MATHS

BOOKS - A N EXCEL PUBLICATION

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Question Bank

1. Express the following in the form a+ib (i) $(4i)\left(\frac{1}{7}i\right)$

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2. Express the following in the form a+ib (ii) (2i) (-3i) $\left(\frac{6}{7}i\right)$

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3. Express each one of the following in the form a+ib. (i) $\frac{5+4i}{4+5i}$

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4. Express each one of the following in the form a+ib. (i) $\frac{i\sqrt{-9}+7i}{1+\sqrt{-1}}$

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5. Prove that the complex number $\frac{3+2i}{2-3i}+\frac{3-2i}{2+3i}$ is purely real

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6. If $(\cos\theta - i\sin\theta)^2 = x - iy$, show that $x^2 + y^2 = 1$

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7. Find the modulus of the complex number $\dfrac{(1+i)(2+i)}{3+i}$

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8. If $(1+i)(1+2i)(1+3i)\ldots(1+ni) = x + iy$,

show that 2.5.10...$(1+n^2)=x^2+y^2$

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9. Express the following complex numbers in a+ib form. (a) (5i)$\left(-\frac{3}{5}i\right)$

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10. Express the following complex numbers in a+ib form. (b) i^9+i^{19}

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11. Express the following complex numbers in a+ib form. (c) i^{-39}

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12. Express the following in $a+ib$ form

$3(7+7i)+i(7+7i)$

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13. Express the following in $a + ib$ form

$$(1 - i) - (-1 + 6i)$$

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14. Express the following in $a + ib$ form

$$\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + \frac{5}{2}i\right)$$

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15. Express the following in a+ib form.

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left[-\frac{4}{3}+i\right]$$

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16. Express the following in a+ib form. $(1-i)^4$

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17. Express the following in $a+ib$ form

$$\left(\frac{1}{3}+3i\right)^3$$

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18. Express the following in a+ib form.
$$\left(-2-\frac{1}{3}i\right)^3$$

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19. Find the multiplicative inverse of the following complex number 4-3i

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20. Find the multiplicative inverse of the following,

$\sqrt{5}+3i$

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21. Find the multiplicative inverse of the following complex number -i

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22. Express the following expression in the form a+ib $\dfrac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$

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23. Write the values of i^2, i^4 and i^6

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24. Show that $1+i^2+i^4+i^6=0$

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25. Using the value of i^2, prove that $\frac{1}{i} = -i$

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26. Prove that $(1+i)^4\left(1+\frac{1}{i}\right)^4 = 16$

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27. Express $\frac{(3-i)^2}{2+i}$ in the form x+iy

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28. Find the conjugate of $\frac{(3-i)^2}{2+i}$

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29. Using the results $|z^2| = |z|^2$ and $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, find the modulus of $\frac{(3-i)^2}{2+i}$

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30. Express $\frac{1}{1-2i} + \frac{3}{1+i}$ in the form x + iy

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31. Find the modulus of the complex number

$$\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$$

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32. Evaluate

$$\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$$

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33. A student writes the formula $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

Then he substitutes a = -1, b = -1 and finds 1 = -1.

Explain where he is wrong?

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34. Evaluate $4x^2 + 8x + 35$, when $x = 2 + \sqrt{-3}$

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35. Consider the complex number $z_1 = 3 + i$ and $z_2 = 1 + i$ .What is the conjugate of z_2?

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36. Consider the complex number $z_1 = 3 + i$ and $z_2 = 1 + i$ .Find $\frac{1}{z_2}$

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37. Consider the complex number $z_1 = 3 + i$ and $z_2 = 1 + i$ .Using the value of $\frac{1}{z_2}$, find $\frac{z_1}{z_2}$

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38. Prove that $\left|\frac{(a+i)^2}{2a-i}\right| = \frac{a^2+1}{\sqrt{4a^2+1}}$

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39. If $\frac{(a+i)^2}{2a-i} = p + iq$, prove that $p^2 + q^2 = \frac{(a^2+1)^2}{4a^2+1}$

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40. If $x = \frac{1+i}{\sqrt{2}}$, prove that $x^2 = i$

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41. If $x = \frac{1+i}{\sqrt{2}}$.Prove that $x^6 + x^4 + x^2 + 1 = 0$

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42. Prove that $\left|\frac{1+i}{1-i}\right| = 1$

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43. If a+ib = $\sqrt{\frac{1+i}{1-i}}$, prove that $a^2+b^2=1$

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44. Prove that $|\cos\theta - i\sin\theta| = 1$

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45. If $(\cos\theta - i\sin\theta)^2 = x - iy$, show that $x^2+y^2=1$

