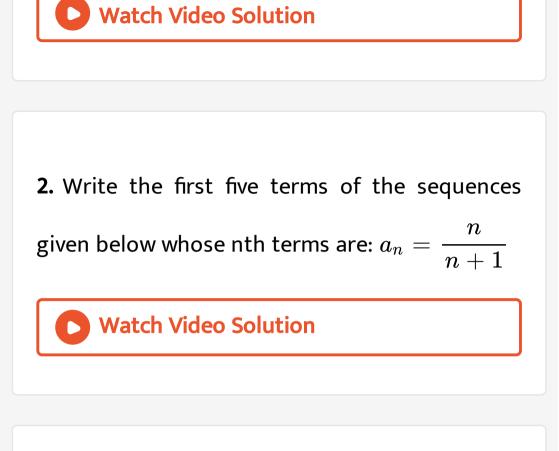# MATHS

BOOKS - PSEB

SEQUENCES AND SERIES

Exercise

1. Write the first five terms of the sequences given below whose nth terms are:

$a_n = n(n+2)$

2. Write the first five terms of the sequences given below whose nth terms are: $a_n = \dfrac{n}{n+1}$
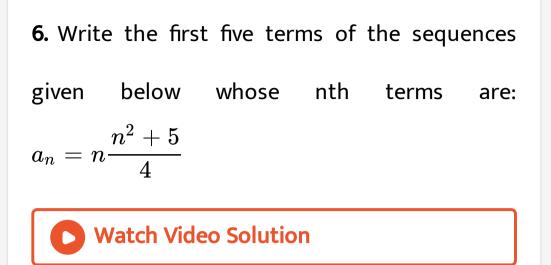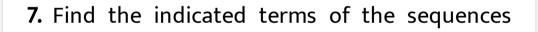
3. Write the first five terms of the sequences given below whose nth terms are: $a_n = 2^n$

4. Write the first five terms of the sequences given below whose nth terms are: $a_n = \dfrac{2n - 3}{6}$
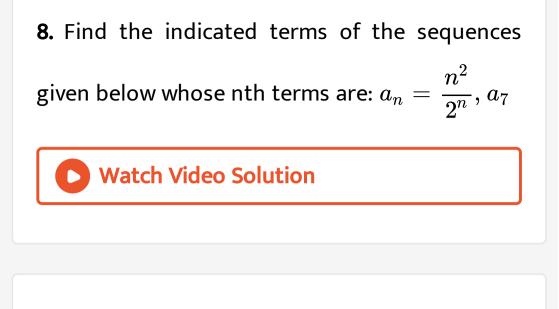
5. Write the first five terms of the sequences given below whose nth terms are:

$a_n = (-1)^{n-1}5^{n+1}$

6. Write the first five terms of the sequences given below whose nth terms are:

$$a_n = n\frac{n^2 + 5}{4}$$

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7. Find the indicated terms of the sequences given below whose nth terms are:

$$a_n = 4n - 3, \ a_{17}, \ a_{24}$$

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8. Find the indicated terms of the sequences given below whose nth terms are: $a_n = \dfrac{n^2}{2^n}$, a_7

9. Find the indicated terms of the sequences given below whose nth terms are:

$a_n = (-1)^{n-1} n^3$, a_9

10. Find the indicated terms of the sequences given below whose nth terms are:

$$a_n = \frac{n(n-2)}{n+3}, \ a_{20}$$

11. Write the first five terms of the sequences given below and obtain the corresponding series: $a_1 = 3, a_n = 3a_{n-1} + 2 \ \forall n > 1$

12. Write the first five terms of the sequences given below and obtain the corresponding series: $a_1 = -1, a_n = \dfrac{a_{n-1}}{n}, n \geq 2$

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13. Write the first five terms of the sequences given below and obtain the corresponding series: $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

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14. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}$, $n > 2$.

Find $\dfrac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

15. Find the sum of odd integers from 1 to 2001.

(A) 100200

(B) 1002001

(C)1000201

(D) 100002

16. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

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17. In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is -112.

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18. How many terms of the A.P. -6, $-\dfrac{11}{2}$, -5 , ...

are needed to give the sum 25?

19. In an A.P., if p^{th} term is $\dfrac{1}{q}$ and q^{th} term is $\dfrac{1}{p}$,

prove that the sum of first pq terms is

$\dfrac{1}{2}(pq + 1)$, where $p \neq q$.

