MATHS

NCERT - FULL MARKS MATHS(TAMIL)

CONTINUITY AND DIFFERENTIABILITY

Examples

1. Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$.

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2. Examine whether the function f given by $f(x) = x^2$ is continuous at x= 0.

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3. Discuss the continuity of the function f given by $f(x) = |x| \quad \text{at} \quad x = 0.$

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4. Show that the function f given by $f(x) = \begin{cases} x^3 + 3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ is not continuous at x=0.

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5. Check the points where the constant function f(x)= k is continuous.

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6. Prove that the identity function on real numbers given by f(x)= x is continuous at every real number.

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7. Is the function defined by $f(x) = |x|$, a continuous function?

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8. Discuss the continuity of the function f given by $f(x) = x^3 + x^2 - 1$.

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9. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0$.

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10. Discuss the continuity of the function f defined by $\begin{cases} x + 2 & \text{if } x \leq 1 \\ x - 2 & \text{if } x > 1 \end{cases}$.

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11. Find all the points of discontinuity of the function f defined by $\begin{cases} x+2 & \text{if } x<1 \\ 0 & \text{if } x=1 \\ x-2 & \text{if } x>1 \end{cases}$.

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12. Discuss the continuity of the function defined by $\begin{cases} x+2 & \text{if } x<0 \\ -x+2 & \text{if } x>0 \end{cases}$

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13. Discuss the continuity of the function f given by $\begin{cases} x & \text{if } x \geq 0 \\ x^2 & \text{if } x<0 \end{cases}$.

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14. Show that a function p is a polynomial function is continuous.

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15. Find all the points of discontinuity of the greatest interger function defined by $f(x) = [x]$, where [x] denote the greatest integer less than or equal to x.

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16. Prove that every rational function is continuous.

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17. Discuss the continuity of sine function.

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18. Prove that the function defined by $f(x) = \tan x$ is a continuous function.

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19. Show that the function defined by $f(x) = \sin\left(x^2\right)$ is a continuous function.

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20. Show that the function f defined by $f(x) = |1 - x + |x\ |\ |$,

where x is any real number, is a continuous function.

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21. Find the derivative of the function given by $f(x) = \sin(x^2)$.

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22. Find the derivative of $\tan(2x + 3)$.

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23. Differentiate $\sin\left(\cos\left(x^2\right)\right)$ with respect to x.

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24. Find $\frac{dy}{dx}$ if $x - y = \pi$.

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25. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$.

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26. Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists.

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27. Find the derivative of f given by $f(x) = \tan^{-1} x$ assuming it exists.

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28. Is it true that $x = e^{\log x}$ for all real x?

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29. Differentiate the following w.r.t. x :

e^{-x}

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30. Differentiate the following w.r.t. x :

$\sin(\log x), x > 0$

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31. Differentiate the following w.r.t. x :

$\cos^{-1}(e^x)$.

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32. Differentiate the following w.r.t. x :

$e^{\cos x}$

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33. Differentiate $\sqrt{\dfrac{(x-3)(x^2+4)}{(3x^2+4x+5)}}$.

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34. Differentiate a^x w.r.t. x, where a is a positive constant.

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35. Differentiate $x^{\sin x}$, $x > 0$ w.r.t. x.

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36. Find $\dfrac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

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37. Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta.$

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38. Find $\frac{dy}{dx}$, if $x = at^2, y = 2at.$

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39. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta), y = a(1 - \cos\theta).$

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40. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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41. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$.

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42. If $y = A \sin x + B \cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$.

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43. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

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44. If $y = \sin^{-1} x$, show that $\left(1 - x^2\right) \frac{d^2y}{dx^2} - x \frac{dy}{dx} 0.$

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45. Verify Rolle's theorem for the function $y = x^2 + 2, a = -2$ and $b = 2.$

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46. Verify Mean Value Theorem for the function $f(x) = x^2$ in the interval [2, 4].

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Miscellaneous Examples

1. Differentiate w.r.t.x, the following functions :

$$\sqrt{3x+2}+\frac{1}{\sqrt{2x^2+4}}.$$

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2. Differentiate w.r.t.x, the following functions :

$$e^{\sec^2 x}+3\cos^{-1}x.$$

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3. Differentiate w.r.t.x, the following functions :

$$\log_7(\log x).$$

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4. Differentiate the following w.r.t.x.

