MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Continuity, Differentiability and Graph of Function

Example

1. Draw the graph of the function $f(x) = x - \left|x - x^2\right|, \ -1 \leq x \leq 1$ and discuss the continuity or discontinuity of f in the interval $-1 \leq x \leq 1$

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2. Let $f(x) = \lim_{n \to \infty} \frac{\log(2+x) - x^{2n} \sin x}{1 + x^{2n}}$. then

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3. Let $f(x+y)=f(x)f(y)$ and $f(x)=1+xg(x)G(x)$ where $\lim_{x\to 0} g(x)=a$ and $\lim_{x\to o} G(x)=b$. Then $f'(x)$ is

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4. Let $f(x)=x^3-x^2-x+1$ and $g(x)=\{\max\{f(t);0\le t\le x\},0\le x\le 1,3-x,1\le x\le 2$ Discuss the continuity and differentiability of the function g (x) in the interval (0, 2).

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Exercise

1. If $f(x)=\dfrac{e^{\frac{1}{x}}-1}{1+e^{\frac{1}{x}}}$ when $x\neq 0\ =0,$ when $x=0$ show that $f(x)$ is discontinuous at $x=0$.

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2. Determine the values of a, b, c for which the function

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & f \text{ or } x < 0c, \\ f \text{ or } x = 0 \dfrac{\sqrt{x + bx^2}}{bx^{3/2}}, \end{cases}$$

is continuous at $x = 0$

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3. If $g(x) = \dfrac{1 - a^x + xa^x \log a}{x^2 \cdot a^x}, x < 0 \quad \dfrac{(2a)^x - x\log(2a) - 1}{x^2}, x > 0$

(where a > 0) then find a and $g(0)$ so that $g(x)$ is continuous at $x = 0$.

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4. Let $f(x) = \begin{cases} \frac{x^2}{2}, & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2}, & 1 \le x \le 2 \end{cases}$

Discuss the continuity of f, f' and f'' on [0, 2].

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5. Discuss the continuity of $f(x) = (\lim)_{n\overrightarrow{\infty}} \frac{x^{2n}-1}{x^{2n}+1}$

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6. The function f(x). defined as $f(x) = \lim_{n\to\infty} \frac{f(x)+x^{2n}g(x)}{1+x^{2n}}$ shall be continuous every where, if

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7. Consider the function f defines by$f(x) = x - [x]$, where[x] denotes the greatest integral function. Show that the function is discontinous for integral values of x and continous for all other values.`

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8. The values of a and b so that the function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x < \pi/4 \\ 2x\cot x + b, & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x, & \pi/2 < x \le \pi \end{cases}$$ is continuous for $x \in [0, \pi]$, are

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9. Let $f(x+y) = f(x) + f(y)$ for all $xandy\cdot$ If the function $f(x)$ is continuous at $x = 0,$ show that $f(x)$ is continuous for all $x\cdot$

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10. Leg $f(x+y) = f(x) + f(y)f$ or $allx, y \in R,$ If $f(x)iscont \in uousatx = 0, showthatf(x)$ is continuous at all $x\cdot$

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11. Let $f(x+y)=f(x)f(y)\,\forall x, y \in R, f(0) \neq 0$ If $f(x)$ is continous at $x=0,$ then $f(x)$ is continuous at :

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12. Let $f:[0,1] \to [0,1]$be a continuous function such that $f(f(x))=1f \text{ or } allx \in [0,1]$then:

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13. The set of all points, where the function $f(x)=\frac{x}{1+|x|}$ is differentiable, is

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14. The set of points where ,$f(x)=x|x|$ is twice differentiable is

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15. Let f(x) be difined in the interval [-2, 2] such that

$$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x - 1, & 0 < x \le 2 \end{cases}$$

and $g(x) = f(|x|) + |f(x)|$.

Test the differentiability of g (x) in (-2, 2).

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16. Let $f(x) = \begin{cases} \frac{x^2}{2}, & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2}, & 1 \le x \le 2 \end{cases}$

Discuss the continuity of f, f' and f'' on [0, 2].

