

MATHS

NCERT - FULL MARKS MATHS(TAMIL)

DIFFERENTIAL EQUATIONS

Example

1. Find the order and degree, if defined, of each of the following

differential equations:

(i)
$$\frac{dy}{dx} - \cos x = 0$$

(ii) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
(iii) $y''' + y^2 + e^{y'} = 0$

2. Verify that the function $y = e^{-3x}$ is a solution of the differential

equation

$$rac{d^2y}{dx^2}+rac{dy}{dx}-6y=0$$

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3. Verify that the function $y=b\cos x+a\sin x,\,\,$ where $a\,\,$ and $b\in R$ is a solution of the differential equation $rac{d^2y}{dx^2}+y=0$



4. From the differential equation representing the family of curves

y = nx, where n is an arbitrary constant.



5. From the differential equation representing the family of curves

 $y = a \sin(x + b)$, where a, b are arbitrary constants.

• Find the differential equations of the family of all the ellipses

having foci on the y-axis and centre at the origin.

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7. Form the differential equation of all circles touching the x-axis

at the origin and center on y-axis.



8. From the differential equation of the family of parabolas having

vertex at the origin and axis along positive direction of x-axis.





13. Find the equation of a curve passing through the point (-2,3), given that the slope of the tangent to the curve at any point (x,y) is $\frac{2x}{2x}$

is
$$\frac{2\pi}{y^2}$$
.

14. In a bank, principal increases continuously at the rate of 5% per

year.In how many years Rs 1000 double itself?



17. Show that the differential equation $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$ is homogeneous and find its particular solution, given that, x = 0 when y = 1.

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18. Show that the family of curves for which the slope of the tangent at any point (x,y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2-y^2=Cx$

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19. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x.$

20. Find the general solution of the differential equation $x \frac{dy}{dx} + 2y = x^2 (x \neq 0).$ Watch Video Solution 21. Find the general solution of the differential equation $y dx - (x + 2y^2) dy = 0.$

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22. Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x (x \neq 0)$ given that y = 0 when $x = \frac{\pi}{2}$.

23. Find the equation of the curve passing through the point (0, 1), if the slope of the tangent to the curve at any point (x, y), is equal to the sum of x coordinate and product of x coordinate and y coordinate of that point.

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Miscellaneous Examples

1. Verify that the function $y = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx$, where c_1, c_2 are arbitrary constants is a solution of the differential equation

$$rac{d^2y}{dx^2}-2arac{dy}{dx}+ig(a^2+b^2ig)y=0$$

2. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

3. Find the particular solution of the differential equations log

$$\left(rac{dy}{dx}
ight)=3x+4y$$
 given that y = 0 when x = 0.

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4. Solve the differential equation

$$(xdy-ydx)y\sin\Bigl(rac{y}{x}\Bigr)=(ydx+xdy)x\cos\Bigl(rac{y}{x}\Bigr).$$



1. Determine order and degree (if defined) of differential equations

given in Exercises 1 to 10

(1)
$$rac{d^4y}{dx^4}+\sin(y')=0$$

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2.
$$y' + 5y = 0$$

3. Determine order and degree (if defined) of differential

equations given in

$$\left(rac{ds}{dt}
ight)^4 + 3srac{d^2s}{dt^2} = 0$$

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4. Determine order and degree (if defined) of differential equations given in

$$\left(rac{d^2y}{dx^2}
ight)^3+\cos\!\left(rac{dy}{dx}
ight)=0$$

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5. Determine order and degree (if defined) of differential

equations given in

$$rac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

6. Determine order and degree (if defined) of differential

equations given in

$$(y^{\prime} \, {}^{\prime} \, {}^{\prime} \, {}^{\prime})^2 + (y^{\prime} \, {}^{\prime})^3 + (y^{\prime})^4 + y^5 = 0$$

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7. Determine order and degree (if defined) of differential equations given in

$$y^{\prime} \, \overset{\prime}{}\, ^{\prime} + 2 y^{\prime} \, \overset{\prime}{}\, + y^{\prime} = 0$$

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8. Determine order and degree (if defined) of differential equations given in

$$y' + y = e^x$$



9.
$$y + (y')^2 + 2y = 0$$

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10.
$$y + 2y' + \sin y = 0$$

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11. The degree of the differential equation
$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + 1 = 0$$
 is

