



# MATHS

# **BOOKS - A N EXCEL PUBLICATION**

# **BINOMIAL THEOREM**



1. Evaluate 
$$\left(2+\sqrt{3}
ight)^7+\left(2-\sqrt{3}
ight)^7$$

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**2.** Using binomial theorem evaluate  $\left(0.99
ight)^5+\left(1.01
ight)^5$ 

3. Write the middle term in the expansion of the following,

$$\left(x+rac{2}{\sqrt{x}}
ight)^{17}$$



**6.** The first term in the expansion of  $(1 + ax)^n$  in ascending powers of x are  $1 + 12x + 64x^2$ , find n and a.

7. If  $C_0, C_1, C_2...C_n$  denote the coefficients in the binomial expansion of  $(1+x)^n$ , prove that  $C_0 + 3C_1 + 5C_2 + ... + (2n+1)C_n = (n+1)2^n$ ) Watch Video Solution **8.** Expand the following expressions  $(1+2x)^5$ Watch Video Solution **9.** Expand the following expressions  $\left(\frac{2}{x} + \frac{x}{2}\right)^{\circ}$ Watch Video Solution **10.** Expand the following expressions  $\left(2x-3
ight)^6$ Watch Video Solution

11. Expand 
$$\left(\frac{x}{3} + \frac{1}{x}\right)^5$$
  
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12. Expand  $\left(x + \frac{1}{x}\right)^6$   
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**13.** Using Binomial theorem evaluate the following  $\left(96
ight)^3$ 

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14. Using Binomial theorem evaluate the following  $\left(102
ight)^5$ 



19. Find
$$(x+1)^6+(x-1)^6$$
. Hence, or otherwise evaluate $\left(\sqrt{2}+1
ight)^6+\left(\sqrt{2}-1
ight)^6$ 

**20.** Show that  $9^{n+1} - 8n - 9$  is divisible by 64.

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**21.** Prove that 
$$\sum_{r=0}^n 3^r \ \hat{} \ nC_r = 4^n$$

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**22.** Find the coefficient of  $x^5$  in  $(x + 3)^8$ .



**27.** Find the  $13^t h$  term in the expansion of

$$\left(9x-rac{1}{3\sqrt{x}}
ight)^{18}$$

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28. Write the middle term in the expansion of the following,

$$\left(3-rac{x^3}{6}
ight)^7$$

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**29.** Find the middle term in the expansion of  $\left(rac{x}{3}+9y
ight)^{10}$ 

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**30.** In the expansion of  $(1+a)^{m+n}$ , prove that the coefficient of  $a^m$  and

 $a^n$  are equal.`

**31.** The coefficients of the  $(r-1)^{th}$ ,  $r^{th}$  and  $(r+1)^{th}$  terms in the expansion of  $(x+1)^n$  are in the ratio 1:3:5. Find n and r

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**32.** Prove that the coefficient of  $x^n$  in the expansion of  $\left(1+x
ight)^{2n}$  is twice

the coefficient of  $x^n$  in the expansion of  $\left(1+x
ight)^{2n-1}$ 

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**33.** Find the positive value of m for which the coefficient of  $x^2$  in the expansion of  $(1+x)^m$  is 6.

**34.** Consider the expansion of  $\left(3x - \frac{x^3}{6}\right)^8$  Find the general term in the

expansion

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**35.** Consider the expansion of 
$$\left(3x - \frac{x^3}{6}\right)^8$$
 How many terms are there in

the expansion?

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**36.** Consider the expansion of 
$$\left(3x-rac{x^3}{6}
ight)^8$$
 Find the middle term in the

expansion.

