MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Differentiation

Example

1. If $\sqrt{1-x^{2n}}+\sqrt{1-y^{2n}}=a^n(x^n-y^n)$, prove that $y^{n-1}\cdot\sqrt{1-x^{2n}}dy=x^{n-1}\sqrt{1-y^{2n}}dx$.

Watch Video Solution

2. If $u=f(x^2), v=g(x^3), f'(x)=\sin x$ and $g'(x)=\cos x$ then find $\frac{du}{dv}$.

Watch Video Solution

3. Find $\frac{dy}{dx}$ at $x = -1$,when

$(\sin y)^{\sin\left(\left(\frac{\pi}{2}\right)x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$

Watch Video Solution

4. If $y = e^{m\sin^{-1}x}(-1 \leq x \leq 1)$, Prove that,

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y.$

Watch Video Solution

5. Let f be a twice differentiable function such that $f^x = -f(x), and f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2.$ Find $h(10)$ if $h(5) = 11$

Watch Video Solution

6. A function $f: R \to R$ satisfies the equation $f(x+y) = f(x)f(y)$ for all values of x and y and for any $x \in R$, $f(x) \neq 0$. Suppose the function is differentiable at $x = 0$ and $f'(0) = 2$, then for all $x \in R$, $f(x) =$

Watch Video Solution

7. Let f(x) be a real valued function not identically zero such that $f\left(x + y^{2n+1}\right) = f(x) + (f(y))^{2n+1}$, $n\varepsilon N$ and $x, y\varepsilon loR$. If $f'(0) \geq 0$, then $f'(6)$ is

Watch Video Solution

8. Find the sum of $\sin x + 3\sin 3x + 5\sin 5x + \ldots\ldots\ldots + (2k-1)\sin(2k-1)x.$

Watch Video Solution

Exercise

1. Differentiate the function w.r.t x : $x^2 \cos x$

Watch Video Solution

2. Differentiate the function w.r.t x :$\cos^{-1}\left(x^{x^2}\right)$

Watch Video Solution

3. Differentiate the function w.r.t x :$\cos^{-1}\left(e^{\sqrt{\tan x}}\right)$

Watch Video Solution

4. Differentiate the function w.r.t x :$\sqrt[3]{\sin x}$

Watch Video Solution

5. Differentiate the function w.r.t x : e^{x^2}

Watch Video Solution

6. Differentiate the function w.r.t x : $\frac{\log x}{x}$

Watch Video Solution

7. Find the derivatives w.r.t x at the indicated points : $x \tan^{-1} x$ at $x = 1$

Watch Video Solution

8. Find the derivatives w.r.t x at the indicated points : $\log|x| at x = -2$

Watch Video Solution

9. Find the derivatives w.r.t x at the indicated points : $|x-1|+|x-3| at x=2$

Watch Video Solution

10. Find the derivatives w.r.t x at the indicated points : $|\cos x| at x=\frac{3\pi}{4}$

Watch Video Solution

11. Find the derivatives w.r.t x at the indicated points : $\sin(\log x)$ at $x=e$

Watch Video Solution

12. If $f(x)=x+\log|x|, x\neq 0$ find $f'(x)$.

Watch Video Solution

13. If $y = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left[\left(\frac{\sqrt{a-b}}{a+b}\right)\frac{\tan x}{2}\right]$, show thatdy/dx= $\frac{1}{a + b\cos x}$ and $\frac{d^2y}{dx^2} = \frac{b\sin x}{(a + bs\cos x)^2}$

Watch Video Solution

14. Show that $y = A\sin x + B\cos x + x\sin x$ is a solution of differential equation $y + \frac{d^2y}{dx^2} = 2\cos x,$ where A, B are constants.