20. If the sum of a certain number of terms of the A.P. 25, 22, 19, .. . is 116. Find the last term.

21. Find the sum to n terms of the A.P., whose k^{th} term is $5k + 1$.

22. If the sum of n terms of an A.P. is $\left(pn + qn^2\right)$, where p and q are constants, find the common
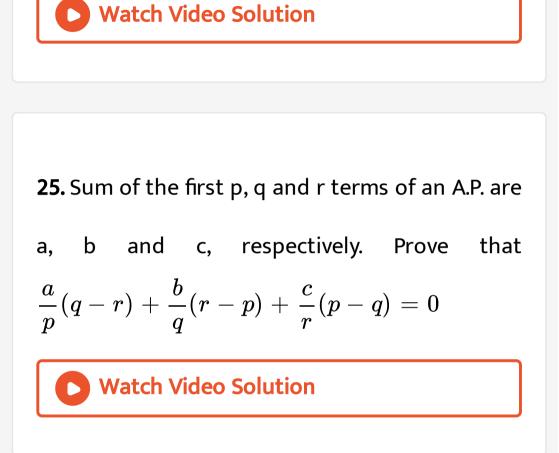
difference.

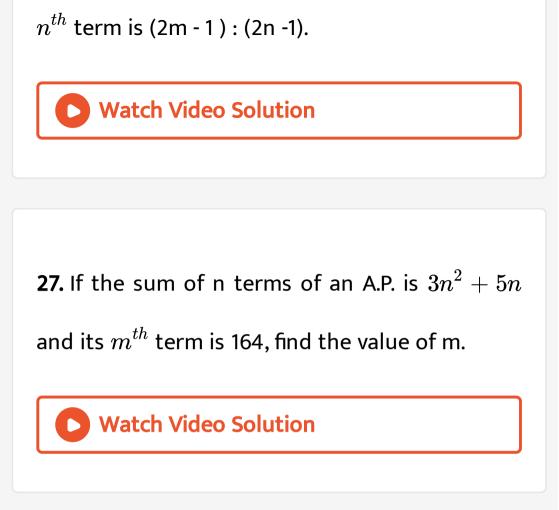**23.** The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18th terms.

24. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the

sum of the first (p+ q) terms.

25. Sum of the first p, q and r terms of an A.P. are a, b and c, respectively. Prove that

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$$

26. The ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio of mth and

n^{th} term is (2m - 1) : (2n -1).

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27. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its mth term is 164, find the value of m.

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28. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

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29. If $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

30. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7th and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m.

31. A man starts repaying a loan as first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30th instalment?

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32. The difference between any two consecutive interior angles of a polygon is $5°$. If the smallest angle is $120°$, find the number of the sides of the polygon.

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33. Find the 20th and nth terms of the G.P.

$$\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots..$$

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34. Find the 12th term of a G.P. whose 8th term

is 192 and the common ratio is 2.

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35. The 5th, 8th and 11th terms of a G.P. are p, q and x, respectively. Show that $q^2 = ps$.
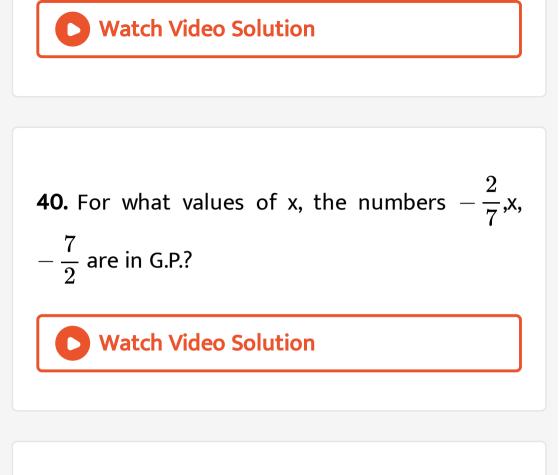
36. The 4th term of a G.P. is square of its second term, and the first term is-3. Determine its 7th term.

37. Which term of the following sequence:-

$2, 2\sqrt{2}, 4,.....$is 128?

38. Which term of the following sequence:-

$\sqrt{3}, 3, 3\sqrt{3},....$is 729?