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46. Complete the following table

Complex number (z)	Re (z)	Im (z)	$\bar{z}$	$\|z\|$
$3+7i$	-----	-----	-----	-----
$1-i^3$	-----	-----	-----	-----
$9+3i^7$	-----	-----	-----	-----
$(1-i)^4$	-----	-----	-----	-----

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47. When two complex numbers are equal? Find the real values of x and y for which 3x+2iy-ix+5y=7+5i

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48. Match the following

Column A	Column B
i^4	-1
i^6	i
i^{11}	$-i$
i^{17}	1
	0

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49. Fill in the blanks by choosing the correct answer from the bracket $\overline{2i^3 - i^2} = \ldots . . , i^{-5}$ = ... $(1-2i, 1+2i, i, -i, 1, -1)$ Also express

$\overline{2i^3 - i^2} + (12i + i^{-5}) - \left(\overline{5 - i^5}\right)$ in the form

x+iy

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50. Suppose $z_1 = 1 - i$ and $z_2 = -2 + 4i$ Find

$z_1 z_2$

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51. Suppose $z_1 = 1 - i$ and $z_2 = -2 + 4i$ Find

$\bar{z}_1$

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52. Suppose $z_1 = 1 - i$ and $z_2 = -2 + 4i$ Find $\text{Im}\left(\frac{z_1 z_2}{z_1}\right)$ and $\text{Re}\left(\frac{z_1 z_2}{z_1}\right)$. Hence, find $\left|\frac{z_1 z_2}{z_1}\right|$

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53. Suppose $z_1 = 1 - i$ and $z_2 = -2 + 4i$ Joseph evaluted $\left|\frac{z_1 z_2}{z_1}\right|$ using the following properties

(a)modulus of a quotient is the quotient of the moduli

Modulus of a product is the product of the moduli

(c) Modulus of $\bar{z}$ =Modulus of z

Write the steps carried out by Joseph

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54. Suppose z = x + iy and w = $\frac{1 - iz}{z - i}$ Find 1 - iz and z - 1 in the standard form of a complex number.

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55. Suppose z = x + iy and w = $\dfrac{1 - iz}{z - i}$ Find $|w|$. If $|w|$ = 1, prove that z is purely real

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56. Consider the complex number $z_1 = 2 - i$ and $z_2 = -2 + i$ Find $\dfrac{z_1 z_2}{z_1}$. Hence, find $\left|\dfrac{z_1 z_2}{z_1}\right|$

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57. Consider the complex number $z_1 = 2 - i$ and $z_1 = -2 + i$ Raju prove that $\left|\dfrac{z_1 z_2}{z_1}\right| = |z|_2$

and using it he derived the value of $\left|\frac{z_1 z_2}{z_1}\right|$.

Write the steps written by Raju.

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58. Consider the complex number $z_1 = 2 - i$ and $z_2 = 2 + i$ Find $\frac{1}{z_1 z_2}$. Hence, prove than $I_m\left(\frac{1}{z_1 z_2}\right) = 0$

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59. Given that for any complex number z, $|z|^2 = z\bar{z}$. Prove that $|z_1 + z_2|^2 = |z_1|^2 + 2Re(z_1\bar{z}_2) + |z_2|^2$

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60. Given that for any complex number z, $|z|^2 = z\bar{z}$. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left[|z_1|^2 + |z_2|^2\right]$ where z_1 and z_2 are any two complex numbers.

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61. Represent the following complex numbers as points in the argand plane 2i

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62. Represent the following complex numbers as points in the argand plane 3 - i

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63. Represent the following complex numbers as points in the argand plane the conjugate of 4 - i

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64. Represent the complex number $z = 1 + i$ in the polar form.

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65. Represent each of the following numbers in polar form. $-\sqrt{3} + i$

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66. Represent each of the following numbers in polar form. 4i

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67. Find the modulus-amplitude form of the complex number $\frac{1+7i}{(2-i)^2}$

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68. Prove that $(\cos 45^\circ + i\sin 45^\circ)^2 = i$

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69. Represent the quotient $\frac{7\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)}{21\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)}$ in polar form.

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70. Show that the points representing the complex numbers 3+2i, 2-i and -7i are collinear.

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71. Find the modulus and argument of the following complex number $z = -1 - i\sqrt{3}$

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72. Represent each of the following numbers in polar form. $-\sqrt{3} + i$

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73. Convert the following complex numbers into polar form 1 - i

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74. Convert the following complex numbers into polar form -1 + i

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75. Convert the following complex numbers into polar form -1 -i

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76. Convert the following complex numbers into polar form -3

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77. Represent the complex number $\sqrt{3}+i$ in the polar form.