A. 3

B. 2

C. 1

D. not defined

Answer: D

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is

A. 2

B. 1

C. 0

D. not defined

Answer: A



Exercise 9 2

1. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation :

$$y=e^x+1\!:\!y'\,{}'-y'=0$$

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2. Verify that the given functions (explicit or implicit) is a solution

of the corresponding differential equation :

 $y = \cos x + C {:} y' + \sin x = 0$

3. Verify that the given functions (explicit or implicit) is a solution

of the corresponding differential equation
$$y = \sqrt{1 + x^2}$$
: $y' = \frac{xy}{1 + x^2}$
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4. Verify the differential equation y = Ax : xy' = y(x
eq 0)



6.
$$xy = \log y + C$$
: $y' = rac{y^2}{1-xy}(xy
eq 1)$

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7.
$$y - \cos y = x \cdot (y \sin y + \cos y + x)y' = y$$

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8.
$$x + y = \tan^{-1} y$$
: $y^2 y' + y^2 + 1 = 0$

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9.
$$y=\sqrt{a^2-x^2}x
eq(-a,a)$$
 : $x+yrac{dy}{dx}=0(y
eq 0)$

10. The number of arbitrary constants in the general solution of a

defferential equation of fourth order are

A. O B. 2 C. 3 D. 4

Answer: D

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11. The number of arbitrary constants in the particular solution of

a defferential equation of third order are

A. 3

B. 2

C	1
L.	I

D. 0

Answer: D

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Exercise 9 3	

1. Form a differential equation representing the given family of

curves by eliminating arbitrary constants a and b.

$$rac{x}{a} + rac{y}{b} = 1$$

2. Form a differential equation representing the given family of

curves by eliminating arbitrary constants a and b.

$$y^{2} = a(b^{2} - x^{2})$$

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3. Form the differential equation from $y = Ae^{3x} + Be^{-2x}$.

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4. $y = e^{2x}(a + bx)$

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5.
$$y = e^x(a\cos x + b\sin x)$$



6. Form the differential equation of the family of parabolas having

vertex at origin and axis along positive y-axis.

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7. Form the differential equation of the family of parabolas having

vertex at origin and axis along positive y-axis.

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8. From the differential equation representing the family of ellipses having focion x-axis and centre at the origin.



9. Form the differential equation of the family of hyperbolas havig

foci on x-axis and centre at origin.

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10. Form the differential equation of the family of circles having centre on y -axis and radius 5 units.	
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11. Which of the following differential equations has $y = c_1 e^x + c_2 e^{-x}$ as the general solution?	

A.
$$rac{d^2y}{dx^2}+y=0$$

B. $rac{d^2y}{dx^2}-y=0$
C. $rac{d^2y}{dx^2}+1=0$

D.
$$\displaystyle rac{d^2y}{dx^2} - 1 = 0$$

Answer: B



12. Which of the following differential equations has y = x as one of its particular solution?

A.
$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$$

B.
$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = x$$

C.
$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$$

D.
$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + xy = 0$$

Answer: C

1. For each of the differential equations in Exercises 1 to 10, find

the general solution:

$$1.\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

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2.
$$rac{dy}{dx} = \sqrt{4-y^2}(-2 < y < 2)$$

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3.
$$rac{dy}{dx}+y=1(y
eq1)$$

4. The solution of $\sec^2x \tan y dx + \sec^2y \tan x dy = 0$ is

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5. For each of the differential equations in

$$ig(e^x+e^{-x}ig)dy-ig(e^x-e^{-x}ig)dx=0$$

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6.
$$rac{dy}{dx}=\left(1+x^2
ight)\left(1+y^2
ight)$$



7. Differential equation $y\log ydx - xdy = 0$

8. Differential equation $x^5 rac{dy}{dx} = -y^5$

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9.
$$\frac{dy}{dx} = \sin^{-1}x$$

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10.
$$e^x an y dx + (1-e^x) \mathrm{sec}^2 y dy = 0$$

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11. For the differential equations, find a particular solution satisfying given condition;

$$ig(x^3+x^2+x+1ig)rac{dy}{dx}=2x^2+x,y=1$$
 when x = 0.



12.
$$x \left(x^2-1
ight) rac{dy}{dx}=1, y=0$$
 when x = 2.

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13.
$$\cos\!\left(rac{dy}{dx}
ight)=a(a
eq R),y=2$$
 when x = 0

14.
$$rac{dy}{dx} = y an x, y = 1$$
 when x = 0

15. Find the equation of a curve passing through the point (0,0) and whose differential equation is $y' = e^x \sin x$.



17. Find the equation of a curve passing through the point (0, -2). Given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

18. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve, given that it passes through (-2, 1).