39. Which term of the following sequence:-

$\dfrac{1}{3}, \dfrac{1}{9}, \dfrac{1}{27},.....$is $\dfrac{1}{19683}$?

40. For what values of x, the numbers $-\dfrac{2}{7}$, x, $-\dfrac{7}{2}$ are in G.P.?

41. Find the sum to indicated number of terms of the geometric progressions given below:-

0.15, 0.015, 0.0015, ... 20 terms.

42. Find the sum to indicated number of terms of the geometric progressions given below:-
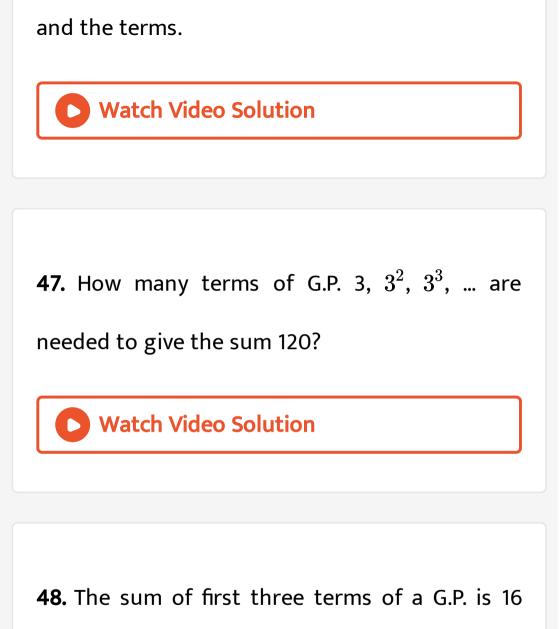
$\sqrt{7}, \sqrt{21}, 3\sqrt{7},.....$n terms.

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43. Find the sum to indicated number of terms of the geometric progressions given below:-

$1, -a, a^2, -a^3, ...$ n terms (if $a \neq -1$).

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44. Find the sum to indicated number of terms of the geometric progressions given below:-

$x^3, x^5, x^7, \ldots$ n terms(if $x \neq \pm 1$).

45. Evaluate $\sum\limits_{k=1}^{11} \left(2 + 3^k\right)$.

46. The sum of first three terms of a G.P. is $\dfrac{39}{10}$ and their product is 1 . Find the common ratio

and the terms.

47. How many terms of G.P. 3, 3^2, 3^3, ... are needed to give the sum 120?

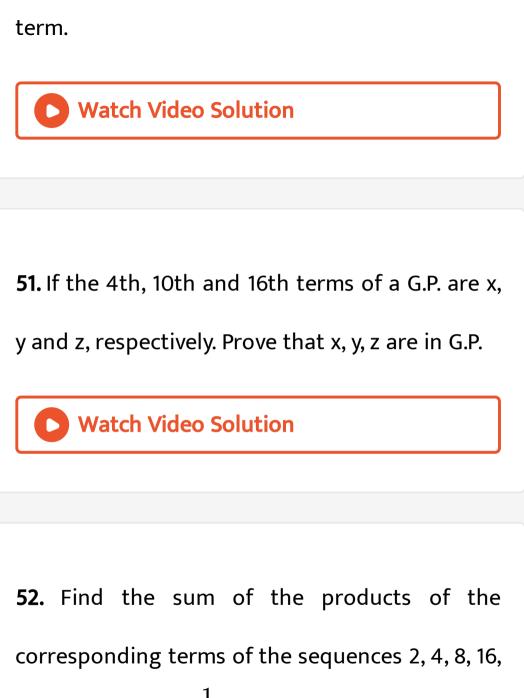**48.** The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128.

Determine the first term, the common ratio and the sum to n terms of the G.P.

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49. Given a G.P. with a= 729 and 7th term 64, determine S_7.

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50. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third

term.