$\cos^{-1}(\sin x)$.

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5. Differentiate the following w.r.t.x.

$\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

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6. Differentiate the following w.r.t.x.

$\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

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7. Find $f'(x)$ if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$.

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8. For a positive constant a find $\frac{dy}{dx}$, where $a^{t+\frac{1}{t}}$, and $x = \left(t + \frac{1}{t}\right)^a$.

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9. Differentiative $\sin^2 xw.\, r.\, t.\, e^{\cos x}$.

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Exercise 5 1

1. Prove that the function $f(x) = 5x - 3$ is continuous at x=0, at $x = -3$ and at $x = 5$.

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2. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

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3. Examin the following functions for continuity.

$f(x) = x - 5$

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4. Examin the following functions for continuity.

$$f(x) = \frac{1}{x-5}, x \neq 5$$

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5. Examin the following functions for continuity.

$$f(x) = \frac{x^2 - 25}{x+5}, x \neq -5.$$

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6. Examin the following functions for continuity.

$$f(x) = |x-5|.$$

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7. Prove that the function $f(x) = x^n$ is continuous at x= n, where n is a positive integer.

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8. Is the function f defined by

$$\begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$$

continuous at x= 0? At x=1? At x=2?.

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9. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x + 3 & \text{if } x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}.$$

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10. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} |x| + 3 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}.$$

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11. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

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12. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|} & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$$

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13. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x + 1 & \text{if } x \geq 1 \\ x^2 + 1 & \text{if } x < 1 \end{cases}$$

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14. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

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15. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} x^{10} - 1 & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

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16. Is the function defined by

$$f(x) = \begin{cases} x + 5 & \text{if } x \leq 1 \\ x - 5 & \text{if } x > 1 \end{cases}$$ a continuous function?

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17. Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 1 \\ 4 & \text{if } 1 < x < 3 \\ 5 & \text{if } 3 \leq x \leq 10 \end{cases}.$$

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18. Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} 2x & \text{if } x < 0 \\ 0 & \text{if } 0 \leq x \leq 1 \\ 4x & \text{if } x \geq 1 \end{cases}.$$

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19. Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ 2x & \text{if } -1 < x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}.$$

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20. Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1 - 2 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$ is continuous at x=3.

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21. For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x) & \text{if } x \leq 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$$

continuous at x= 0? What about continuity at x =1?

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22. Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

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23. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

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24. Discuss the continuity of the following functions :

$f(x) = \sin x + \cos x$

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25. Discuss the continuity of the following functions :

$f(x) = \sin x - \cos x$

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26. Discuss the continuity of the following functions :

$f(x) = \sin x . \cos x .$

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27. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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28. Find all points of dicontinuity of f, where

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}.$$

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29. Determine if f defined by

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$ is a continuous function?

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30. Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}.$$

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31. Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}.$$

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32. Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases} \quad \text{at } x = 2.$$

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33. Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases} \quad \text{at } x = \pi.$$

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34. Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases} \quad \text{at } x = 5.$$

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35. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$$ is a continuous function.

A. $a = 3, b = 1$

B. $a = 1, b = 1$

C. $a = 1, b = 2$

D. $a = 2, b = 1$

Answer: D

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36. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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37. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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38. Examine that $\sin|x|$ is a continuous function.

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39. Find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$.

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Exercise 5 2

1. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\sin\left(x^2+5\right)$

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2. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\cos(\sin x)$

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3. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\sin(ax + b)$

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4. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

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5. Differentiate the functions with respect to x.

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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6. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\cos x^3 . \sin^2\left(x^5\right).$

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7. Differentiate the functions with respect to x in Exerecises 1 to 8.

$2\sqrt{\cot\left(x^2\right)}$

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8. Differentiate the functions with respect to x in Exerecises 1 to 8.

$\cos\left(\sqrt{x}\right).$

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9. Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at x= 1.

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10. Prove theat the greatest interger function defined by $f(x) = |x|, 0 < x < 3$ is not differentiable at x=1 and x= 2.

