the pth term.
5. If the sum of n terms of an A.P. is $n P+\frac{1}{2} n(n-1) Q$ , where $P$ and $Q$ are constants, find the common difference.

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6. The sum of n terms of two arithmetic progressions are in the ratio $(3 n+8):(7 n+15)$. Find the ratio of their $12^{\text {th }}$ terms.

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7. The income of a person is Rs. $3,00,000$ in the first year and he receives an increases of Rs. 10,000 to his income per year for the next 19 years. Find the total amount he received in 20 years.

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8. Example 8 Insert 6 numbers between 3 and 24 such that the resulting sequence is an A.P.

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9. Find the $10^{\text {th }}$ and $n^{\text {th }}$ terms of the G.P . 5, 25,125

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10. Which term of the G.P., $2,8,32$, ... up to $n$ terms is

131072 ?

## (D) Watch Video Solution

11. In a GP the 3rd term is 24 and 6th term is 192 . Find the 10th term.

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12. Find the sum of first $n$ terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+\ldots \ldots \ldots .$.

## - Watch Video Solution

13. How many terms of the G.P. $3, \frac{3}{2}, \frac{3}{4} \ldots \ldots . . . .$. are needed to give the sum $\frac{3069}{512}$ ?

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14. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1 . Find the common ratio and the terms.
15. Find the sum of the sequence $7,77,777,7777, \ldots$ to $n$ terms.

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16. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on.Find the number of his ancestors during the ten generations preceding his own.

- Watch Video Solution

17. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

## D Watch Video Solution

18. If A.M. and G.M. of two positive numbers $a$ and $b$ are 10 and 8 , respectively, find the numbers.

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19. Find the sum to n terms of the series : $5+11+19+29+41 \ldots \ldots \ldots$.
20. Find the sum to n terms of the series whose $n^{\text {th }}$ term is $n(n+3)$.

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## Miscellaneous Examples

1. If $p^{\text {th }}, q^{\text {th }}, r^{\text {th }}$ and $s^{t h}$ terms of an A.P. are in G.P, then show that $(p-q),(q-r),(r-s)$ are also in G.P.
2. If $a, b, c$ are in geometric progression, and if $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$, then prove that $\mathrm{x}, \mathrm{y}, \mathrm{z}$ are in arithmetic progression.

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3. If $a, b, c, d$ and $p$ are different real numbers such that $\left(a^{2}+b^{2}+c^{2}\right) p^{2}-2(a b+b c+c d) p+\left(b^{2}+c^{2}+d^{2}\right) \leq 0$
, then show that $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are in GP.

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4. If $p, q, r$ are in G.P. and the equations, $p x^{2}+2 q x+r=0$ and $d x^{2}+2 e x+f=0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

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## Exercise 91

1. Write the first five terms of each of the sequences whose $n^{\text {th }}$ terms are:

$$
a_{n}=n(n+2)
$$

2. Write the first five terms of each of the sequences whose $n^{\text {th }}$ terms are:
$a_{n}=\frac{n}{n+1}$

## D Watch Video Solution

3. Write the first five terms of each of the sequences whose $n^{\text {th }}$ terms are:
$a_{n}=2^{n}$

## D Watch Video Solution

4. Write the first five terms of each of the sequences
whose $n^{\text {th }}$ terms are:
$a_{n}=\frac{2 n-3}{6}$

## - Watch Video Solution

5. Write the first five terms of each of the sequences whose $n^{\text {th }}$ terms are:
$a_{n}=(-1)^{n-1} 5^{n+1}$

## D Watch Video Solution

6. Write the first five terms of each of the sequences
whose $n^{\text {th }}$ terms are:
$a_{n}=n \frac{n^{2}+5}{4}$
7. Find the indicated terms in each of the sequences whose $n^{\text {th }}$ terms are:
$a_{n}=4 n-3, a_{17}, a_{24}$

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8. Find the indicated terms in each of the sequences
whose $n^{\text {th }}$ terms are:
$a_{n}=\frac{n^{2}}{2^{n}}, a_{7}$
9. Find the indicated terms in each of the sequences
whose $n^{\text {th }}$ terms are:
$a_{n}=(-1)^{n-1} n^{3}, a_{9}$