Watch Video Solution

15. Show that, $v = \frac{A}{r} + B$ satisfies the differential equation $\frac{d^2v}{dr^2} + \frac{2}{r}.\frac{dv}{dr} = 0$

Watch Video Solution

16. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$ then $\frac{y'}{y} =$

Watch Video Solution

17. If $y = \sqrt{a^2 - x^2} + \frac{a}{2}\log\left(\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}}\right)$, then find the value of $\frac{dy}{dx}$

Watch Video Solution

18. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

Watch Video Solution

19. if $y = \cos^{-1}\left(\frac{5\cos x - 12\sin x}{13}\right)$, where $x \in \left(0, \frac{\pi}{2}\right)$, then $\frac{dy}{dx}$ is $-$

Watch Video Solution

20. Find dy/dx if y is equal to : $(x \log x)^{\log \log x}$

Watch Video Solution

21. Find dy/dx if y is equal to : $x^{x^x} + (\cot 4x)^x$

Watch Video Solution

22. Find dy/dx if y is equal to : $(\tan x)^{\sin x} + (\log x)^x$

Watch Video Solution

23. Find dy/dx if y is equal to : $x^{x^n} + x^{n^x}$

Watch Video Solution

24. Find dy/dx if y is equal to : $\left\{(\tan x)^{\tan x}\right\} atx = \frac{\pi}{4}$

Watch Video Solution

25. Find the derivative with respect to x of $\left((\log)_{\cos x} \sin x\right)\left((\log)_{\sin x} \cos x\right)^{-1} + arc\frac{\sin(2x)}{1+x^2}$ at $x = \frac{\pi}{4}$.

Watch Video Solution

26. Find dy/dx if :$x^{\sin y} + y^{\cos x} = 1$

Watch Video Solution

27. If $y^x + x^y = (x+y)^{x+y}$ find $\frac{dy}{dx}$

Watch Video Solution

28. Find $\frac{dy}{dx}$ if : $y = \frac{x}{a + \frac{x}{b + \frac{x}{a + \frac{x}{b + \ldots}}}}$

Watch Video Solution

29. Find dy/dx if :$y = \left(x^{x^{x} \cdots \to \propto}\right)$

Watch Video Solution

30. If $\sqrt{1-x^4} + \sqrt{1-y^4} = k\left(x^2 - y^2\right)$, prove that $\frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$

Watch Video Solution

31. Find the derivative of $f(x) = \log_x \sin x^2 + \left(\sin x^2\right)^{\log_e x}$ w.r.t. $\phi(x) = \log_e x$

Watch Video Solution

32. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x - \frac{1}{2}$.

Watch Video Solution

33. Find the differentiation of $\log_{(1-\sqrt{x})}\left(\sin^{-1}\left(1-\sqrt{x}\right)\right)$ with respect to $2^{2(1-\sqrt{x})}$ and also find its value at $x = 0.25$.

Watch Video Solution

34. If $y = \sqrt{\frac{t-\alpha}{\beta-t}}$ and $x = \sqrt{(t-\alpha)(\beta-t)}$ then find $\frac{dy}{dx}$.

Watch Video Solution

35. If $x = f(t)$ and $y = \phi(t)$, prove that $\frac{d^2y}{dx^2} = \frac{f_1\phi_2 - f_2\phi_1}{f_1^3}$ where suffixes denote differentiation $w.\ r.\ t.\ t.$

Watch Video Solution

36. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

Watch Video Solution

37. If $x = \cos\theta, y\sin^3\theta$, prove that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta(5\cos^2\theta - 1)$.

Watch Video Solution

38. If $x = 2\cos t - \cos 2t, y = 2\sin t - \sin 2t$, then the value of $\left|\frac{d^2y}{dx^2}\right|_{t=\pi/2}$ is

Watch Video Solution

39. Find $\frac{dx}{dt}$ when $x = \sin^{-1}\left(t.\sqrt{1-t} + \sqrt{t}\sqrt{1-t^2}\right)$.

Watch Video Solution

Watch Video Solution

40. If $x^2 + y^2 = t$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then prove that $\frac{dy}{dx} = \frac{1}{x^3 y}$.