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Exercise 5 3

1. Find $\frac{dy}{dx}$ in the following :

$2x + 3y = \sin x$

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2. Find $\frac{dy}{dx}$ in the following :

$2x + 3y = \sin y.$

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3. Find $\frac{dy}{dx}$ in the following :

$ax + by^2 = \cos y.$

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4. Find $\frac{dy}{dx}$ in the following :

$xy + y^2 = \tan x + y.$

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5. Find $\frac{dy}{dx}$ in the following :

$x^2 + xy + y^2 = 100$

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6. Find $\frac{dy}{dx}$ in the following :

$x^3 + x^2y + xy^2 + y^3 = 81.$

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7. Find $\frac{dy}{dx}$ in the following :

$\sin^2 y + \cos xy = k.$

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8. Find $\frac{dy}{dx}$ in the following :

$\sin^2 x + \cos^2 y = 1.$

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9. Find $\frac{dy}{dx}$ in the following :

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right).$

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10. Find $\frac{dy}{dx}$ in the following :

$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \ -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}.$

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11. Find $\frac{dy}{dx}$ in the following :

$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1.$

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12. Find $\frac{dy}{dx}$ in the following :

$y = \sin^{1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1.$

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13. Find $\frac{dy}{dx}$ in the following :

$y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), \ -1 < x < 1.$

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14. Find $\frac{dy}{dx}$ in the following :

$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}.$

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15. Find $\frac{dy}{dx}$ in the following :

$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}.$

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Exercise 5 4

1. Differentiate the following w.r.t. x :

$$\frac{e^x}{\sin x}$$

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2. Differentiate the following w.r.t. x :

$$e^{\sin^{-1} x}$$

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3. Differentiate the following w.r.t. x :

$$e^{x^3}.$$

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4. Differentiate the following w.r.t. x :

$\sin\left(\tan^{-1} e^{-x}\right)$

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5. Differentiate the following w.r.t. x :

$\log(\cos e^{x})$

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6. Differentiate the following w.r.t. x :

$e^{x} + e^{x^2} + \ldots\ldots\ldots\ldots + e^{x^5}.$

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7. Differentiate the following w.r.t. x :

$\sqrt{(3)^{\sqrt{x}}}, x > 0.$

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8. Differentiate the following w.r.t. x :

$\log(\log x), x > 1.$

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9. Differentiate the following w.r.t. x :

$\frac{\cos x}{\log x}, x > 0.$

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10. Differentiate the following w.r.t. x :

$\cos(\log x + e^x), x > 0.$

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Exercise 5 5

1. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$\cos x . \cos 2x . \cos 3x.$

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2. Differentiate the functions given in w.r.t. x.

$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}.$

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3. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$(\log x)^{\cos x}$

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4. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$x^x - 2^{\sin x}$.

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5. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$(x+3)^2.(x+4)^3.(x+5)^4.$

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6. Differentiate the functions given in w.r.t. x.

$(\log x)^x + x^{\log x}.$

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7. Differentiate the functions given in w.r.t. x.

$(\log x)^x + x^{\log x}.$

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8. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$(\sin x)^x + \sin^{-1}\sqrt{x}$

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9. Differentiate the functions w.r.t. x.

$x^{\sin x} + (\sin x)^{\cos x}$.

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10. Differentiate the functions given in Exercises 1 to 11 w.r.t. x.

$(\log x)^{\cos x}$

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11. Differentiate the functions w.r.t. x.

$(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$.

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12. Find $\frac{dy}{dx}$ of the functions.

$x^y + y^x = 1$.

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13. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$x^y = y^x$.

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14. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$(\cos x)^y = (\cos y)^x$.

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15. Find $\frac{dy}{dx}$ of the functions given in Exercises 12 to 15.

$xy = e^{(x-y)}$.

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16. Find the derivative of the functions given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find f'(1).

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17. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below :

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial.

(iii) by logarithmic differentiation.

Do they all give the same answer?

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18. If u, v and w are functions of x, then show that

$$\frac{d}{dx}(u.\,v.\,w) = \frac{du}{dx}v.\,w + u.\,\frac{dv}{dx}.\,w + u.\,v\frac{dw}{dx}$$

in two ways-first by repeated application of product rule, second by logarithmic differentiation.

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Exercise 5 6

1. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = 2at^2, y = at^4.$

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2. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\cos\theta, y = b\cos\theta.$

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3. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \sin t, y = \cos 2t.$

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4. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = 4t, y = \frac{4}{t}.$

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5. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta.$

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6. If x and y are connected parametrically by the equations, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta).$

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7. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the

parameter, Find $\frac{dy}{dx}$.