D Watch Video Solution
10. Find the indicated terms in each of the sequences whose $n^{\text {th }}$ terms are:
$a_{n}=\frac{n(n-2)}{n+3}, a_{20}$
11. Write the first five terms of each of the sequences and obtain the corresponding series:
$a_{1}=3, a_{n}=3 a_{n-1}+2$ for all $n>1$

## D Watch Video Solution

12. Write the first five terms of each of the sequences and obtain the corresponding series:
$a_{1}=-1, a_{n}=\frac{a_{n-1}}{n}, n \geq 2$
13. Write the first five terms of each of the sequences and obtain the corresponding series: $a_{1}=a_{2}=2, a_{n}=a_{n-1}-1, n>2$

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14. The Fibonacci sequence is defined by $1=a_{1}=a_{2}$ and $\quad a_{n}=a_{n-1}+a_{n-2}, n>2$. Find $\frac{a_{n+1}}{a_{n}}$, for $n=1,2,3,4,5$,

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1. Find the sum of odd integers from 1 to 2001.

## - Watch Video Solution

2. Find the sum of all natural numbers lying between

100 and 1000 , which are multiples of 5 .

## - Watch Video Solution

3. In an A.P., the first term is 2 and the sum of the first
five terms is one-fourth of the next five terms. Show that $20^{\text {th }}$ term is -112.
4. How many terms of the A.P. $-6,-\frac{11}{2},-5, \ldots$ are needed to give the sum -25 ?

## - Watch Video Solution

5. In an A.P. if $p^{\text {th }}$ term is $\frac{1}{q}$ and $q^{\text {th }}$ term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(p q+1)$, where $p \neq q$.

## D Watch Video Solution

6. If the sum of a certain number of terms of the A.P. 25 ,
$22,19, \ldots$ is 116 . Find the last term.
7. Find the sum to $n$ terms of the A.P., whose $k^{\text {th }}$ term is $5 \mathrm{k}+1$.
(D) Watch Video Solution
8. If the sum of n terms of an A.P. is $\left(p n+q n^{2}\right)$, where p and q are constants, find the common difference.

D Watch Video Solution
9. The sums of n terms of two arithmetic progressions are in the ratio $5 n+4: 9 n+6$. Find the ratio of their $18^{\text {th }}$ terms.

## - Watch Video Solution

10. If the sum of first $p$ terms of an A.P. is equal to the sum of the first $q$ terms, then find the sum of the first $(p+q)$ terms.

- Watch Video Solution

11. Sum of the first $p, q$ and $r$ terms of an A.P. are $a, b$ and c, respectively. Prove that
$\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

## D Watch Video Solution

12. The ratio of the sum of $m$ and $n$ terms of an A.P. is $m^{2}: n^{2}$. Show that the ratio of $m^{\text {th }}$ and $n^{\text {th }}$ term is $2 m-1: 2 n-1$.
13. If the sum of $n$ terms of an A.P. is $3 n^{2}+5 n$ and its $m^{\text {th }}$ term is 164 , find the value of $m$.

## (D) Watch Video Solution

14. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

## - Watch Video Solution

15. If $\frac{a^{n}+b^{n}}{a^{n-1}+b^{n-1}}$ is the A.M. between $a$ and b , then find the value of $n$.
16. Between 1 and $31, \mathrm{~m}$ numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of $7^{\text {th }}$ and $(m-1)^{\text {th }}$ numbers is $5: 9$. Find the value of $m$.

## D Watch Video Solution

17. A man starts repaying a loan as first installment of

Rs. 100. If he increases the installment by Rs 5 every month, what amount he will pay in the $30^{\text {th }}$ installment?
18. The difference between any two consecutive interior angles of a polygon is $5^{\circ}$.If the smallest angle is $120^{\circ}$, find the number of the sides of the polygon.

## D Watch Video Solution

## Exercise 93

1. Find the 20th and nth term of the GP.
$5 \quad 5 \quad 5$
$\overline{2}, \overline{4}, \overline{8} \ldots \ldots \ldots$

## - Watch Video Solution

2. Find the 12th term of a G.P. whose 8 th term is 192 , and the common ratio is 2 .

## D Watch Video Solution

3. The $5^{\text {th }}, 8^{\text {th }}$ and $11^{\text {th }}$ terms of a G.P are $\mathrm{p}, \mathrm{q}$ and s , respectively. Show that $q^{2}=p s$.

