MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Differential Equation of the First Order

Example

1. Find the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$ where A,B are arbitrary constants. Also write its order and degree.

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2. Solve $\log_e\left(\frac{dy}{dx}\right) = ax + by$

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3. Solve: $\frac{dy}{dx} + y \cdot f'(x) = f(x) \cdot f'(x)$, where $f(x)$ is a given function.

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4. Solve the following differential equation

$x dy - y dx = \sqrt{x^2 + y^2} dx$

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5. Solve $\left(x e^{\frac{y}{x}} - y \sin \frac{y}{x}\right) dx + x \sin \frac{y}{x} dy = 0$

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6. Show that the differential equation $y^3 dy - (x + y^2) dx = 0$ can be reduced to a homogenous equation.

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7. Solve $(y - 3x + 3)\frac{dy}{dx} = 2y - x - 4$

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8. $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

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9. Solution of the equation $xdy - \left[y + xy^3(1 + \log x)\right]dx = 0$ is :

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10. Find the equation of the curve passing through (1,2) whose differential equation is $y\left(x + y^3\right)dx = x\left(y^3 - x\right)dy$

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11. If y_1 and y_2 are two solutions to the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$. Then prove that $y = y_1 + c(y_1 - y_2)$ is the general solution to the equation where c is any constant.

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12. Let $u(x)$ and $v(x)$ satisfy the differential equation $\frac{du}{dx} + p(x)u = f(x)$ and $\frac{dv}{dx} + p(x)v = g(x)$ are continuous functions. If $u(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y), where $x > x_1$, does not satisfy the equations $y = u(x)$ and $y = v(x)\cdot$

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13. Solve: $x\left(\frac{dy}{dx}\right)^2 + (y - x)\frac{dy}{dx} - y = 0$

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14. A solution of $y = 2x\left(\frac{dy}{dx}\right) + x^2\left(\frac{dy}{dx}\right)^4$ is

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15. Solve $1 + \left(\frac{dy}{dx}\right)^2 = x\frac{dy}{dx}$

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16. Solve $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$.

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17. Find the general solution of $\frac{dy}{dx} \cdot \frac{d^3y}{dx^3} = 3\left(\frac{d^2y}{dx^2}\right)^2$.

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18. The rate of cooling of a substance in moving air is proportional to the difference of temperatures of the substance and the air. A substance cools from 36 degree celsius to 34 degree celsius in 15 minutes. Find when the substance will have the temperature 32 degree celsius , it being known that the constant temperature of air is 30 degree celsius.

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19. A and B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at the time. One hour after the water is released, the quantity of water is reservoir A is $1\frac{1}{2}$ times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water?

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20. Let $y = f(x)$ be a curve passing through $(1, 1)$ such that the triangle formed by the coordinate axes and the tangent at any point of the curve lies in the first quadrant and has area 2. Form the differential equation and determine all such possible curves.

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21. A curve passing through $(1, 2)$ has its slope at any point (x, y) equal to $\frac{2}{y-2}$ then the area of the region bounded by the curve and the line $y = 2x - 4$ is

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22. The orthogonal trajectories of the circle $x^2 + y^2 - ay = 0$, (where a is a parameter), is

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Exercise

1. Find the differential equation corresponding to the family of curves $y = c(x - c)^2$ where c is an arbitrary constant.

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2. Obtain the differential equation of all conic sections $ax^2 + by^2 = 1$ where a, b are parameters.

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3. Form the differential equation of the family of circles touching the x-axis at origin.