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17. Draw the graph of the function and discuss the continuity and differentiability at x = 1 for, $f(x) = \begin{cases} 3^x, & \text{when} \quad -1 \le x \le 1 \\ 4 - x, & \text{when} \quad 1 < x < 4 \end{cases}$

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18. Given the function $f(x) = \frac{1}{1-x}$, The points of discontinuity of the composite function f[f{f(x)}] are given by

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19. If $f(x) = \begin{cases} ax^2 - b, \text{ for } & 0 \leq x < 1 \\ 2, \text{ for } & x = 1 \\ x + 1, \text{ for } & 1 < x \leq 2 \end{cases}$ is continuous at $x = 1$, then the most suitable values of a, b are

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20. If $f(x) = \begin{cases} \frac{(1-\sin^3 x)}{3\cos^2 x}, & x < \frac{\pi}{2} \\ a, & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2}, & x > \frac{\pi}{2} \end{cases}$

is continuous at $x = \frac{\pi}{2}$, then the value of $\left(\frac{b}{a}\right)^{5/3}$ is

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21. Let $f(x) = \frac{x^2 + 3x - 10}{x - 2}$, x=!2. The value f(2) = _____ will make the function f(x) continous at x=2.`

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22. If f(x) is continuous in $[0, 1]$ and $f\left(\frac{1}{2}\right) = 1$. prove that $\lim_{n \to \infty} f\left(\frac{\sqrt{n}}{2\sqrt{n+1}}\right) = 1$

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23. If $f(x) = \frac{\sin\left(e^{x-2} - \right)}{\log(x - 1)}$ then $\lim_{x \to 2} f(x)$ is given by

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24. If $f(x) = \begin{cases} \frac{\sqrt{1+px} - \sqrt{1-px}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-2}, & 0 \leq x \leq 1 \end{cases}$ is continuous in [-1,1] then p is equal to

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25. The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is not difined at x = 0. The value which should be assigned to f at x = 0, so that it is continuous at x = 0, is

A. a-b

B. a+b=0

C. $\log_e(ab)$

D. none of these

Answer:

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26. The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \left(\frac{x^2}{4}\right) - \left(\frac{3x}{2}\right) + \frac{13}{4}, & x < 1 \end{cases}$ is

A. is continuous at x=1

B. is continuous at x=3

C. is differentiable at x=1

D. f'(3) exists

Answer:

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27. The range of the function $f(x) = \frac{\tan(\pi[x+1])}{x^4+1}$ (where, $[.]$ is the greatest integer function) is

A. is discontinuous at some x

B. f'(x)exists for all x

C. f'(x) xists for all but f"(x) does not exist

D. is continuous for all x but f"(x) does not exist for some x

Answer:

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28. The function $f(x) = 1 + |\sin x|$, is

A. is continuous nowhere

B. is continuous everywhere

C. is differentiable nowhere

D. f'(0) does not exist

Answer:

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29. If $x + |y| = 2y$, then y as a function of x is

A. defined for all x

B. continuous at x=0

C. differentiable for all x

D. such that dy/dx=1/3

Answer:

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30. The graph of the function, $\cos x \cos(x+2) - \cos^2(x+1)$ is

A. a $straightl \in epas \sin gthrough\left(0, -\sin^{21}\right) withslope 2$

B. A staight line passing through (0,0)

C. ` a straight line passing through the point (pi/2, -sin^21)

D. $aparabolawithvertex\left(1, -\sin^{21}\right)$

Answer:

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31. यदि $f(x) = x\left[\sqrt{x} - \sqrt{(x+1)}\right]$ हो तो-

A. s(x) is continous but not differerntiable ar x=0

B. f'(0) exists

C. f(x) is non differentiable at x=0

D. none of these

Answer:

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32. Let $f(x) = \lim_{n \to \infty} \frac{1 - x^n}{1 + x^n}$. Then

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33. If both $f(x)$ and $g(x)$ are non-differentiable at $x = a$ then $f(x) + g(x)$ may be differentiable at $x = a$

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34. The function f defined by $f(x) = [2x^2 + 3f \text{ or } x \leq 1 \text{ and } 3x + 2f \text{ or } x < 1$ is neither differentiable nor continuousat $x = 1$.

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35. If $(\lim)_{x \to a}[f(x)g(x)]$ exists, then both $(\lim)_{x \to a} f(x) and (\lim)_{x \to a} g(x)$ exist.

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36. $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$

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37. Let f(x) be a continuous function defined on $[1, 3]$. If f(x) takes only rational values for all x and $f(2) = 10$, then $f(2.5) =$

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38. If $f(x) = \{x^2$ if x is rational and 0, if x is irrational$\}$, then

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