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19. The volume of spherical balloon being inflated changes at a constant rate.If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

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20. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years $(\log_e 2 = 0.6931).$



21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years $(e^{0.5} = 1.648)$.

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22. In a culture, the bacteria count is 1, 00, 000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2, 00, 000 if the rate of growth of bacteria is proportional to

the number present ?



23. The general solution of the differential equation $rac{dy}{dx}=e^{x+y}$

is

A.
$$e^x + e^{-y} = C$$

B. $e^x + e^y = C$
C. $e^{-x} + e^y = C$
D. $e^{-x} + e^{-y} = C$

Answer: A



1. Show that the given differential equation is homogeneous and

solve each of them.

$$ig(x^2+xyig)dy=ig(x^2+y^2ig)dx$$



2. Solve the given differential equation.

$$y'=rac{x+y}{x}$$

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3.
$$(x-y)dy - (x+y)dx = 0$$

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4.
$$ig(x^2-y^2ig)dx+2xydy=0$$

5.
$$x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$$



6.
$$xdy-ydx=\sqrt{x^2+y^2}dx$$

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7.

$$\Big\{x\cos\Big(rac{y}{x}\Big)+y\sin\Big(rac{y}{x}\Big)\Big\}ydx=\Big\{y\sin\Big(rac{y}{x}\Big)-x\cos\Big(rac{y}{x}\Big)\Big\}xdy$$

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8.
$$x rac{dy}{dx} - y + x \sin \Bigl(rac{y}{x} \Bigr) = 0$$

9. Show that the given differential equation is homogeneous and

solve each of them.

 $ydx+x\log\Bigl(rac{y}{x}\Bigr)dy-2xdy=0$

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10.
$$\left(1+e^{rac{x}{y}}
ight)dx+rac{e^{x}}{y}\left(1-rac{x}{y}
ight)dy=0$$

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11. For each of the differential equations in find the particular solution satisfying the given condition :

$$(x+y)dy+(x-y)dx=0,y=1$$
 when $x=1$

12.
$$x^2 dy + ig(xy+y^2ig) dx = 0, y = 1$$
 when x = 1



13.
$$\Big[x\sin^2\Big(rac{y}{x}\Big)-y\Big]dx+xdy=0,$$
 $y=rac{\pi}{4}$ when x = 1

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14. For each of the differential equations in find the particular solution satisfying the given condition :

$$rac{dy}{dx} - rac{y}{x} + \cos ec \Big(rac{y}{x}\Big) = 0, y = 0$$
 when $x = 1$

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15. For each of the differential equations in find the particular solution satisfying the given condition :

$$2xy+y^2-2x^2rac{dy}{dx}=0,y=2$$
 when $x=1$

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16. A homogeneous differential equation of the from $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution.

A. y = vx

B. v = yx

C. x = vy

Answer: C

17. Which of the following is a homogeneous differential equation?

A.
$$(4x+6y+5)dy-(3y+2x+4)dx=0$$

B. $(xy)dx-(x^3+y^3)dy=0$
C. $(x^3+2y^2)dx+2xydy=0$
D. $y^2dx+(x^2-xy-y^2)dy=0$

Answer: D

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Exercise 96

1. Find the general solution :

$$\cdot rac{dy}{dx} + 2y = \sin x$$

2.
$$rac{dy}{dx}+3y=e^{-2x}$$

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3. solve each of the differential equations given in

$$rac{dy}{dx}+rac{y}{x}=x^2$$

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4. For each of the differential equations given in

$$rac{dy}{dx} + \sec x. \ y = an x \Big(0 \leq x < rac{\pi}{2} \Big)$$

5.
$$\cos^2 x rac{dy}{dx} + y = an x \Big(o \leq x \leq rac{\pi}{2} \Big)$$

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6. For each of the differential equations given in

$$xrac{dy}{dx}+2y=x^2\log x$$

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7.
$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$$

8.
$$ig(1+x^2ig) dy + 2xy dx = \cot x dx (x
eq 0)$$



9. Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \cos ecx (x \neq 0)$, given that y = 0 when $x = \frac{\pi}{2}$.

