MATHS

NCERT - FULL MARKS MATHS(TAMIL)

RELATIONS AND FUNCTIONS

Example

1. Let A be the set of all students of a boys school. Show that the relation R in A given by $R = \{(a, b) : a$ is sister of b$\}$ is the empty relation and $R' = \{(a, b) :$ the difference between heights of a and b is less than 3 meters $\}$ is the universal relation.

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2. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1, T_2) : T_1$ is congruent to $T_2\}$ Show that R is an equivalence relation.

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3. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is perpendicular to L_2 }. Show that R is symmetric but neither reflexive nor transitive.

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4. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive.

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5. Show that the relation R in the set Z of intergers given by

$R = \{(a, b) : 2$ divides a-b }

is an equivalence relation.

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6. Let R be the realtion defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) :$ both a and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

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7. Let A be the set of all 50 students of Class X in a school Let $f : A \to N$ be function defined by $f(x) =$ roll number of the student x. Show that f in one-one but not onto.

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8. Show that the function $f: N \rightarrow N$, given by $f(x) = 2x$, is one-one but not onto.

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9. Prove that the function $f: R \rightarrow R$, given by $f(x) = 2x$, is one-one and onto.

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10. Show that the function $f: N \rightarrow N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one-one.

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11. Show that the function $f: R \to R$.

defined as $f(x) = x^2$, is neither one-one nor onto.

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12. Show that $f: N \to N$, given by

$f(x) = x + 1$, if x is odd,

$f(x) = x - 1$, if x is even

is both one-one and onto.

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13. Show that an onto function $f: \{1, 2, 3\} \to \{1, 2, 3\}$ is always one-one.

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14. Show that a one-one function $f: \{1, 2, 3\} \to \{1, 2, 3\}$ must be onto.

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15. Let $f: \{2, 3, 4, 5\} \to \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \to \{7, 11, 15\}$ be function defined as $f(2) = 3, f(3) = 4, f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g($

Find g (f (x))

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16. Find gof and fog, if $f: R \to R$ and $g: R \to R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that gof $\neq$ fog.

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17. Show that if $f: R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ is defined by $g(x) = \frac{7x+4}{5x-3}$, then fog $= I_A$ and gof $= I_B$, where,

$A = R - \left\{\frac{3}{5}\right\}, B = R - \left\{\frac{7}{5}\right\}, I_A(x) = x, \forall x \in A, I_B(x) = x, \forall x \in B$

are called identity functions on sets A and B, respectively.

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18. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one, then gof $: A \rightarrow C$ is also one-one.

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19. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto, then gof$: A \rightarrow C$ is also onto.

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20. Consider functions f and g such that composite gof is defined and is one one Are f and g both necessarily one-one.

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21. Are f and g both necessarily onto, if gof is onto ?

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22. Let $f: \{1, 2, 3\} \to \{a, b, c\}$ be one-one and onto function given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Show that there exists a function $g: \{a, b, c\} \to \{1, 2, 3\}$ such that gof $= I_x$ and fog $= I_y$, where, $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

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23. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$, where, $Y = \{y \in N: y = 4x + 3$ for some $x \in N\}$. Show that f is invertible. Find the inverse.

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24. Let $Y = \{n^2 : n \in N\} \subset N$. Consider $f : N \rightarrow Y$ as $f(n) = n^2$.

Show that f is invertible. Find the inverse of f.

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25. Let $f : N \rightarrow R$ be a function defined as $f'(x) = 4x^2 + 12x + 15$.

Show that $f : N \rightarrow S$. where, S is the range of f, is invertible. Find the inverse of f.

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26. Consider $f : N \rightarrow N, g : N \rightarrow N$ and $h : N \rightarrow R$ defined as $f(x) = 2x, g(h) = 3y + 4$ and $h(z = \sin z, \forall x, y$ and $z$in N. Show that h(gof) = (hog) of.

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27. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$ and $g: \{a, b, c\} \to$ {apple, ball, cat} defined as $f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a)$ = apple,$g(b) =$ ball and g (c)= cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(\text{gof})^{-1}$ and show that $(\text{gof})^{-1} = f^{-1} o g^{-1}$.

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28. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary opertion on R. Further, show that division is binary opertion on the set R, of nonzero real numbers.

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29. Show that subtraction and division are not binary opertions on N.

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30. Show that $+ : R \times R \to$ and $x : R \times R \to R$ are communtative binary opertions , but $- : R \times R \to R$ and $\div : R_* \times R_*$ are not commutative.

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31. Show that addition and multiplication are associative binary opertion on R. But substraction is not associative on R. Division is not associative on R_*.

