PHYSICS

BOOKS - PSEB

Systems of Particles and Rotational Motion

Exercise

1. Give the location of the centre of mass of a :-

sphere.

2. Give the location of the centre of mass of a :- cylinder.

3. Give the location of the centre of mass of a :- ring.

4. Give the location of the centre of mass of a

:- cube, each of uniform mass density. Does the

centre of mass of a body necessarily lie inside

the body ?

5. In the HC1 molecule, the separation between

the nuclei of the two atoms is about $1.27\overset{\circ}{A}$ (

$1\overset{\circ}{A} = 10^{-10}m$). Find the approximate location

of the CM of the molecule, given that a

chlorine atom is about 35.5 times as massive

as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

6. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system ?

7. Show that the area of the triangle contained between the vectors a and b is one half of the magnitude of $a \times b$.

8. Show that $a. (b \times c)$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors , a, b and c.

9. Find the components along the x, y, z axes of the angular momentum 1 of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

10. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

11. A non-uniform bar of weight W is suspended at rest by two strings of negligible

weight as shown in Fig.7.39. The angles made by the strings with the vertical are $36.9°$ and $53.1°$ respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.

12. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level

ground on each front wheel and each back wheel.

13. Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

14. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

15. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. Hie cylinder

is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

16. A solid cylinder of mass 20 kg rotates about its axis with angular speed $100 rads^{-1}$. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of

the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

17. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of $40 rev/\min$. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to $2/5$

times the initial value ? Assume that the turntable rotates without friction.

18. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping.

19. To maintain a rotor at a uniform angular speed of $200 \, rads^{-l}$, an engine needs to transmit a torque of 180 N m. What is the power required by the engine ? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

20. From a uniform disk of radius R, a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locale the centre of gravity of the resulting flat body.

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21. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm

mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

22. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination:- Will it reach the bottom with the same speed in each case?

23. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination:- Will it take longer to roll down one plane than the other?

24. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination:- Will it reach the bottom with the same speed in each case?

25. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of $20 cm/s$. How much work has to be done to stop it?

26. The oxygen molecule has a mass of $5.30 \times 10^{-26} kg$ and a moment of inertia of $1.94 \times 10^{-46} kgm^2$ about an axis through its

centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is $500 m/s$ and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

27. A solid cylinder rolls up an inclined plane of angle of inclination $30°$. At the bottom of the inclined plane the centre of mass of the

cylinder has a speed of $5m/s$:- How far will the cylinder go up the plane?

28. A solid cylinder rolls up an inclined plane of angle of inclination $30°$. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of $5m/s$:- How long will it take to return to the bottom?

29. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to $7.6 kgm^2$:- What is his new angular speed? (Neglect friction.)

30. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to $7.6kgm^2$:- Is kinetic energy conserved in the

process? If not, from where does the change come about?

31. A bullet of mass 10 g and speed $500m/s$ is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.

32. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

33. Prove the theorem of parallel axes.

**34.** Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by $v^2 = \dfrac{2gh}{1 + k^2 / R^2}$ using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

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35. A disc rotating about its axis with angular speed ω_0 is placed lightly (without any translational push) 011 a perfectly frictionless table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig. 7.41? Will the disc roll in the direction indicated ?
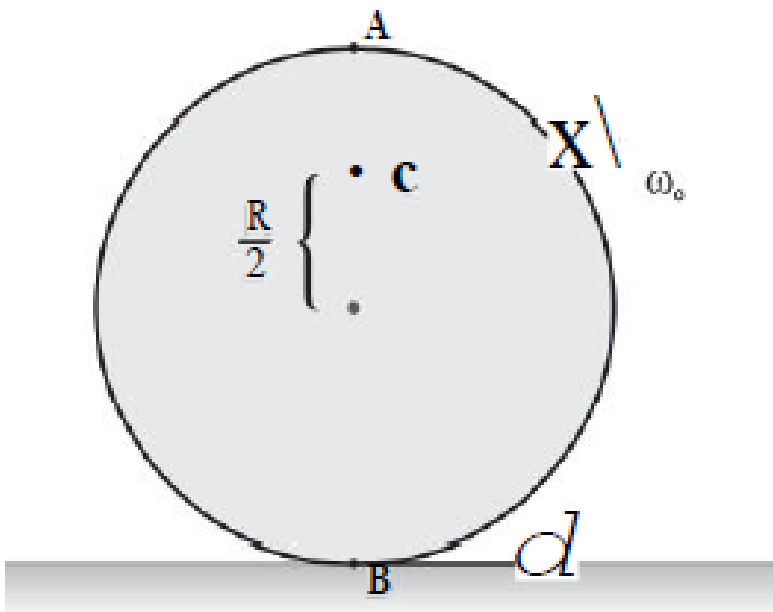
36. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10\pi\, rads^{-1}$. Which of the two will start to roll earlier ? The co-efficient of kinetic friction is $\mu_k = 0.2$.

37. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination $30°$. The co-efficient of static friction $\mu s = 0.25$:- How much is the force of friction acting on the cylinder ?

38. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination $30°$. The co-efficient of static friction $\mu s = 0.25$:-

What is the work done against friction during rolling ?

39. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination $30°$. The co-efficient of static friction $\mu s = 0.25$:- If the inclination θ of the plane is increased, at what value of θ does the cylinder begin to skid, and not roll perfectly ?

40. Read each statement below carefully, and state, with reasons, if it is true or false :-

During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

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41. Read each statement below carefully, and state, with reasons, if it is true or false :- The

instantaneous speed of the point of contact during rolling is zero.

42. Read each statement below carefully, and state, with reasons, if it is true or false :- The instantaneous acceleration of the point of contact during rolling is zero.

43. Read each statement below carefully, and state, with reasons, if it is true or false :- For perfect rolling motion, work done against friction is zero.

44. Read each statement below carefully, and state, with reasons, if it is true or false :- A wheel moving down a perfectly frictionless

inclined plane will undergo slipping (not rolling) motion.

45. Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass :- Show

(dL')/(dt)=(sumr_i') XX (dp')/(dt)

$$Further, showt\dfrac{\widehat{dL'}}{dt} = \text{tau'\_(ext)} where$$

tau'_(ext)` is the sum of all external torques

acting on the system about the centre of mass.

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