MATHS

BOOKS - MODERN PUBLICATION

MATHEMATICAL INDUCTION

Example

1. If P (n) is the statement : "n (n + 1) (n + 2) is divisible by 12", prove that P(3) and P(4) are true, but P(5) is not true.

2. If P (n) is the statement : $2^{3n} - 1$ is an integral multiple of 7", prove that P(1), P(2) and P (3) are true.

3. P (n) is the statement $''2^n > 3n''$ and if P(r) is true, then prove that P(r + 1) is also true, n $\in$ N.

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4. Let P (n) be the statement $''3^n > n''$.

Is P (1) true?

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5. Let P (n) be the statement $''3^n > n''$.

What is P(n+1) ?

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6. Let P (n) be the statement $''3^n > n''$.

If P (n) is true, prove that P(n + 1) is true.

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7. If P(n) is the statement that the sum of first n natural numbers is divisible by n +1, prove that P(2r) is true for all r = 1, 2, 3,...... .

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8. Let P(n) be the statement : "$n^2 + n$ is even".

Prove that P(n) is true for all n $\in$ N by Mathematical Induction.

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9. By Principle of Mathematical Induction, prove that : $2^n > n$ for all n $\in$ N.

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10. Use principle of mathematical induction to prove that: $1 + 2 + 3 + \dots\dots\dots + n = \frac{n(n+1)}{2}$

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11. $1^2 + 2^2 + 3^3 + \dots. + n^2 = \frac{n(n+1)(2n+1)}{6}$

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12. Prove the following by using the principle of mathematical induction for all $n \in N$:- $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$.

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13. Using principle of mathematical induction, prove that

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots.+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

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14. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\dots+\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}.$$

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15. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

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16. Use the Principle of Mathematical Induction to prove that

n(n + 1) (2n + 1) is divisible by 6 for all n $\in$ N.

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17. Prove, by Principle of Mathematical Induction, that the sum of the cubes of

three consecutive natural numbers is divisible by 9.

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18. By the Principle of Mathematical Induction, prove that for all n $\in$ N, 3^{2n} when divided by 8, the remainder is 1 always.

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19. Prove the rule of exponents , $(ab)^n = a^n b^n$ by using Principle of Mathematical Induction for every natural number.

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20. Prove the following by using the principle of mathematical induction for all $n \in N$:- $1 + 2 + 3 + \ldots + n < \frac{1}{8}(2n+1)^2$.

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21. Prove by the principle of mathematical induction $10^{2n-1} + 1$ is divisible by 11.

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22. Prove by Principle of Mathematical Induction, that :

$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots\ldots. + \sin n\theta = \frac{\sin(n+1)\frac{\theta}{2}\sin\frac{n\theta}{2}}{\sin\frac{\theta}{2}}$ for all n $\in$ N.

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23. Using Principle of Mathematical Induction, prove that : $\cos\alpha\cos 2\alpha\cos 4\alpha............\cos\left(2^{n-1}\alpha\right)=\dfrac{\sin(2^n\alpha)}{2^n\sin\alpha}$ for all n $\in$ N.

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24. Prove by Principle of Mathematical Induction, that $\left(\dfrac{n^5}{5}+\dfrac{n^3}{3}+\dfrac{7n}{15}\right)$ is a natural number for all n $\in$ N.

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25. For the proposition P(n), given by , $1+3+5+.........+(2n-1)=n^2+2$, prove that P(k) is true $\Rightarrow P(k+1)$ is true. But, P(n) is not true for all n $\in$ N.

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26. Prove by Induction, that $2^n<n!$ for all $n\geq 4$.

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27. Prove, by Induction, that the number of all the subsets of a set containing n distinct elements, is 2^n.

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Exercise

1. Let P (n) be the statement : "P(n) : 10n + 3 is prime". Is P(3) true ?

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2. Let P(n) be the statement $''2^n > 1''$. Is P(1) true ?

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3. If P(n) is the statement "n(n + 1) is even", then what is P(4) ?

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4. Let P(n) be the statement $''n^3+n$ is divisible by 3".

Is the statement P(3) true ?

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5. Let P(n) be the statement $''n^3+n$ is divisible by 3".

Is the statement P(4) true ?

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6. If P(n) is the statement $``n^2>100"$ prove that P(r + 1) is true whenever P(r) is true.

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7. If P(n) is the statement "$2^n \geq n$", prove that P(r +1) is true whenever P(r) is true.

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8. Let P(n) be the statement "$4^n > n$". If P(r) is true, prove that P(r +1) is also true.