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32. Show that $* R \times R \to R$ given by $a * b \to a + 2b$ is not associative.

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33. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the opertions $\div R \times R \to R$ and $\div : R_* \times R_* \to R_*$.

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34. Show that -a is not the inverse of $a \in N$ for the addition opertion + on N and $\frac{1}{a}$ is not the inverse of $a \neq N$ for multiplication opertion $\times$ on N , for $a \neq 1$.

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35. If R_1 and R_2 are equivalence rrelations in a set A show that $R_1 \cap R_2$ is also an equivalence relation.

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36. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by

$R_1 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}\}$ or $\{x, y\} \subset \{2, 5, 8\}$ or $\{x, y\} \subset \{3, 6, 9$

Show that $R_1 = R_2$.

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37. Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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38. Show that number of equivalence relation in the set $\{1, 2, 2\}$ containing $(1, 2)$ and $(2, 1)$ is two.

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39. Show that the number of binary opertions on $\{1, 2\}$ having 1 as identity and having 2 as the inverse of 2 is exactly one.

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40. Consider the identity function $I_N : N \to N$ defined as $I_N(x) = x \, \forall x \in N$. Show that although I_N is onto but $I_N + I_N : N \to N$ defined as

$(I_N + I_N)(x) = I_N(x) + I_N(x)x + x = 2x$ is not onto.

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41. Consider a function $f : \left[0, \frac{\pi}{2}\right] \to R$ given by $f(x) = \sin x$ and $g : \left[0, \frac{\pi}{2}\right] \to R$ given by $g(x) = \cos x$, Show that f and g are one-one but f + g is not one-one.

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Exercise 1 1

1. Determine w hether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as

$R = \{(x, y): 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as

$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\} as$

$R = \{(x, y): y$ is divisible by x}

(iv) Relation R in the set Z of allintegers defined as

$R = \{(x, y): x - y$ is an integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y): x \text{ and }$ y work at the same place}

(b) $R = \{(x, y): x \text{ and } y$ live in the same locality]

(c) $R = \{(x, y): x$ is exactly 7 cm taller than y}

(d)$R = \{(x, y): x$ is wife of y}

(e) $R = \{(x, y): x$ is father of y}

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2. Check whether the realtion R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

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3. Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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4. Show that each of the relation R in set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |a - b|$ is a multiple of 4}

(ii) $R = \{(a, b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

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5. Show that the relation R defined in the set A of all triangles as $R = \{T_1 \quad T_2) : T_1$ is similar to $T_2\}$ is equivalence relation. Consider three right angle triangles T_1 with sides $3, 4, 5, T_2$ with sides $5, 12, 13$ and T_3 with sides $6, 8, 10.$ Which triangles among T_1, T_2 and T_3 are related ?

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6. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5 ?

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7. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

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8. Let R be the relation in the set $\{(1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1)(4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.

A. R is reflexive symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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9. Let R be the relation in the set N given by $R = \{(a, b), a = b - 2, b > 6\}$. Choose the correct answer.

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $((8, 7) \in R$

Answer: B

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Exercise 1 2

1. Show that the function $f: R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero numbes. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_* ?

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2. Check the injectivty and surjectiveity of the following functions :

(i) $f: N \rightarrow N$ given by $f(x) = x^2$

(ii) $f: Z \rightarrow Z$ given by $f(x) = x^2$

(iii) $f: R \rightarrow R$ given by $f(x) = x^2$

(iv) $f: N \rightarrow N$ given by $f(x) = x^3$

(v) $f: Z \rightarrow Z$ given by $f(x) = x^3$

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3. In each of the following cases, state whether the function is one-one onto or bijective. Justify your answwer.

(i)$f: R \rightarrow R$ defined by $f(x) = 3 - 4x$

(ii) $f: R \rightarrow R$ defined by $f(x) = 1 + x^2$

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4. Let $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$.

State whether the function f is bijective. Justify your answer.

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5. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

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6. Let $f: R \rightarrow R$ be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer: D

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7. Let $f: R \rightarrow R$ be defined as $f(x) = 3x.$ Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer: A

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Exercise 1 3

1. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Find gof and fog, if

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$.

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3. State with reason wheher following functins have inverse

(i) $f: \{1, 2, 3, 4\} \rightarrow \{10\}$ with

$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h:\{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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4. Show that $f:[-1, 1] \rightarrow R$, given by $f(x) = \frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f:[-1, 1] \rightarrow$ Rangle f.

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5. Consider $f:R \rightarrow R$ given by $f(x) = 4x = 4x + 3$. Show that f is invertible. Find the inverse of f.