Watch Video Solution

41. Prove that $\frac{d}{dx}\begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{vmatrix} = \begin{vmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_4 & v_4 & w_4 \end{vmatrix}$ where u, v, w are functions of x and $\frac{du}{dx} = u_1, \frac{d^2u}{dx^2} = u_2$, etc.

Watch Video Solution

42. If $f_r(x), g_r(x), h_r(x), r = 1, 2, 3$ are polynomials such that $f_r(a) = g_r(a) = h, (a), r = 1, 2, 3 and$

$F(x) = |f_1(x) f_2(x) f_3(x) g_1(x) g_2(x) g_3(x) h_1(x) h_2(x) h_3(x)|$ then

$F'(x) atx = a$ is_____

Watch Video Solution

43. If $y = \cos ax$, prove that $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} = 0$ where $y_r = \frac{d^r}{dx^r} . y$

Watch Video Solution

44. Let $\Delta(x) = \begin{vmatrix} x^2 - 1 & x + 1 & x - 2 \\ 2x^2 - 1 & 3x & 3x - 3 \\ x^2 + 4 & 2x - 1 & 2x - 1 \end{vmatrix}$. Prove, by using calculus, that $\Delta(x)$ is a first degree polynomial.

Watch Video Solution

45. If $y = \sin\left(2\sin^{-1} x\right)$ show that

$$\left(1 - x^2\right)\frac{d^2y}{dx^2} = x\frac{dy}{dx} - 4y$$

Watch Video Solution

46. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_2 + xy_1 + y = 0$.

Watch Video Solution

47. If $y = x^{n-1}\log x$, prove that $\left(x^2y_2\right) + (3-2n)xy_1 + (n-1)^2$.y=0 where$y_1 = \dfrac{dy}{dx}$ and $y_2 = \dfrac{d^2}{dx^2}$.

Watch Video Solution

48. Let $y = \tan^{-1}\sqrt{x^2-1}$. Prove that $\left(2x^2-1\right)\left(\dfrac{dy}{dx}\right) + x\left(x^2-1\right)\left(\dfrac{d^2y}{dx^2}\right) = 0.$

Watch Video Solution

49. If $y = \dfrac{ax+b}{x^2+c}$, prove that $(2xy_1+y)y_3 = 3(xy_2+y_1)y_2$.

Watch Video Solution

50. Let $y = f(x).\phi(x)$ and $z = f'(x).\phi'(x)$. prove that $\frac{1}{y}\cdot\frac{d^2y}{dx^2} = \frac{1}{f}\cdot\frac{d^2f}{dx^2} + \frac{1}{\phi}\cdot\frac{d^2\phi}{dx^2} + \frac{2z}{f\phi}$.

Watch Video Solution

51. Let $f(x+y) = f(x)f(y)$ for all $x and y$. Suppose $f(5) = 2 and f'(0) = 3$. Find $f'(5)$.

Watch Video Solution

52. Let $f'(x)$ exists for all $x \neq 0$ and $f(xy) = f(x) + f(y)$ for all real x, y. Prove that $f(x) = k\log x$ where k is aconstant.

Watch Video Solution

53. A function f is defined such that for all real x, y (a) $f(x+y) = f(x).f(y)$ (b) $f(x) = 1 + xg(x)$ where $\lim_{x\to 0} g(x) = 1$.

prove that $f;(x) = f(x)$ and $f(x) = e^x$

Watch Video Solution

54. A function f is defined such that for all real x, y (a) $f(x+y) = f(x).f(y)$ (b) $f(x) = 1 + xg(x)$ where $\lim_{x \to 0} g(x) = 1.$ prove that $f;(x) = f(x)$ and $f(x) = e^x$

Watch Video Solution

55. Let $f(x+y) = f(x) + f(y)$ for all real x, y and $f'(0)$ exists. Prove that $f'(x) = f'(0)$ for all $x \in R$ and $2f(x) = xf(2).$

Watch Video Solution

56. Let $f\left(\frac{x+y}{2}\right) = \frac{1}{2}|f(x) + f(y)|$ for all real x and y, if $f'(0)$ exists and equal to $(-1),$ and $f(0) = 1$ then $f(2)$ is equal to-

Watch Video Solution

57. A function $f(x)$ is so defined that for all real x, $\{f(x)\}^n = f(nx)$. Prove that $f(x) \cdot f'(nx) = f'(x) \cdot f(nx)$.