## - Watch Video Solution

4. The $4^{\text {th }}$ term of a G.P. is square of its second term, and the first term is -3 .Determine its $7^{\text {th }}$ term.
5. Which term of the following sequences:
(a) $2,2 \sqrt{4}, 4 \ldots \ldots \ldots$ is 128 ? (b) $\sqrt{3}, 3,3 \sqrt{3}, \ldots \ldots \ldots \ldots$ is 729 ?
(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots \ldots$ is $\frac{1}{19683}$ ?

## - Watch Video Solution

6. For what values of x , the numbers $-\frac{2}{7}, x,-\frac{7}{2}$ are in G.P ?

Watch Video Solution
7. Find the sum to indicated number of terms in each of
the geometric progressions in
$0.15,0.015,0.0015, \ldots . . . . . . .20$ terms .

## D Watch Video Solution

8. Find the sum to indicated number of terms in each of
the geometric progressions in
$\sqrt{7}, \sqrt{21} 3 \sqrt{7}, \ldots \ldots . . . n$ terms

- Watch Video Solution

9. Find the sum to indicated number of terms in each of
the geometric progressions in Exercises
$1,-a, a^{2},-a^{3}, \ldots \ldots . n$ terms (if $a \neq-1$ )

## D Watch Video Solution

10. Find the sum to indicated number of terms in each of the geometric progressions in
$x^{3}, x^{5}, x^{7}, \ldots \ldots \ldots . n$ terms (if $x \neq \pm 1$ )

## - Watch Video Solution

11. Evaluate $\sum_{k-1}^{11}\left(2+3^{k}\right)$

## (-) Watch Video Solution

12. The sum of first three terms of a G.P. is and their product is 1 . Find the common ratio and the terms.

## - Watch Video Solution

13. How many terms of G.P. $3,3^{2}, 3^{3}, \ldots$ are needed to give the sum 120 ?

- Watch Video Solution

14. the sum of the first six terms of a GP is 9 times the sum of the first three terms. The common ratio is

## D Watch Video Solution

15. If the first term of a G.P. is 729 and its $7^{\text {th }}$ term is 64 ,
then the sum of first seven terms is

## D Watch Video Solution

16. Find a G.P. for which sum of the first two terms is -4
and the fifth term is 4 times the third term.
17. If the 4th, 10th and 16th terms of a G.P. are $x, y$ and $z$, respectively. Prove that $x, y, z$ are in GP.

## D Watch Video Solution

18. Find the sum to $n$ terms of the sequence, $8,88,888$, 8888... .

## D Watch Video Solution

19. Find the sum of the products of the corresponding
terms of the sequences $2,4,8,16,32$ and $128,32,8,2$

## D Watch Video Solution

20. Show that the products of the corresponding terms of the sequences $\mathrm{a}, \mathrm{ar}, a r^{2}, \ldots a r^{n-1}$ and $\mathrm{A}, \mathrm{AR}$, $\mathrm{AR}^{22}, \ldots \mathrm{AR}^{n-1}$ form a G.P, and find the common ratio.

## - Watch Video Solution

21. Find four numbers forming a geometric progression
in which the third term is greater than the first term by
9 , and the second term is greater than the $4^{\text {th }}$ by 18.
22. If the $p^{\text {th }}, q^{\text {th }}$ and $r^{\text {th }}$ terms of a G.P. are $\mathrm{a}, \mathrm{b}$ and c , respectively. Prove that $a^{q-r} b^{r-p} c^{P-q}=1$.

## D Watch Video Solution

23. If the first and the $n^{\text {th }}$ term of a G.P. are a and b , respectively, and if P is the product of n terms, prove that $P^{2}=(a b)^{n}$.

## D Watch Video Solution

24. Show that the ratio of the sum of first $n$ terms of a G.P. to the sum of terms from $(n+1)^{\text {th }}$ to $(2 n)^{\text {th }}$ term is $\frac{1}{r^{n}}$.

## D Watch Video Solution

25. If $a, b, c$ and $d$ are in G.P. show that $\left(a^{2}+b^{2}+c^{2}\right)\left(b^{2}+c^{2}+d^{2}\right)=(a b+b c+c d)^{2}$

## D Watch Video Solution

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.
27. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^{n}+b^{n}}$ may be the geometric mean between a and b .