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78. express the complex number i in the polar form.

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79. What is the conjugate of 2 + i?

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80. Express $(2+3i)^2$ in the form x + iy

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81. Express $\frac{(2+3i)^2}{2+i}$ in the form x + iy

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82. Solve $\frac{1}{z}+\frac{1}{2+i}=\frac{1}{1+3i}$

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83. If $|a+ib|=1$ then what is the value of a^2+b^2?

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84. If $(a^2 + b^2) = 1$,Prove that

$$\frac{1+b+ai}{1+b-ai} = b + ai$$

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85. Match the following

z	$\bar{z}$
$1 + i^{27}$	-2
$6 + i^3$	0
$i^2 - i^4$	$1 + i$
$1 + i^{22} + i^{220} - i^{1000}$	$6 + i$
	2

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86. If $x = 4 + \sqrt{7}i$, find x^2 and x^3.Hence, find the value of $x^3 - 4x^2 - 9x + 97$

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87. If $\frac{(a+i)^2}{2a-i} = p + iq$, prove that $p^2 + q^2 = \frac{(a+1)^2}{4a^2+1}$

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88. Sheeba proved the same relation stated above by expressing $\frac{(a+i)^2}{2a-i}$ in x+iy form. Write the steps written by Sheeba

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89. Match the following

Complex Number	Polar form
$4\sqrt{3}+4i$	$2\left(\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)\right)$
$-\sqrt{3}+i$	$8\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
$-1-i\sqrt{3}$	$2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$
$\frac{1+2i}{1-3i}$	$2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
	$\frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

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90. Suppose $z_1 = 2(\cos 60^\circ + i\sin 60^\circ)$ and $z_2 = 4(\cos 30^\circ + i\sin 30^\circ)$ Find $z_1 z_2$

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91. Suppose $z_1 = 2(\cos 60^\circ + i\sin 60^\circ)$ and $z_2 = 4(\cos 30^\circ + i\sin 30^\circ)$ find $\frac{z_1}{z_2}$

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92. Suppose $z_1 = 2(\cos 60^\circ + i\sin 60^\circ)$ and $z_2 = 4(\cos 30^\circ + i\sin 30^\circ)$ Find $z_1^2 z_2^3$

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93. Express $\frac{1-3i}{1+2i}$ in the form x+iy

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94. Find the polar form of the complex number

$$\frac{1-3i}{1+2i}$$

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95. Convert the following complex numbers into polar form 1 - i

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96. Represent the complex number $\sqrt{3}+i$ in the polar form.

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97. Express $\dfrac{1-i}{\sqrt{3}+i}$ in polar form

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98. Express $(1-i)(\sqrt{3}+i)$ in polar form

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99. Plot the points represented by the complex numbers 1+i, 2+i, 2+3i, 1+3i

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100. What are the cartesian co-ordinates of the points 1+i, 2+i, 2+3i, 1+3i?

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101. If two points P and Q are represented by the complex numbers z_1 and z_2 prove that $PQ = |z_1 - z_2|$

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102. If the points P,Q,R,S are representing the complex numbers -1, 3i, 3+2i and 2-i respectively on the argand plane, prove that PQRS is a square

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103. If z = x+iy, prove that arg(z -1) = $\tan^{-1}\left(\frac{y}{x-1}\right)$ and arg(z+1) = $\tan^{-1}\left(\frac{y}{x+1}\right)$

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104. If $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 - 1 = 0$

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105. Express $\sqrt{3} - i$ in polar form using the polar form of $\sqrt{3} + i$

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106. Use De-Moivere's theorem to find $\left(\sqrt{3}-i\right)^9$ and $\left(\sqrt{3}-i\right)^{-1}$ in rectangular form (x+iy form)

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107. Determine also, for which values of $\ne \psi lon Z, \left(\sqrt{3}-i\right)^n$ is real

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108. Express $1+i$ in modulus amplitude form

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109. Prove that $(1+i)^4 = -4$

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110. Represent the complex number $1+\sqrt{3}i$ in the polar form.

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111. What is the polar form of $1-\sqrt{3}i$?

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112. Prove that

$$\left(1+\sqrt{3}i\right)^{n}+\left(1-\sqrt{3}i\right)^{n}=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$$

for any positive integer n

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113. Solve : $x^2-2x+4=0$

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114. If α and β are the roots of $x^2 - 2x + 4 = 0$, Prove that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$

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115. Express $\frac{1+i}{\sqrt{3}} + i$ in the form x + iy. Hence, find the polar form of $\frac{1+i}{\sqrt{3}} + i$

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116. Nisha derived the polar form of $\frac{1+i}{\sqrt{3}}+i$ by using the polar forms of (1+i) and $\sqrt{3}+i$.