51. If the 4th, 10th and 16th terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

52. Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and $128, 32, 8, 2, \dfrac{1}{2}$.
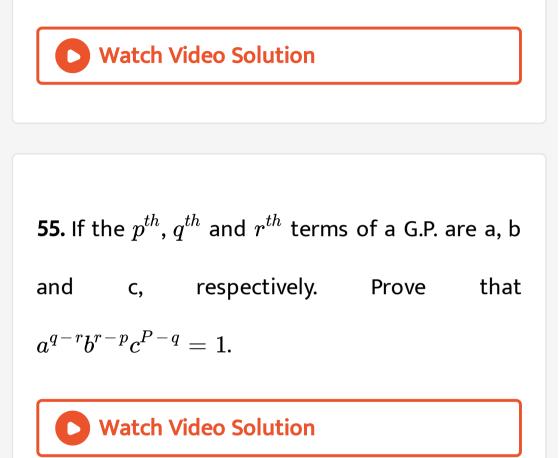
53. Show that the products of the corresponding terms of the sequences a, ar, ar^2 ,...ar^{n-1} and A, AR, AR^2,.....AR^{n-1} form a G.P, and find the common ratio.

54. Find four numbers forming a geometric progression in which the third term is greater

than the first term by 9, and the second term is greater than the 4th by 18.

55. If the pth, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{P-q} = 1$.

56. If the first and the nth term of a G.P. are a and b. respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

57. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from (n+ 1)th to (2n)th term is $\dfrac{1}{r^n}$.

58. If a, b, c and d are in G.P. show that $\left(a^2 + b^2 + c^2\right)\left(b^2 + c^2 + d^2\right) = (ab + bc + cd)^2$

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59. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

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60. Find the value of n so that $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

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61. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$.

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62. If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}.$

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63. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour ?

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64. What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

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65. If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

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66. Find the sum to n terms of the series given below:- $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + ...$
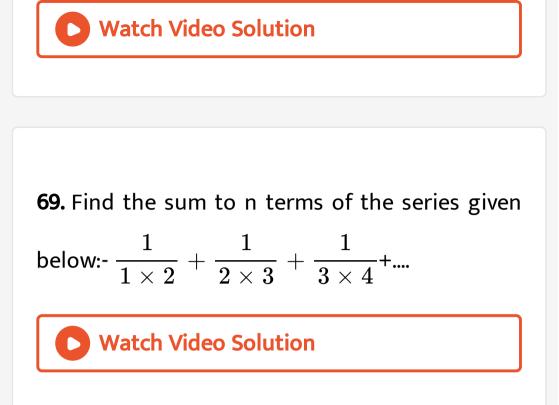
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67. Find the sum to n terms of the series given below:- $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + ...$

▶ **Watch Video Solution**

68. Find the sum to n terms of the series given below:- $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + ...$

69. Find the sum to n terms of the series given below:- $\dfrac{1}{1 \times 2} + \dfrac{1}{2 \times 3} + \dfrac{1}{3 \times 4} + ...$

70. Find the sum to n terms of the series given below:- $5^2 + 6^2 + 7^2 + ... + 20^2$

71. Find the sum to n terms of the series given below whose nth terms is given by $n(n+1)(n+4)$.

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72. Find the sum to n terms of the series given below whose nth terms is given by $n^2 + 2^n$

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73. Find the sum to n terms of the series given below whose nth terms is given by $(2n - 1)^2$

74. Show that the sum of $(m + n)^{th}$ and $(m - n)^{th}$ terms of an A.P. is equal to twice the mth term.

75. If the sum of three numbers in A.P., is 24 and their product is 440, find the numbers.

76. Let the sum of n, 2n, 3n terms of an A.P. be S_1, S_2 and S_3, respectively, show that $S_3 = 3(S_2 - S_1)$

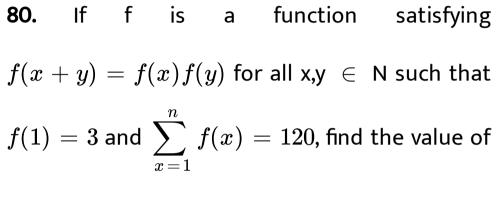**77.** Find the sum of all numbers between 200 and 400 which are divisible by 7.

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78. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

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79. Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

80. If f is a function satisfying $f(x + y) = f(x)f(y)$ for all x,y $\in$ N such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$, find the value of n.

81. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2,

respectively. Find the last term and the number of terms.

82. The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.

83. The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

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84. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

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85. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

86. If $\dfrac{a + bx}{a - bx} = \dfrac{b + cx}{b - cx} = \dfrac{c + dx}{c - dx} (x \neq 0),$ then show that a, b, c and d are in G.P.