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6. Consider $f:\{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a, f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

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7. If $f: R \rightarrow R$ be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$, then fof (x) is

A. $x^{\frac{1}{3}}$

B. x^3

C. x

D. $\left(3 - x^3\right)$.

Answer: C

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8. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g: Range $f \rightarrow R\left\{-\frac{4}{3}\right\}$ given by

A. $g(y) = \frac{3y}{3 - 4y}$

B. $g(y) = \frac{4y}{4 - 3y}$

C. $g(y) = \frac{4y}{3 - 4y}$

D. $g(g) = \frac{3y}{4 - 3y}$

Answer: B

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Exercise 1 4

1. Determine whether or not each of the defination of $*$ given below gives a binary opertion. In the even that $*$ is not a binary opertion, give justification for this.

(i) On Z^+, define $*$ by $a * b = a - b$

(ii) On Z^+, define $*$ by$a * bb = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^+, define $*$ by $a * b = |a - b|$

(v) On Z^+, define $*$ by $a * b = a$

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2. For each opertion $*$ difined below, determine whether $*$ isw binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^+, define $a * b = 2^{ab}$

(v) On Z^+, define $a * b = a^b$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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3. Consider the binary opertion $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ min $\{a, b\}$. Write the opertion table of the opertion $\wedge$.

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4. Let $*'$ be the binary opertion on the set $\{1, 2, 3, 4, 5\}$ defined by $a\&\&'$ b = H.C.F. of a and b. Is the opertion $*'$ same as the opertion $*$ defined in above ? Your answer

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5. Let $*$ be the binary opertion on N given by $a * b = L. C. M.$ of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $*$ commutative ?

(iii) Is $*$ associative ?

(iv) Find the identity of $*$?

(v) Which elements of N are invertible for the opertion $*$?

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6. Is $*$ defined on the set $\{1, 2, 3, 4, 5\} by a * b = L. C. M.$ of a and b a binary opertion ? Justified your answer.

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7. Let $*$ be the binary opertion on N defined by $a * b = H.C.F.$ of a and b. Is $*$ commutative ? Is $*$ associative ? Does there exist identity for this binary opertion on N ?

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8. Let $*$ be a binary opertion on the set Q of rational numbers as follows:

(i) $a \star b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

Find which of the binary opertions are commutative and which are associative.

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9. Find which of the opertions given above has identity.

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10. Let $A = N \times N$ and $*$ be the binary opertion on A defined by

$(a, b) * (c, d) = (a + c, b + d)$

Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

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11. State whether the following statements are true or false, Justify.

(i) For an arbitraty binary opertion $*$ on a set $N, a * a = a \forall a \in N$.

(ii) If $*$ is a commutative binary opertion on N, then $a * (b * c) = (c * b) \cdot 8a$

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12. Consider a binary opertion $*$ on N defined as $a * b = a^3 + b^3$.

Choose the correct answer.

A. Is $*$ both associative and commutative ?

B. Is $*$ commutative but not associative ?

C. Is $*$ associative but not commutative ?

D. Is $*$ neither comutative nor associative ?

Answer: B

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Miscllellaneous Exercise On Chapter 1

1. Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = f0g = 1_R$.

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2. Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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4. Given a non empty set X, consider P (X) which is the set of all subsets of X.

Define the relation R in P(X) as follows :

For subsets A, B in P(X), ARB if and only if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

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5. Find the number of all onto functins from the set $\{1, 2, 3. . , n\}$ to itself.

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6. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following F from S to T, if exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$

(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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7. Consider the binary opertions $* : R \times R \rightarrow R$ and $o : R \times R \rightarrow R$ defined as $a * b = |a - b|$ and $aob = a, \forall a, b \in R$. Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that $\forall a, b, c \in R, a * (boc) = (a * b)o(a * c)$. [If it is so, we say that the opertion $*$ distributes over the opertion 0]. Does o distribute over $*$? Justify your answer.

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8. Let $A = \{-1, 0, 1, 2\}, B = \{-4, -2, 0, 2\}$ and $f, g, A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$ Are f and g equal ? Justify your answer.

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9. Let $A = \{1, 2, 3\}$. Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive ans symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

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10. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing $(1, 2)$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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11. Let $f: R \to R$ be the Signumb Function defined as

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

and $g: Ro \to R$ be the Greatest Integer Function given by $g(x) = [x]$, where $[x]$ is greatest integer less than or equal to x. Then does fog and gof coincide in $(0, 1]$?

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12. Number of binary opertions on the set $\{a, b\}$ are

A. 10

B. 16

C. 20

D. 8

Answer: B

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