## D Watch Video Solution

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio

$$
(3+2 \sqrt{2}):(3-2 \sqrt{2})
$$

## - Watch Video Solution

29. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

## D Watch Video Solution

30. The number of bacteria in a certain culture doubles
every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of $2^{\text {nd }}$ hour, $4^{\text {th }}$ hour and $n^{\text {th }}$ hour ?

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31. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of $10 \%$ compounded annually?

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32. If A.M. and G.M. of roots of a quadratic equation are

8 and 5, respectively, then obtain the quadratic equation.

## D Watch Video Solution

1. Find the sum to $n$ terms of each of the series in $1 \times 2+2 \times 3+3 \times 4+4 \times 5+\ldots$.

## - Watch Video Solution

2. Find the sum to $n$ terms of each of the series in
$1 \times 2 \times 3+2 \times 3 \times 4+3 \times 4 \times 5+\ldots$

## D Watch Video Solution

3. Find the sum to $n$ terms of each of the series in
$3 \times 1^{2}+5 \times 2^{2}+7 \times 3^{2}+\ldots \ldots \ldots$
4. Find the sum to $n$ terms of each of the series in

$$
\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\frac{1}{3 \times 4}+\ldots \ldots
$$

## D Watch Video Solution

5. Find the sum to $n$ terms of each of the series in $5^{2}+6^{2}+7^{2}+\ldots+20^{2}$

## - Watch Video Solution

6. Find the sum to $n$ terms of each of the series in $3 \times 8+6 \times 11+9 \times 14+\ldots$.
7. Find the sum to $n$ terms of each of the series in
$1^{2}+\left(1^{2}+2^{2}\right)+\left(1^{2}+2^{2}+3^{2}\right)+\ldots$

## D Watch Video Solution

8. Find the sum to n terms of the series in whose $n^{\text {th }}$ terms is given by
$n(n+1)(n+4)$

D Watch Video Solution
9. Find the sum to n terms of the series in whose $n^{\text {th }}$ terms is given by
$n^{2}+2^{n}$

## D Watch Video Solution

10. Find the sum to n terms of the series in whose $n^{\text {th }}$
terms is given by
$(2 n-1)^{2}$

- Watch Video Solution

1. Show that the sum of $(m+n)^{t h}$ and $(m-n)^{t h}$ term of an A.P is equal to twice the $m^{\text {th }}$ term.

## - Watch Video Solution

2. If the sum of three numbers in A.P., is 24 and their product is 440 , find the numbers.

## D Watch Video Solution

3. The sum of first $n, 2 n$ and $3 n$ terms of an A.P. are $S_{1}, S_{2}, S_{3}$ respectively. Prove that $S_{3}=3\left(S_{2}-S_{1}\right)$.
4. Find the sum of all numbers between 200 and 400 which are divisible by 7 .

## D Watch Video Solution

5. Find the sum of integers from 1 to 100 that are divisible by 2 or 5 .

## - Watch Video Solution

6. Find the sum of all two digit numbers which when divided by 4 , yields 1 as remainder.
7. If $f$ is a function satisfying $f(x+y)=f(x) f(y)$ for all $x, y \in N$ such that $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$, find the value of $n$.

## D Watch Video Solution

8. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2 , respectively. Find the last term and the number of terms.
9. The first term of a G.P. is 1 . The sum of the third term and fifth term is 90 . Find the common ratio of G.P.

## D Watch Video Solution

10. The sum of three numbers in G.P. is 56 . If we subtract $1,7,21$ from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

## D Watch Video Solution

11. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms
occupying odd places, then find its common ratio.

## - Watch Video Solution

12. The sum of the first four terms of an A.P. is 56 . The sum of the last four terms is 112 . If its first term is 11 , then find the number of terms.

## - Watch Video Solution

13. If $\frac{a+b x}{a-b x}=\frac{b-c x}{b-c x}=\frac{c+d x}{c-d x}(x \neq 0)$ then show that $\mathrm{a}, \mathrm{b}, \mathrm{c}$ and d are in G.P.
14. Let $S$ be the sum, $P$ the product and $R$ the sum of reciprocals of n terms in a G.P. Prove that $P^{2} R^{n}=S^{n}$.