87. Let S be the sum, P the product and R the sum of reciprocals of n terms in a G.P. Prove that $P^2 R^n = S^n$.

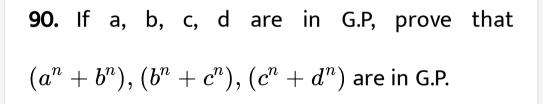**88.** The pth, q^{th} and r^{th} terms of an A.P. are a, b, c, respectively.   Show   that

$$(q - r)a + (r - p)b + (p - q)c = 0$$

89. If $a\left(\dfrac{1}{b} + \dfrac{1}{c}\right), b\left(\dfrac{1}{c} + \dfrac{1}{a}\right), c\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

are in A.P, prove that a, b, c are in A.P.

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90. If a, b, c, d are in G.P, prove that
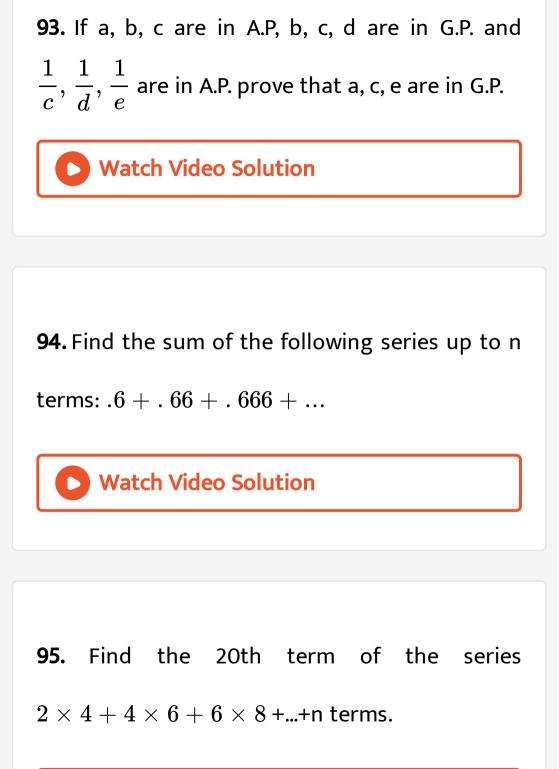
$(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

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91. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a GP. Prove that (q + p) : (q -p)= 17:15.

92. The ratio of the A.M. and G.M. of two positive numbers a and b, is m : n. Show that $a:b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

93. If a, b, c are in A.P, b, c, d are in G.P. and $\dfrac{1}{c}, \dfrac{1}{d}, \dfrac{1}{e}$ are in A.P. prove that a, c, e are in G.P.

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94. Find the sum of the following series up to n terms: $.6 + .66 + .666 + ...$

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95. Find the 20th term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + ... + n$ terms.

96. Find the sum of the first n terms of the series: $3 + 7 + 13 + 21 + 31 + \ldots$
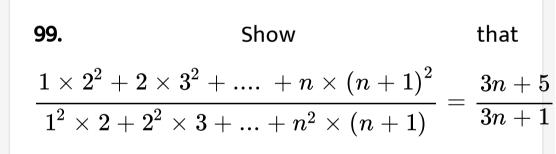
97. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

98. Find the sum of the following series up to n terms: $\dfrac{1^3}{1} + \dfrac{1^3 + 2^3}{1 + 3} + \dfrac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + ...$

99. Show that

$$\frac{1 \times 2^2 + 2 \times 3^2 + ... + n \times (n + 1)^2}{1^2 \times 2 + 2^2 \times 3 + ... + n^2 \times (n + 1)} = \frac{3n + 5}{3n + 1}$$

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100. A farmer buys a used tractor for Rs 12000. He pays Rs 6000 cash and agreesto pay the balance in annual instalments of Rs 500 plus 12% interest on the unpaid amount. How much will the tractor cost him?

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101. Shamshad Ali buys a scooterfor Rs 22000. He pays Rs 4000 cash and agreesto pay the balance in annual instalment of Rs 1 000 plus

10% interest on the unpaid amount. How much will the scooter cost him?

102. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8th set of letter is mailed.

103. Aman deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15th year since he deposited the amount and also calculate the total amount after 20 years.

104. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will

depreciate each year by 20%. Find the estimated value at the end of 5 years.

105. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.