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9. If P(n) is the statement "$2^{3n} - 1$ is an integral multiple of 7", prove that P(r + 1) is true whenever P(r) is true.

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10. If P(n) is the statement "sum of first n natural numbers is divisible by n + 1", prove that P(r + 1) is true if P(r) is true.

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11. Give an example of a statement P(n), which is true for all $n \geq 4$, but P(1), P(2), P(3) are not true.

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12. Give an example of the following statement : P(n) such that it is true for all n $\in$ N.

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13. Give an example of the following statement : P(n) such that P(3) is true, but P(4) is not true.

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14. If P(n) is the statement : $'\,{}^{n}C_r \leq n!$ for $1 \leq r \leq n$ ", then : find P(n +1) .

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15. If P(n) is the statement : $'\,{}^{n}C_r \leq n!$ for $1 \leq r \leq n$ ", then : show that P(3) is true.

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16. Prove that the Principle of Mathematical Induction does not apply to the following :

P(n) : " $n^3 + n$ is divisible by 3" .

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17. Prove that the Principle of Mathematical Induction does not apply to the following :

P(n) : " $n^3 > 100$".

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18. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

The nth term of an A.P. whose first term is 'a' and common difference 'd' is a + (n-1) d.

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19. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$1 + 3 + 5 + \ldots\ldots + (2n - 1) = n^2$ i.e. the sum of first » odd natural numbers is n^2 .

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20. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$4+8+12+\ldots\ldots+4n=2n(n+1)$.

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21. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$5+15+45+\ldots.+5(3)^{n-1}=\frac{5}{2}(3^n-1)$.

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22. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$1+4+7+\ldots.+(3n-2)=\frac{n(3n-1)}{2}$.

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23. Prove the following by using the principle of mathematical induction for all $n \in N$:- $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

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24. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2 > \frac{n^3}{3}$.

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25. Prove the following by using the principle of mathematical induction for all $n \in N$:- $1 + 3 + 3^2 + \ldots. + 3^{n-1} = \frac{(3^n - 1)}{2}$.

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26. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}$$

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27. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$3.6 + 6.9 + 9.12 + \dots\dots + 3n(3n+3) = 3n(n+1)(n+2).$$

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28. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.2 + 2.3 + 3.4 + \dots + n.\ (n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$$

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29. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$1.2 + 2.2^2 + 3.2^3 + \ldots. + n.2^n = (n-1)2^{n+1} + 2.$

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30. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$1.3 + 2.3^2 + 3.3^3 + \ldots. + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}.$

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31. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$

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32. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$$

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33. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{(3n+1)}.$$

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34. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}.$$

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35. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)...\left(1+\frac{(2n+1)}{n^2}\right)=(n+1)^2.$$

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36. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right).......\left(1+\frac{1}{n}\right)=(n+1).$$

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37. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$a+(a+d)+(a+2d)+....+[a+(n-1)d]=\frac{n}{2}[2a+(n-1)d].$$

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38. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$b + br + br^2 + ... + br^{n-1} = \frac{b(1 - r^n)}{1 - r}, r < 1.$$

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39. Prove the following by using the principle of mathematical induction for all $n \in N$:- $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$.

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40. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$x^n - y^n$ is divisible by (x-y) for every n $\in$ N, $x - y \neq 0$.

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41. Prove the following by using the principle of mathematical induction for all $n \in N$:- $x^{2n} - y^{2n}$ is divisible by$x + y$.

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42. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$2^{3n} - 1$ is divisible by 7 .

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43. By the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$3^{2n} - 1$ is divisible by 8 .

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44. By the Principle of Mathematical Induction, prove the following for all

$n \in N$:

$5^{2n} - 1$ is divisible by 24.

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45. By the Principle of Mathematical Induction, prove the following for all

$n \in N$:

$4^{n} - 3n - 1$ is divisible by 9.

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46. By the Principle of Mathematical Induction, prove the following for all

$n \in N$:

$4^{n} + 15n - 1$ is divisible by 9.

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47. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$10^{2n-1}+1$ is divisible by 11.

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48. Prove the following by using the principle of mathematical induction for all $n \in N$:- $3^{2n+2}-8n-9$ is divisible by 8.

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49. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$3^{4n+1}+2^{2n+2}$ is divisible by 7.

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50. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$a^{2n-1} - 1$ is divisible by a-1.

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51. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$15^{2n-1} + 1$ is multiple of 16.

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52. Prove the following by using the principle of mathematical induction for all $n \in N$:- $41^n - 14^n$ is a multiple of 27.