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**37.** Find the coefficient of middle term in the expansion of  $(1 + a)^8$ 

**38.** Find the coefficient of the middle term in the expansion of  $\left(1+a\right)^7$ 

**39.** Consider 
$$\left(\sqrt{2}+3^{1/5}
ight)^{10}$$
 Find the  $r+1^{th}$  terms in the expansion of

the given expression

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**40.** Consider  $\left(\sqrt{2}+3^{1/5}
ight)^{10}$  If the  $r+1^{th}$  term is rational , find r. Hence ,

find the sum of the rational terms in the expansion of  $\left(\sqrt{2}+3^{1/5}
ight)^{10}$ 

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**41.** Write the first three terms in the expansion of  $(1+ax)^n, n\in N$ 

**42.** If the first three terms in the expansion of  $(1 + ax)^n$  are 1,6x and  $16x^2$ , find a and n

			$\begin{pmatrix} 1 \\ 23 \end{pmatrix}$	
<b>43.</b> If $t_r$ denotes the $r^t$	<sup>h</sup> term in the	expansion of	$=\left(x+rac{1}{x} ight)$	,find $t_{12}$

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and t_{13} Hence ,prove that x^2 t_{13} = t_{12}
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**44.** Let  $c_r$  denote the binomial coefficient  ${}^nC_r$  Write the factorial formula

for  $c_r$ 

**45.** Prove that 
$$rac{{}^nC_r}{{}^nC_{r-1}}=rac{n-r+1}{r}$$

**46.** Let  $c_r$  denote the binomial coefficient  ${}^nC_r$  Hence, show that  $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3. \frac{C_3}{C_2} + ... + n. \frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$ Watch Video Solution

47. Given that  ${}^nC_r = {}^nC_{n-r}$  or  $C_r = C_{n-r}$  Prove that  $aC_0 + (a+b)C_1 + ... + (a+nb)C_n = (2a+nb)2^{n-1}$  Hence, prove that  $C_0 + 4C_1 + 7C_2 + ... + (3n+1)C_n = (3n+2)2^{n-1}$ 

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**48.** Given that  ${}^{n}C_{r} = {}^{n}C_{n-r}$  or  $C_{r} = C_{n-r}$  Prove that  $aC_{0} + (a+b)C_{1} + ... + (a+nb)C_{n} = (2a+nb)2^{n-1}$  Hence, prove that  $C_{0} + 5C_{1} + 9C_{2} + ... + (4n+1)C_{n} = (4n+2)2^{n-1}$ 

**49.** What is the  $\left(r+1
ight)^{th}$  term in the expansion of  $\left(3x+2y
ight)^8$ ?



50. Using the  $(r+1)^{th}$  term, find the 3rd,4th and 5th terms in the expansion of  $(3x+2y)^8$ 

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**51.** find the middle term in the expansion of  $\left(3x+2y
ight)^8$ 

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52. Expand 
$$(1+x)^4$$
,  $(1-x)^3$  and  $(1-x)^2$ 

**53.** Expand 
$$\left(1-x+x^2
ight)^4$$

**54.** Consider the expansion of 
$$\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$$
 Find the general term in

the expansion

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**55.** Consider the expansion of 
$$\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$$
 Find the 7th term in the

expansion

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56. Consider the expansion of 
$$\left(rac{4x}{5}-rac{5}{4x}
ight)^8$$
 Find the  $(r+1)^{th}$  term in

the expansion

**57.** Consider the expansion of 
$$\left(\frac{4x}{5} - \frac{5}{4x}\right)^8$$
 Find the number of terms

in the expansion

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**58.** Consider the expansion of 
$$\left(\frac{4x}{5} - \frac{5}{4x}\right)^8$$
 The middle term in the

expansion is ...{4th,5th,6th]

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**59.** Consider the expansion of 
$$\left(\frac{4x}{5} - \frac{5}{4x}\right)^8$$
 Find the fourth term from

the end in the expansion.

**60.** Consider the expansion of  $\left(3x^3+rac{2}{x^2}
ight)^{40}$  Find the general term in

the expansion

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**61.** Consider the expansion of  $\left(3x^3 + \frac{2}{x^2}\right)^{40}$  Assuming the term containing  $x^{20}$  as the  $(r+1)^{th}$ term, find the coefficient of  $x^{20}$  in the expansion of  $\left(3x^3 + \frac{2}{x^2}\right)^{40}$ 

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**62.** Consider the expansion of  $\left(3x^2-rac{1}{2x^3}
ight)^{10}$  Find the  $\left(r+1
ight)^{th}$  term in

the expansion





in the expansion

**71.** Consider the expansion of  $(x + a)^n$  If these are 112,7 and  $\frac{1}{4}$  respectively, find x, a, n



**72.** Expand  $(1 + ax)^n$  upto the third term if n is a positive integer

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73. If the first three terms in the expansion of  $(1 + ax)^n$  are 1, -21xand  $216x^2$  respectively, prove that  $a = -\frac{3}{7}$  and n=49.