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4. Solve $x^2dy + y^2dx = xy(xdy - ydx)$

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5. Solve the equation: $\left(1+x^{2}\right)\frac{\tan^{-1}(dy)}{dx}+y=0$

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6. Solve $\sec^{2}x\tan y dx+\sec^{2}y\tan x dy=0$

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7. Solve $\frac{dy}{dx}+x\tan(x+y)+1=0$

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8. Solve $\left(x^{2}+y^{2}+2xy\right)\frac{dy}{dx}=a^{2}$

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9. Solve $x + y = \sin^{-1}\left(\frac{dy}{dx}\right)$

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10. Solve $\frac{dy}{dx}\sqrt{1 + x + y} = x + y - 1$

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11. Solve $x\,dy - y\,dx = \sqrt{x^2 + y^2}dx$

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12. Solve $y^2dx + x(x + y)dy = 0$

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13. Solve $\frac{x}{y} \cdot \frac{dy}{dx} = \frac{x - 2y}{x - 3y}$

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14. $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

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15. Solve $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$

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16. Solve $(4x-6y+3)dy+(2x-3y-1)dx=0$

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17. Solve $\frac{dy}{dx}=\frac{3x-4y+2}{4x-5y+3}$

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18. Solve $(2x + 4y + 1\}dy = (2y + x - 1)dx$

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19. Solve $x\left(x^2 + 1\right)\frac{dy}{dx} = y\left(1 - x^2\right) + x^2 \ln x$

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20. Solve $\frac{dy}{dx} = \frac{1 + y^2}{y(x + y^2 + 1)}$

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21. Solve the differential equation: (i) $\left(1 + y^2\right) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$

(ii) $x\frac{dy}{dx} + \cos^2 y = \tan y \frac{dy}{dx}$

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22. If $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$, find $y(x)$.

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23. Solve $x\left(1 - x^2\right)dy + \left(2x^2y - y - 5x^3\right)dx = 0$

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24. Solution of the equation `cos^2x(dy)/(dx)-(tan2x)y=cos^4x ,|x|

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25. Solve $\frac{dy}{dx} + \frac{y}{\sqrt{x^2 + 4}} = 3x$ where $y(0) = 4$

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26. Solve $\sin y \frac{dy}{dx} = \cos(x + y) + \cos(x - y) - \frac{1}{2}\sin x \cdot \sin 2x$

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27. Solve $\left(1+x^2\right)dy = y(x-y)dx$

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28. Solve $(x-1)dy + ydx = x(x-1)y^{\frac{1}{3}}dx$.

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29. Solve: $ydx - xdy + xy^2dx = 0$

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30. Solve $xdy = y(xy+1)dx$

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31. The solution of $xdx + ydy + (x^2 + y^2)dy = 0$

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32. The solution of $\frac{xdd + ydy}{xdy - ydx} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$ is (a) *[Math Processing Error]* (dd) (ee) *[Math Processing Error]* (hhh) (iii)

$(jjj)(kkk)\sqrt{(lll)(mmm)\left(\bigcap\right)x^{(ooo)2(ppp)}(qqq) + (rrr)y^{(sss)2(ttt)}(uuu)}$

$= \left(\tan\left\{si(\times x)n^{(yyy)(zzz)-1(aaaa)}(bbbb)\left((cccc)(dddd)(eeee)\frac{y}{ffff}x(g\right.\right.\right.$

(kkkk) (llll)

$(mmmm)\left(\bigcap n\right)y$

$+ x\tan\left((oooo)(pppp)c + (qqqq)(rrrr)\sin^{(ssss)(tt)-1(uuuu)}(vvvv)\sqrt{(wwv}\right.$

$(jjjjj)$

(kkkkk)

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33. Solution of the differential equation $y\cos\frac{y}{x}(xdy - ydx) + x\sin\frac{y}{x}(xdy + ydx) = 0$ which satisfies $y(1) = \frac{\pi}{2}$ is

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34. Find the paricular solution of the equation $y\frac{dy}{dx}\sin x = \cos x\left(\sin x - \frac{y^2}{2}\right)$ if $y = 1$ when $x = \frac{\pi}{2}$

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35. Solve $\left(\frac{dy}{dx}\right)^2 - (e^x + e^{-x})\frac{dy}{dx} + 1 = 0$