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53. By the Principle of Mathematical Induction, prove the following for all

n $\in$ N :

$n^2 - n + 41$ is prime.

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54. Prove by Induction, for all n $\in$ N :

$2^n > n.$

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55. Prove by Induction, for all n $\in$ N :

$2^n < 3^n.$

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56. Prove the following by using the principle of mathematical induction for all $n \in N$:- $(2n+7) < (n+3)^2$.

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57. Prove by Induction, for all n $\in$ N :

$(n+3)^2 \leq 2^{n+3}$.

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58. Prove by Induction, that $(2n+7) \leq (n+3)^2$ for all n $\in$ N. Using this, prove by induction that : $(n+3)^2 \leq 2^{n+3}$ for all n $\in$ N.

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59. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1+\frac{1}{(1+2)}+\frac{1}{(1+2+3)}+\ldots+\frac{1}{(1+2+3+\ldots n)}=\frac{2n}{(n+1)}.$$

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60. Use method of induction, prove that : If $n^3+(n+1)^3+(n+2)^3$ is divisible by 9 for every n $\in$ N

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61. Using mathematical induction , show that $n(n+1)(n+5)$ is a multiple of 3.

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62. Use method of induction, prove that : $n(n+1)(n+2)$ is divisible by 6.

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63. Use method of induction, prove that :

$n^3 + 3n^2 + 5n + 3$ is divisible by 3.

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64. Prove, by Induction, on the inequality $(1+x)^n \geq 1 + nx$

for all natural numbers n, where x > - 1.

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65. Let P(n) be the statement "$n^2 - n + 41$" is prime. Prove that P(1), P(2) and P(3) are true. Also prove that P(41) is not true. How does this not contradict the Principle of Induction ?

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66. Let P(n) be the statement : "the arithmetic mean of n and (n + 2) is the same as their geometric mean"". Prove that P(1) is not true. Also prove that if P(n) is true, then P(n + 1) is also true. How does this contradict the principle of Induction ?

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67. If n straight lines in a plane are such that no two of them are parallel and no three of them are concurrent, prove that they intersect each other in $\frac{n(n-1)}{2}$ points.

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68. Let P (n) denote the statement : "$2^n \geq n!$". Show that P(1), P(2) and P(3) are true but P(4) is not true.

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69. By using the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\dots\dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}.$$

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70. By using the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$x + 4x + 7x + \dots\dots + (3n-2)x = \frac{1}{2}n(3n-1)x.$$

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71. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right]$$

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72. Prove the following by using the principle of mathematical induction for all $n \in N$:-

$$1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

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73. By using the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$$1.4.7 + 2.5.8 + 3.6.9 + \ldots\ldots. + n(n+3)(n+6) = \frac{n}{4}(n+1)(n+6)(n+$$

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74. By using the Principle of Mathematical Induction,

prove the following for all n $\in$ N :

$7 + 77 + 777 +$........ to n terms = $\frac{7}{81}\left(10^{n+1} - 9n - 10\right)$.

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75. By using the Principle of Mathematical Induction, prove the following for all n $\in$ N :

$1.1! + 2.2! + 3.3! + \ldots\ldots + n.n! = (n+1)! - 1.$

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76. Prove, by Mathematical Induction, that for all n $\in$ N,

$3^{2n} - 1$ is divisible by 8.

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77. $10^{n} + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

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78. Prove, by Mathematical Induction, that for all n $\in$ N,

$2.7^{n} + 3.5^{n} - 5$ is divisible by 24 .

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79. Prove, by Mathematical Induction, that for all n $\in$ N,

$n(n+1)(n+2)(n+3)$ is a multiple of 24.

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80. By Mathematical Induction, prove the following :

$(4^n + 15n - 1)$ is divisible by 9 .

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81. By Mathematical Induction, prove the following :

$12^n + 25^{n-1}$ is divisible by 13.

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82. Prove the following by using induction for all $n \in N$.

$11^{n+2} + 12^{2n+1}$ is divisible by 133.

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83. For all n $\in$ N, prove that : $\frac{n^2}{7} + \frac{n^5}{5} + \frac{2}{3}n^2 - \frac{n}{105}$ is an integer.

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84. Prove that :

$\cos A \cos 2A \cos 2^2 A \cos 2^3 A \ldots\ldots.\cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$.

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85. Let $U_1 = 1, U_2 = 1$ and $U_{n+2} = U_{n+1} + U_n$ for $n \geq 1$. Use Mathematical Induction to show that: $U_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ for all $n \geq 1$.

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