Watch Video Solution

58. Prove that the derivative of (a) an odd function is an even function;

Watch Video Solution

59. Prove that the derivative of a periodic function of period T is a periodic function of period T.

Watch Video Solution

60. Let $f(x)$ be a function satisfying the condition $f(-x) = f(x)$ for all real x. If $f'(0)$ exists, then its value is equal to

Watch Video Solution

61. A function $f: R \to R$ satisfies the relation $f\left(\frac{x+y}{3}\right) = \frac{1}{3}|f(x) + f(y) + f(0)|$ for all $x, y \in R$. If $f'(0)$ exists, prove that $f'(x)$ exists for all $x, \in R$.

Watch Video Solution

62. Find the sum of series $\sum_{r=1}^{n} r.\, x^{r-1}$, using calculus.

Watch Video Solution

63. Differential coefficient of $\log_2(\log_2 x)$w.r.t x is_____

Watch Video Solution

64. If $f(x) = \log_x(\log_e x)$ then $f'(e)$=_____

Watch Video Solution

65. If $x = a\left(\cos t + \log\left(\tan\left(\frac{t}{2}\right)\right)\right), y = a\sin t, then \frac{dy}{dx} =$

Watch Video Solution

66. Let $f'(x) = \sin(x^2)$ and $y = f(x^2+1)$ then $\frac{dy}{dx}$ at $x = 1$ is

Watch Video Solution

67. If $x = \cos t, y = \log_e t$ then at $t = \frac{\pi}{2}, \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 =$

Watch Video Solution

68. If $f'(x) = \sin x^2$ and $y = f(x^2+1) then \frac{dy}{dx}$=____

Watch Video Solution

69. If $x = \cos t, y = \log_e t$ then at $t = \frac{\pi}{2}, \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 =$

Watch Video Solution

70. If $\tan y = \frac{2t}{1-t^2}$ and $\sin x = \frac{2t}{1+t^2}$ then $\frac{dy}{dx}$=_____

Watch Video Solution

71. If $xe^{xy} = y + \sin^2 x$ then at $x = 0\ \frac{dy}{dx} =$

Watch Video Solution

72. If $x^2y + y^3 = 2$, the value of $\frac{dy}{dx}$ at the point $(1, 1)$ is

Watch Video Solution

73. Let

$f(x) = \left\{(\tan)\frac{\pi}{4} + \tan x\right\}\left\{(\tan)\frac{\pi}{4} + (\tan)\left(\frac{\pi}{4} - x\right)\right\}$ and $g(x) = x^2 +$

Then $g\{f(x)\} + f'(x) =$

Watch Video Solution

74. IF $y = \frac{1}{1 + x + x^2 + x^3}$, then value of $\frac{d^2y}{dx^2}$ at $x = 0$ is

Watch Video Solution

75. IF $y = \frac{1}{1 + x + x^2 + x^3}$, then value of $\frac{d^2y}{dx^2}$ at $x = 0$ is

Watch Video Solution

76. If $y = \log(x + 5), x \neq -5$ then $\frac{dy}{dx}$=

Watch Video Solution

77. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$?