$x = \sin t, y = \cos 2t.$

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8. If x and y are connected parametrically by the equations, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\left(\cos t + \frac{\log \tan(t)}{2}\right) y = a \sin t.$

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9. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a \sec \theta, y = b \tan \theta.$

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10. If x and y are connected parametrically by the equations given in Exercises 1 to 10, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta).$

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11. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

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Exercise 5 7

1. Find the second order derivatives of the functions given in Exercises 1 to 10.

$x^2 + 3x + 2$

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2. Find the second order derivatives of the functions given in Exercises 1 to 10.

x^{20}

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3. Find the second order derivatives of the functions given in Exercises 1 to 10.

$x \cdot \cos x$

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4. Find the second order derivatives of the functions given in Exercises 1 to 10.

$\log(\log x)$.

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5. Find the second order derivatives of the functions given in Exercises 1 to 10.

$x^3 \log x$.

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6. Find the second order derivatives of the functions given in Exercises 1 to 10.

$e^x \sin 5x$.

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7. Find the second order derivatives of the functions.

$e^{6x} \cos 3x$.

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8. Find the second order derivatives of the functions given in Exercises 1 to 10.

$\tan^{-1} x$.

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9. Find the second order derivatives of the functions given in Exercises 1 to 10.

$\log(\log x)$.

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10. Find the second order derivatives of the functions given in Exercises 1 to 10.

$\sin(\log x)$.

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11. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

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12. If $y = \cos^{-1} x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

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13. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$.

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14. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

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15. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$.

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16. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

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17. If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2$.

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Exercise 5 8

1. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$.

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2. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

$f(x) = [x] \quad \text{for} \quad x \in [5, 9]$.

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3. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

$f(x) = [x] \quad \text{for} \quad x \in [-2, 2].$

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4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say some thing about the converse of Rolle's theorem from these example?

$f(x) = x^2 - 1 \quad \text{for} \quad x \in [1, 2].$

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5. If $f:[-5,5] \to R$ is a differentiable function function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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6. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a,b], where a=1 and b= 4.

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7. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a,b], where a=1 and b=3. Find all $c \in (1,3)$ for which f'(c)=0`.

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Miscellaneous Exercise On Chapter 5

1. Differentiate w.r.t.x the function in Exercises 1 to 11.

$$\left(3x^2 - 9x + 5\right)^9$$

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2. Differentiate w.r.t.x the function.

$$\sin^3 x + \cos^6 x$$

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3. Differentiate w.r.t.x the function

$(5x)^{3\cos 2x}$

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4. Differentiate w.r.t.x the function in Exercises 1 to 11.

$\sin^{-1}\left(x\sqrt{x}\right), 0 \leq x \leq 1.$

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5. Differentiate w.r.t.x the function in Exercises 1 to 11.

$\frac{\cos^{-1}\frac{1}{2}}{\sqrt{2x+7}}, \ -7 < x < 2.$

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6. Differentiate w.r.t.x the function.

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], 0 < x < \frac{\pi}{2}.$$

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7. Differentiate w.r.t.x the function.

$(\log x)^{\log x}, x > 1.$

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8. Differentiate w.r.t.x the function in Exercises 1 to 11.

$\cos(a\cos x + b\sin x)$, for some constant a and b.

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9. Differentiate w.r.t.x the function.

$(\sin x - \cos x)^{\sin x - \cos x}, \frac{\pi}{4} < x < \frac{3\pi}{4}.$

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10. Differentiate w.r.t.x the function.

$x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0.$

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11. $x^{x^2-3} + (x-3)^{x^2}$, for $x > 3.$

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12. Find

$$\frac{dy}{dx}, \quad \text{if} \quad y = 12(1 - \cos t), x = 10(t - \sin t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

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13. Find

$$\frac{dy}{dx}, \quad \text{if} \quad y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}, 0 < x < 1.$$

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14. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for , $-1 < x < 1$,

prove that

$$\frac{dy}{dx} = -\frac{1}{(1+x)^2}.$$

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15. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

is a constant independent of a and b.

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16. If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$.

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17. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

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18. If $f(x) = |x|^3$, show that f''(x) exists for all real x and find it.

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19. Derivative of $f(x) = x^n$ is nx^{n-1} for any positive integer n.

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20. Using the fact the that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines.

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21. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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22. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$.

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23. If $y = e^{a\cos^{-1}x}, -1 \leq x \leq 1,$ show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

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