Write the steps followed by Nisha.

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117. Using the polar form and x + iy form of $\frac{1+i}{\sqrt{3}}+i$, prove that $\frac{\cos\pi}{12}=\frac{\sqrt{3}+1}{2}\sqrt{2}$ and $\frac{\sin\pi}{12}=\frac{\sqrt{3}-1}{2}\sqrt{2}$

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118. Fill in the blank by choosing the correct answer from the bracket. If z is any complex number, $z\bar{z} = ...\left(|z|^2, |z|, 0, 1\right)$

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119. Suppose α and β are two complex numbers so that $|\beta|$ = 1. Raju proved $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$ = 1 in the following way. Fill in the blanks and write the complete solution.

$$\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right| = \left|\frac{\beta-\alpha}{\beta\bar{\beta}-\overline{\alpha}\beta}\right| = \left|\frac{\beta-\alpha}{\beta(....)}\right| = \frac{|...|}{|\beta||...|}$$

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120. Match the following

Complex number	Polar form
$1-i$	$3(\cos\pi + i\sin\pi)$
-3	$\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$
$-1+i$	$\sqrt{2}\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)$
$\sqrt{3}+i$	$\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
	$\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

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121. Express the following in a+ib form. $(1-i)^4$

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122. prove the following. $(\sqrt{3}+i)^{6} = -64$

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123. prove that $(-1+i)^{2} = -2i$

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124. Solve $x^{2}+3=0$

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125. Solve the equation x^2+3=0

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126. Solve $2x^2 + x + 1 = 0$

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127. Solve: $x^2 + 3x + 9 = 0$

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128. Solve: $-x^2 + x - 2 = 0$

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129. Solve $x^2 + 3x + 5 = 0$

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130. Solve: $x^2 - x + 2 = 0$

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131. Solve: $\sqrt{2}x^2 + x + \sqrt{2} = 0$

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132. Solve: $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

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133. Solve: $x^2 + x + \frac{1}{\sqrt{2}} = 0$

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134. Solve: $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

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135. Solve: $x^2 + x + 1 = 0$

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136. Solve: $\sqrt{5}x^2 + x + \sqrt{5} = 0$

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137. Solve: $\sqrt{2}x^2 - x - \sqrt{2} = 0$

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138. Solve: $x^2 + \frac{x}{\sqrt{3}} + 1 = 0$

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139. Evaluate $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

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140. For any two complex numbers z_1 and z_2 prove that

$$Re(z_1 z_2) = Re(z_1)Re(z_2) - Im(z_1)Im(z_2)$$

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141. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form

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142. If $x - iy = \sqrt{\dfrac{a - ib}{c - id}}$, prove that $(x^2 + y^2)^2 = \dfrac{a^2 + b^2}{c^2 + d^2}$

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143. Convert the following in to the polar form $\dfrac{1 + 7i}{(2 - i)^2}$

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144. Consider the complex number $z = \frac{1+3i}{1-2i}$

.Write z in polar form.

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145. Solve the following equations

$$3x^2 - 4x + \frac{20}{3} = 0$$

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146. Solve the following equations

$$x^2 - 2x + \frac{3}{2} = 0$$

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147. Solve the following equations $27x^2 - 10x + 1 = 0$

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148. Solve the equation $21x^2 - 28x + 10 = 0$

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149. If $z_1 = 2 - i, z_2 = 1 + i$

Hence find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$

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150. If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

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151. Let $z_1 = 2 - i, z_2 = -2 + i$. Find Re $\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

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152. Let $z_1 = 2 - i, z_2 = -2 + i$. Find Im $\left(\frac{1}{z_1 \bar{z}_1}\right)$

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153. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$

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154. Find the real numbers x and y if (x-iy)(3+5i) is the conjugate of - 6 - 24i

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155. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$

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156. If $(x+iy)^3 = u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4(x^2-y^2)$

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157. If α and β are different complex numbers with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right|$

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158. Find the number of non zero integral solutions of the equation $|1 - i|^x = 2^x$

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159. If (a+ib) (c+id) (e+if) (g+ih) = A+iB, then show that

$$\left(a^2 + b^2\right)\left(c^2 + d^2\right)\left(e^2 + f^2\right)\left(g^2 + h^2\right) = A^2 + B^2$$

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160. If $\left(\frac{1+i}{1-i}\right)^m = 1$ then find the least integral value of m

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