Watch Video Solution

78. If $x = e^t \cos t, y = e^t \sin t$ then $\frac{dy}{dx}$=_____

Watch Video Solution

79. If $\phi(x) = \begin{vmatrix} 1 & 2x & 3x^2 \\ x & x^2 & x^4 \\ 0 & 2 & 6x \end{vmatrix}$ then $\phi'(1)$=

Watch Video Solution

80. The value of $\left(\frac{dy}{dx}\right)_{(1,1)}$ for the curve $2y = 3 - x^2$ is

A. 1

B. -1

C. 0

D. 2

Answer:

Watch Video Solution

81. Let $y = \log_e \sin\left(x^2\right), 0 < x \leq \frac{\pi}{2}$. The value of $\frac{dy}{dx}$ at $x = \sqrt{\frac{\pi}{2}}$ is

A. 0

B. 1

C. $\frac{\pi}{4}$

D. $\sqrt{\pi}$

Answer: A

Watch Video Solution

82. If $f(x) = \log_e x$ then the differential coefficent of $f(\log_e x)$ with respect to x is

A. $\frac{x}{\log_e x}$

B. $\frac{\log_e x}{x}$

C. $\frac{1}{x \log_e x}$

D. none of these

Answer: C

Watch Video Solution

83. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^2 \frac{d^2 y}{dx^2}\right)$ is equal to $P^x + P'(x)$ (b) $P^x \dot{P}^x$ $P(x)\dot{P}^x$ (d) a constant

A. $P'''(x) + P'(x)$

B. $P(\mathrm{x}).\mathrm{P}'(x)$

C. $P(x).\ P'(x)$

D. a constant

Answer:

Watch Video Solution

84. If $f(x) = \tan^{-1}\left\{\frac{1}{x}\left(\sqrt{1+x^2}\right)\right\}$ then $f'(0)$ is

A. -1

B. 0

C. 1

D. does not exist

Answer: A

Watch Video Solution

85. If $t = e^x$ and $y = t^2 - 1$ then $\left(\frac{dy}{dx}\right)$ at $t = 1$ is

Watch Video Solution

86. $x^3 - 2x^2y^2 + 5x + y - 5 = 0,$ then find $\frac{d^2y}{dx^2}$ at $x = 1, y = 1$

A. -30.444444444444

B. -25.75

C. 8

D. 22/7

Answer:

Watch Video Solution

87. Let $f(x) = \left|x^3 \sin x \cos x 6 - 10pp^2p^3\right|, wherep$ is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is p (b) $p - p^3$ $p + p^3$ (d) independent of p

A. p

B. $p + p^2$

C. $p + p^3$

D. independent of p

Answer:

Watch Video Solution

88. The derivative of an even function is always an odd function.

Watch Video Solution

89. If both $f(x)$ and $g(x)$ are non-differentiable at $x = a$ then $f(x) + g(x)$ may be differentiable at $x = a$

Watch Video Solution

90. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, find $\frac{dy}{dx}$.

Watch Video Solution

91. Q.14 Let f(x) = (ax + b) cosx + (cx + d) sinx andf'(x) = x cosx for all xer, then(a) a = -1(b) b=1(c) c= 1(d) d = -1

Watch Video Solution

92. If $f(x) = |x-1|$ and $g(x) = f(f(f(x)))$ then for $x > 2$, $g'(x) =$

Watch Video Solution

93. $\frac{d\left(x^{\log x}\right)}{d(\log x)} =$

Watch Video Solution

94. Find $\frac{dy}{dx}$ in terms of x if $t = \frac{x}{1+x^2}, y = x^2 + t^2.$

Watch Video Solution

95. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then g'(x) is equal to

Watch Video Solution

96. If $y = f(x), p = \frac{dy}{dx}$ and $q = \frac{d^2y}{dx^2}$, then what is $\frac{d^2x}{dy^2}$ equal to ?

Watch Video Solution

97. Let $f(x+y) = f(x) + f(y)$ for all real x, y and $f'(0)$ exists. Prove that $f'(x) = f'(0)$ for all $x \in R$ and $2f(x) = xf(2).$

Watch Video Solution

98. Transform the differential equation $(1-x^2)\left(\frac{d^2y}{dx^2}\right) = x\left(\frac{dy}{dx}\right)$ by substituting $x = \cos t$.

Watch Video Solution