10. For each of the differential equations given in

$$(x+y)rac{dy}{dx} = 1$$

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11.
$$ydx+ig(x-y^2ig)dy=0$$

12.
$$\left(x+3y^2
ight)rac{dy}{dx}=y(yrt0).$$



13. For each of the differential equations given in find a particular

solution satisfying the given condition :

 $rac{dy}{dx}+2y an x=\sin x,y=0$ when $x=rac{\pi}{3}$

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14. For each of the differential equations given in find a particular solution satisfying the given condition :

$$ig(1+x^2ig)rac{dy}{dx}+2xy=rac{1}{1+x^2},y=0$$
 when $x=1$

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15.
$$\frac{dy}{dx} = 3y \cot x = \sin 2x, y = 2$$
 when $x = \frac{\pi}{2}$

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16. Find the equation of a curve passing through the origin, given that the slope of the tangent of the curve at any point (x, y) is equal to tha sum of the coordinates of the point.

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17. Find the equation of a curve passing through the point (0, 2), given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.



18. The integrating factor of the fifferential equation

$$x \frac{dy}{dx} - y = 2x^2$$
 is
A. e^{-x}
B. e^{-y}
C. $\frac{1}{x}$
D. x

Answer: C

19. The integrating factor of the fifferential equation
$$(1-y^2)rac{dy}{dx}+yx=ay(-1< y'<1)$$
 is A. $rac{1}{y^2-1}$

B.
$$\displaystyle rac{1}{\sqrt{y^2-1}}$$

C. $\displaystyle rac{1}{1-y^2}$
D. $\displaystyle rac{1}{\sqrt{1-y^2}}$

Answer: D



Miscellaneous Exercise

1. For each of the differential equations given below, indicate its

order and degree(if defined).

(i)
$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$$

(ii)
$$\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$$

(iii)
$$\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3} = 0\right)$$

2. For each of the exercises given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

$$xy = ae^x + be^{-x} + x^2 {:} x rac{d^2y}{dx^2} + 2 rac{dy}{dx} - xy + x^2 - 2 = 0$$

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3. Form the differential equation representing the family of curves

given by $\left(x-a
ight)^2+2y^2=a^2$, where a is an arbitrary constant.

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4. Prove that $x^2 - y^2 = c(x^2 + y^2)^2$ is the general solution of differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$, where c is a parameter.



 $rac{dy}{dx}+rac{y^2+y+1}{x^2+x+1}=0$ is given by (x+y+1)=A(1-x-y-2xy), where A is parameter.

8. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is sin x cox y dx + cos x sin y dy = 0.



9. Find the particular solution of the differential equation

$$ig(1+e^{2x}ig)dy+ig(1+y^2ig)e^xdx=0.$$
 Given that $y=1$ when $x=0.$

10. Solve the differential equation
$$y \frac{e^x}{y} dx = \left(x \frac{e^x}{y} + y^2\right) dy (y
eq 0).$$

11. Find a particular solution of the differential equation (x-y)(dx+dy) = dx - dy. Given that y = -1, when x = 0.



12. Solve the differential equation
$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right]\frac{dx}{dy} = 1(x \neq 0).$$

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13. Find a particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \cos ecx (x \neq 0).$ Given that y = 0 when $x = \frac{\pi}{2}$. 14. Find a particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, given that y = 0 when x = 0.

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15. The population of a village increases continuously at the rate proportional to the number of its Inhabitants present at any time. It the population of the village was 20, 000 in 1999 and 25, 000 in the year 2004, what will be the population of the village in 2009 ?

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16. The general solution of the differential equation $\frac{ydx - xdy}{y} = 0$ is

A. xy = CB. $x = Cy^2$ C. y = CxD. $y = Cx^2$

Answer: C

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17. The general solution of a differential equation of the type $rac{dy}{dx} + P_1 x = Q_1$ is

A.
$$Ye^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C$$

B. $y. e^{\int P_1 dx} = \int (Q_1 e^{\int P_1 dx}) dx + C$
C. $xe^{\int P_1 dx} = \int (Q_1 e^{\int P_1 dx}) dx + C$
D. $xe^{\int P_1 dx} = \int (Q_1 e^{\int P_1 dx}) dx + C$

Answer: C



18. The general solution of the differential equation $e^{x}dy + (ye^{x} + 2x)dx = 0$ is A. $xe^{y} + x^{2} = C$ B. $xe^{y} + y^{2} = C$ C. $ye^{x} + x^{2} = C$ D. $ye^{y} + x^{2} = C$

Answer: C