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36. Solve $\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} = 3x^2$

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37. Solve $xy\left\{\left(\frac{dy}{dx}\right)^2 - 1\right\} = (x^2 - y^2)\frac{dy}{dx}$

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38. Solve $x\left\{\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}\right\} = y$

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39. Solve $y = 2px + y^2p^3$ where $p = \frac{dy}{dx}$

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40. Obtain all the solutions of $(y+1)\left(\frac{dy}{dx}\right) + 2 = x\left(\frac{dy}{dx}\right)^2$

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41. solve the differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0.$

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42. Reduce the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x$ using the transformation $y=v(x).\,e^x.$ Hence solve the equation when $y=1,\ \frac{dy}{dx}=0$, for $x=0$

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43. A radioactive substance is subject to the law of natural decay $\frac{dv}{dt}=-kv$, where v is the volume of the substance at time t and k is a positive constant. The half-life of the substance is the time it takes for half the substance to disappear. Calcualte the half-life if 20 % of the substance disappeared in 15 years.

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44. A country has a food deficit of 10%. Its population grows continuously at a rate of 3% per year. Its annual food production every year is 4% more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after n years, where n is the smallest integer bigger than or equal to $\dfrac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}$

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45. Show that the equation of the curve passing through the point $(1, 0)$ and satisfying the differential equation $(1 + y^2)dx - xydy = 0$ is $x^2 - y^2 = 1$

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46. Prove that the equation of a curve whose slope at (x, y) is $-\dfrac{x+y}{x}$ and which passes through the point $(2, 1)$ is $x^2 + 2xy = 8$

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47. Find the equation of the curve such that the square of the intercept cut off by any tangent from the y-axis is equal to the product of the co-ordinate of the point of tangency.

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48. Find the curves for which the length of normal is equal to the radius vector.

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49. Find the cartesian equation of the curves for which the length of the tangent is of constant length.

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50. Find the equation of the curve for which the length of the normal is constant and the curves passes through the point (1,0).

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51. Find the equation of the curve which is such that the area of the rectangle constructed on the abscissa of and the initial ordinate of the tangent at this point is a constanta $= a^2$.

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52. The equation of the curve in which the portion of the tangent included between the coordinate axes is bisected at the point of contact, is

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53. The equation of the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point is

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54. The normal PG to a curve meets the x-axis in G. If the distance of G from the origin is twice the abscissa of P, prove that the curve is a rectangular hyperbola.

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55. Find the equation of curves for which the cartesian subtangent varies as the abscissa.

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56. Find the equation of the curve in which the subnormal varies as the square of the ordinate.

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57. Find the orthogonal trajectory of $y^2 = 4ax$ (a being the parameter).

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58. Prove that the family of curves $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter, is self orthogonal.

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59. For the differential equation $\sqrt{1 + \frac{d^2y}{dx^2}} = x$, the degree=______and order=______.

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60. If the general solution of a differential equation is $(y+c)^2 = cx$, where c is an arbitrary constant then the order of the differential equation is ______.

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61. The general solution of $\frac{dy}{dx} + \frac{y(x+y)}{x^2} = 0$ is ________.

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62. The solution of $(x + \log y)dy + ydx = 0$ when y(0) =1 is :

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63. Let $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^4 2\frac{e^{\sin(x^2)}}{x}dx = F(k) - F(1)$, then possible value of k is:

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64. If $x(t)$ is a solution of $\frac{(1+t)dy}{dx} - ty = 1$ and $y(0) = -1$ then $y(1)$ is (a) $(b)(c) - (d)\frac{1}{e}2(f)(g)(h)$ (i) (b) $(j)(k)e + (l)\frac{1}{m}2(n)(o)(p)$ (q) (c) $(d)(e)e - (f)\frac{1}{g}2(h)(i)(j)$ (k) (d) $(l)(m)(n)\frac{1}{o}2(p)(q)(r)$ (s)

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