## - Watch Video Solution

15. The $p^{\text {th }}, q^{\text {th }}$ and $r^{\text {th }}$ terms of an A.P. are $\mathrm{a}, \mathrm{b}, \mathrm{c}$, respectively.

Show
that
$(q-r) a+(r-p) b+(q-p) c=0$

## D Watch Video Solution

16. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., prove that $\mathrm{a}, \mathrm{b}, \mathrm{c}$, are in A.P.

## - Watch Video Solution

17. If $a, b, c, d$ are in G.P, prove that $\left(a^{n}+b^{n}\right),\left(b^{n}+c^{n}\right),\left(c^{n}+d^{n}\right)$ are in G.P.

## - Watch Video Solution

18. If a and b are the roots of $x^{2}-3 x+p=0$ and $\mathrm{c}, \mathrm{d}$ are roots of $x^{2}-12 x+q=0$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ form a G.P

Prove that $(q+p):(q-p)=17: 15$

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19. The ratio of the A.M. and G.M. of two positive numbers $a$ and $b$, is $m: n$. Show that
$a: b=\left(m+\sqrt{m^{2}-n^{2}}\right):\left(m-\sqrt{m^{2}-n^{2}}\right)$.

## - Watch Video Solution

20. If $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are in A.P., $\mathrm{b}, \mathrm{c}, \mathrm{d}$ are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a,c,e are in GP.

## D Watch Video Solution

21. Find the sum of the following series up to $n$ terms:
(i)

$$
\begin{equation*}
5+55+555+\ldots \ldots \tag{ii}
\end{equation*}
$$

$.6+.66 .+.666+\ldots \ldots \ldots \ldots$

## - Watch Video Solution

22. Find the $20^{\text {th }}$ term of the series
$2 \times 4+4 \times 6+6 \times 8+\ldots+n$ terms.

- Watch Video Solution

23. Find the sum of the first $n$ terms of the series: $3+7$
$+13+21+31+. . . . .$.

Watch Video Solution
24. If $S_{1}, S_{2}, S_{3}$ are the sum of first n natural numbers,
their squares and their cubes, respectively, show that $9 S_{2}^{2}=S_{3}\left(1+8 S_{1}\right)$

## D Watch Video Solution

25. Find the sum of the following series up to $n$ terms:
$\frac{1^{3}}{1}+\frac{1^{3}+2^{3}}{1+3}+\frac{1^{3}+2^{3}+3^{3}}{1+3+5}+\ldots \ldots$.

## - Watch Video Solution

26. 

Show
that
$\frac{1 \times 2^{2}+2 \times 3^{2}+\ldots+n \times(n+1)^{2}}{1^{2} \times 2+2^{2} \times 3+\ldots+n^{2} \times(n+1)}=\frac{3 n+5}{3 n+1}$

## D Watch Video Solution

27. A farmer buys a used tractor for Rs 12000. He pays

Rs 6000 cash and agrees to pay the balance in annual instalments of Rs 500 plus $12 \%$ interest on the unpaid amount. How much will the tractor cost him?

## - Watch Video Solution

28. Shamshad Ali buys a scootor for Rs 22000. He pays

Rs 4000 cash and agrees to pay the balance in annual
instalment of Rs 1000 plus $10 \%$ interest on the unpaid amount. How much will the scootor cost him?
29. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move
the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter.

Find the amount spent on the postage when 8th set of letter is mailed.

## D Watch Video Solution

30. A man deposited Rs 10000 in a bank at the rate of
$5 \%$ simple interest annually. Find the amount in $15^{\text {th }}$
year since he deposited the amount and also calculate the total amount after 20 years.

## D Watch Video Solution

31. A manufacturer reckons that the value of a machine, which costs him Rs. 15625 ,will depreciate each year by $20 \%$. Find the estimated value at the end of 5 years.

## - Watch Video Solution

32. 150 workers were engaged to finish a piece of work
in a certain number of days. Four workers dropped
from the work on the second day. Four workers
dropped on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed. [Let the no.of days to finish the work is ' $r$ ' then
$150 x=\frac{x+8}{2}[2 \times 150+(x+8-1)(-4)]$

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