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**74.** What is the coefficient of the  $(r + 1)^{th}$  term in the expansion of  $(1 + x)^n$ ?









76. If they are in A.P, find the possible values of n

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77. Expand 
$$2^{3n}$$
 by writing  $2^{3n} = (1+7)^n$ 

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78. Using binomial theorem prove that  $2^{3n} - 7n - 1$  is divisible by 49,

where  $n \in N$ 



**79.** prove the result 
$$(n + 1)C_r =^n C_{r-1} +^n C_r$$

80. Prove that 
$${}^{n+1}C_{r+1}=rac{n+1}{r+1}{}^nC_r$$

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81.

Prove

that

 $(C_0+C_1)(C_1+C_2)(C_2+C_3)...(C_{n-1}+C_n)=rac{C_0C_1C_2...C_{n-1}(n+1)^n}{n!}$ 

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**82.** Prove that 
$${}^{n+1}C_{r+1}=rac{n+1}{r+1}{}^nC_r$$

83. Prove that 
$$C_0 + \frac{C_2}{3} + \frac{C_4}{5} + \dots = \frac{2^n}{n+1}$$
  
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84. Given that  $\frac{C_r}{C_{r-1}} = \frac{n-r+1}{r}$  Evaluate  $\frac{C_1}{C_0}, \frac{C_2}{C_1}$  and  $\frac{C_3}{C_2}$   
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85. Given that  $\frac{C_r}{C_{r-1}} = \frac{n-r+1}{r}$  Prove that  $\left(1 + \frac{C_1}{C_0}\right) \left(1 + \frac{C_2}{C_1}\right) \dots \left(1 + \frac{C_n}{C_{n-1}}\right) = \frac{(n+1)^n}{n!}$   
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86. Given that  $C_0+C_1x+C_2x^2+\ldots+C_nx^n=(1+x)^n$  prove that  $C_0+5.\ C_1+5^2.\ C_2+\ldots+5^n.\ C_n=6^n$ 





**96.** Prove that 
$$C_0C_2 + C_1C_3 + \ldots + C_{n-2}C_n = {}^{2n}C_{n-2}$$

**97.** Given that 
$$\frac{1}{r+1}{}^{n}C_{r} = \frac{1}{n+1}{}^{n+1}C_{r+1}$$
  
Prove that  $2^{10}C_{0} + \frac{2^{2}}{2}{}^{10}C_{1} + \frac{2^{3}}{3}{}^{10}C_{2} + \dots + \frac{2^{11}}{11}{}^{10}C_{10} = \frac{3^{11}-1}{11}$ 

**98.** Find a, b and n in the expansion of  $\left(a+b
ight)^n$  if the first three terms of

the expansion are 729,7290 and 30375 respectively.

**99.** Find a if the coefficients of  $x^2$  and  $x^3$  in the expansion of  $(3 + ax)^9$ 

are equal



**100.** Find the coefficient of  $x^5$  in the product  $(1+2x)^6(1-x)^7$  using binomial theorem.

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101. If a and b are distinct integers prove that a-b is a factor of  $a^n-b^n$ 

whenever n is a positive integer.



102. Evaluate 
$$\left(\sqrt{3}+\sqrt{2}
ight)^6-\left(\sqrt{3}-\sqrt{2}
ight)^6$$

103. Find the value of 
$$(a^2 + \sqrt{a^2 - 1})^4 + (a^2 - \sqrt{a^2 - 1})^4$$
  
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104. Find an approximation of  $(0.99)^5$  using the first three terms of its expansion  
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105. Expand using Binomial theorem  $(1 + \frac{x}{2} - \frac{2}{x})^4, x \neq 0$   
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106. Find the expansion of  $(3x^2 - 2ax + 3a^2)^3$  using Binomial theorem