MATHS

BOOKS - V PUBLICATION

PRINCIPLE OF MATHEMATICAL INDUCTION

Question Bank

1. For all $n > 1$, prove that $1^2 + 2^2 + 3^2 + 4^2 + \dots\dots\dots. + n^2 = \frac{n(n+1)(2n+1)}{6}$

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2. Prove that $2^n > n$ for all positive integtrs n.

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3. For all $n \geq 1$, prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

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4. For every positive integer n,prove that $7^n - 3^n$ is divisible by 4 using principle of mathematical induction.

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5. Prove that $(1+x)^n \geq 1 + nx$, for all natural number 'n', where $x > -1$.

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6. For all $n \geq 1$, prove that $p(n) : 2.7^n + 3.5^n - 5$ is divisible by 24.

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7. For all $n \geq 1$, prove that

$$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 > \frac{n^3}{3}$$

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8. Prove that the rule of exponents $(ab)^n = a^n b^n$ by using principle of mathematical induction for every natural number.

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9. Prove by using the principal of Mathematical Induction $P(n) = 1 + 3 + 3^2 + \ldots\ldots + 3^{n-1} = \frac{3^n - 1}{2}$ is true for all $n \in N$

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10. Using mathematical induction prove that

$$1^3 + 2^3 + 3^3 + \ldots . + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

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11. For all $n \geq 1$, prove that

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \ldots\ldots + \frac{1}{(1+2+3+\ldots . + n)} = \frac{2n}{(n+1)}$$

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12. For all $n \geq 1$, prove that

$$1.2.3 + 2.3.4 + \ldots\ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

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13. Using mathematical induction prove

$$1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \ldots . + n \cdot 3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$ for all

$n \in N$

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14. Prove that by $1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ by using the principle of mathematical induction for all $n \in N$.

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15. Using mathematical induction prove that

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \ldots\ldots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$

true for all $n \in N$

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16. Using mathematical induction prove that

$1 \cdot 2 + 2 \cdot 2^2 + \ldots\ldots + n \cdot 2^n = (n-1)2^{n+1} + 2$ for all $n \in N$

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17. A statement $p(n)$ for a natural number n is given by

$$p(n): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$$

Verify that $p(1)$ is true.

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18. Using mathimatical induction prove that

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+\ldots\ldots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$ for all $n \in N$.

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19. Using mathematical induction prove that

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\ldots\ldots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$$ for all $n \in N$.

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20. Consider the following statement:

$$P(n): a + ar + ar^2 + \ldots\ldots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

Prove that $P(1)$ is true.

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21. Using mathematical induction prove that

$$\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right)\ldots\ldots\left(1 + \frac{2n+1}{n^2}\right) = (n+1)^2$$ for all $n \in N$

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22. Using mathematical induction prove that

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\ldots\ldots\left(1 + \frac{1}{n}\right) = n + 1$$ for all $n \in N$

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23. Using mathematical induction prove that

$$1^2 + 3^2 + 5^2 + \ldots. + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \text{ for all } n \in N$$

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24. Using mathematical induction prove that

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots\ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1} \text{ for all } n \in N$$

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25. Using mathematical induction prove that

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots\ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)} \text{ for all } n \in N$$

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26. For all $n \geq 1$,prove that

$1+2+3+\ldots\ldots+n<\frac{1}{8}(2n+1)^2$

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27. Using principle of mathematical induction prove that $n(n+1)(n+5)$ is a multiple of 3 for all $n \in N$.

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28. Using mathematical induction prove that $10^{2n-1}+1$ is divisible by 11 for all $n \in N$

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29. Using mathematical induction prove that $x^{2n}-y^{2n}$ is divisible by x+y for all $n \in N$

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30. Consider the statement

$P(n) = 3^{2n+2} - 8n - 9$ is divisible by 8

Verify the statement for n=1.

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31. Using mathematical induction prove that

$41^n - 14^n$ is a multiple of 27 for all $n \in N$

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32. Using mathematical induction prove that $(2n+7) < (n+3)^2$ for all $n \in N$

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33. Using mathematical induction prove that

$P(n) = 1 + 3 + 5 + \ldots\ldots\ldots. + 2n - 1 = n^2$

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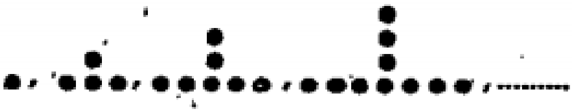
34. Observe the following pattern of dots



i) Find the number of dots in the n^{th} pattern.

ii) Find the sum of number of dots upto n^{th} pattern and the mathematical statement using these facts.

iii)Prove the statement obtained in question (ii) using principle of mathematical induction.

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35. If $P(n)$ is the statement $n(n+1)$ is even then what about P(6)?

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36. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisiblé by 12, prove that $P(3)$ and $P(4)$ are true, but $P(5)$ is not true

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37. Consider the statement $P(n): n(n+1)(2n+1)$ is divisible by 6. By assume that $P(k)$ is true for a natural number k, Verify that $P(k+1)$ is true.

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38. Prove 'that $10^{n}+3 \times 4^{n+2}+5$ is divisible by 9 .

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39. Prove by mathematical induction.

$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$,

where $i = \sqrt{-1}$

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40. Prove that

$1(1)! + 2(2)! + 3(3)! + \ldots\ldots\ldots\ldots + n(n)! = (n+1)! - 1$

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41. Prove that $1 + \frac{1}{4} + \frac{1}{9} + \ldots\ldots\ldots. + \frac{1}{n^2} < 2 - \frac{1}{n}, (n > 1)$

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42. Using principle of mathematical induction, prove that the product of two consecutive natural numbers is an even number

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43. Using mathematical induction prove that

$$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \ldots.. + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

true for all $n \in N$

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