MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Ellipse and Hyberbola

Example

1. Find the equation to the ellipse whose foci are $(4, 0)$ and $(-4, 0)$ and eccentricity is $\frac{1}{3}$.

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2. Find the equation of the hyperbola whose one focus is $(-1, 1)$, eccentricity = 3 and the equation of the corresponding directrix is $x - y + 3 = 0$.

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3. Find the latus rectum, eccentricity, coordinates of the foci and the length of axes

of that ellipses : $9x^2 + 5y^2 - 30y = 0$

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4. Find the centre, eccentricity, foci and directrices of the hyperbola : $16x^2 - 9y^2 + 32x + 36y - 164 = 0$

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5. Find the centre and eccentricity of the ellipse $4(x - 2y + 1)^2 + 9(2x + y + 2)^2 = 5.$

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6. If angle subtended by any chord of a rectangular hyperbola at the centre is α and angle between the tangents at ends of chord is β, then

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7. If $\left(0, 3+\sqrt{5}\right)$ is a point on the ellipse whose foci and (2, 3) and $(-2, 3)$, then the

length of the semi - major axis is

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8. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If $F_1 \& F_2$ are the two foci of the ellipse, then show the $(PF_1 - PF_2)^2 = 4a^2\left[1-\frac{b^2}{d^2}\right]$.

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9. An ellipse has eccentricity $\frac{1}{2}$ and one focus at the point $P\left(\frac{1}{2}, 1\right)$. Its one directrix is the comionand tangent nearer to the point the P to the hyperbolaof $x^2 - y^2 = 1$ and the circle $x^2 + y^2 = 1$.Find the equation of the ellipse.

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10. If the tangent drawn at point $\left(t^2, 2t\right)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$ on

the ellipse $4x^2 + 5y^2 = 20$, then

$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$t = -\frac{2}{\sqrt{5}}$ (d) $t = -\frac{1}{\sqrt{5}}$

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11. The line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

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12. If the normals at four points $P(x_i y_i), i = 1, 2, 3, 4$ on the rectangular hyperbola $xy = c^2$, meet at the point Q(h, k), prove that

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13. Find the equation of that diameter which bisects the chord $7x + y - 2 = 0$ of the hyperbola $\frac{x^2}{3} - \frac{y^2}{7} = 1.$

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14. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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15. A point P moves such that the tangents PT_1 and PT_2 from it to the hyperbola $4x^2 - 9y^2 = 36$ are mutually perpendicular. Find the locus of P.

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16. From points on the circle $x^2 + y^2 = a^2$ tangents are drawn to the hyperbola $x^2 - y^2 = a^2$. Then, the locus of mid-points of the chord of contact of tangents is:

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17. A tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $PandQ$.

Show that the locus of the midpoint of PQ is

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2}{a^2}-\frac{y^2}{b^2}.$$

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Exercise

1. Find the equation to the ellipse whose one vertex is (3,1), the nearer focus is (1, 1) and eccentricity is 2/3.

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2. Find the equation of the ellipse whose centre is (-2,3) and whose semi axis are 3 and 2 when major axis is i. parallel to x-axis ii. parallel to y-axis.

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3. Find the equation of the hyperbola whose foci are $(6, 4) and (-4, 4)$ and eccentricity is 2. Vertices are $(-8, -1) and (16, -1)$ and focus is $(17, -1)$ foci are $(4, 2) and (8, 2)$ and

eccentricity is 2. vertices are at (0 ± 7) and foci at $\left(0, \pm \frac{28}{3}\right)$. vertices are at $(\pm 6, 3)$ and one of the directrices is $x = 4$. Foci at $(\pm 2, 0)$ and eccentricity is 3/2.

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4. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its conjugate hyperbola, prove that $\frac{1}{e^2} + 1 + e'^2 = 1$

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5. Find the latus rectum, eccentricity and foci of the curve $4x^2 + 9y^2 - 8x - 36y + 4 = 0$

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6. Find the centre, eccentricity, foci and directrices of the hyperbola : $x^2 - 3y^2 - 2x = 8.$

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7. PQ is a chord of the ellipse through the centre. If the square of its length is the harmonic mean ofthe squares of lengths major and minor axes then its inclination with the major axis is

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8. Length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is

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9. If α and β are eccentric angles of the ends of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\frac{\tan\alpha}{2}.\frac{\tan\beta}{2}$ is (A) $\frac{1-e}{1+e}$ (B) $\frac{e+1}{e-1}$ (C) $\frac{e-1}{e+1}$ (D) none of these

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10. The hyperbola $\frac{x^2}{a^2}-\frac{y^2}{a^2}-\frac{y^2}{b^2}=1$ passes through the point of intersection of the lines

$7x + 13y - 87 = 0$ and $5x - 8y + 7 = 0$

and its latus rectum is $32\frac{\sqrt{2}}{5}$. Find a and b.

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11. PN is the ordinate of any point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\forall'$ is its transvers axis. If Q divides AP in the ratio $a^2 : b^2$, then prove that NQ is perpendicular to $A'P$.

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12. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

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13. Prove that the product of the perpendicular from the foci on any tangent to an ellipse is equal to the square of the semi-minor axis.

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14. Prove that if any tangent to the ellipse is cut by the tangents at the endpoints of the major axis at $TandT'$, then the circle whose diameter is $\top$ ' will pass through the foci of the ellipse.

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15. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P&Q.

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16. Find the equations of the tangents from the point (2,2) to the ellipse $4x^2 + 9y^2 = 36$. Also find the angle between the tangents.

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17. If the normal at an end of a latus rectaum of an ellipse passes through an extremity of the minor axis then the eccentricity of the ellispe satisfies .

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18. The locus of the point of intersection of tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the points whose eccentric angles differ by $\pi/2$, is

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19. If the normal at any point P on the ellipse cuts the major and mirror axes in G and g respectively and C be the centre of the ellipse, then

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20. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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21. A variable straight line of slope 4 intersects the hyperbola $xy = 1$ at two points. The locus

of the point which divides the line segment between these two points in the ratio $1:2$ is

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22. Prove that the locus of the middle-points of the chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which pass through a fixed point (α,β) is a hyperbola whose centre is $\left(\frac{\alpha}{2},\frac{\beta}{2}\right)$.

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23. A normal to the hyperbola $x^2 - 4y^2 = 4$ meets the x and y axes at A and B. The locus of the point of intersection of the straight lines drawn through A and B perpendicular to the x and y-axes respectively is

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24. A circle cuts two perpendicular lines so that each intercept is of given length. The

locus of the centre of the circle is conic whose eccentricity is

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25. A circle with centre $(3\alpha, 3\beta)$ and of variable radius cuts the rectangular hyperbola $x^2 - y^2 = 9a^2$ at the points P, Q, S, R. Prove that the locus of the triangle PQR is $(x - 2\alpha)^2 - (y - 2\beta)^2 = a^2$.

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26. The locus of the point of intersection of the tangents at the end-points of normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is

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27. Find the locus of the-mid points of the chords of the circle $x^2 + y^2 = 16$, which are tangent to the hyperbola $9x^2 - 16y^2 = 144$

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28. If α and β be the angles subtended by the major axis to an ellipse at the extremities of a pair of conjugate diameters then $\cot^2\alpha + \cot^2\beta$ is equal to

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29. The equation

$$x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$$

represents a _____having its centre at_____.

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30. The eccentricity of a rectangular hyperbola, is

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31. Equation of the ellipse whose focus is $(6, 7)$ directrix is $x + y + 2 = 0$ and $e = \frac{1}{\sqrt{3}}$ is-

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32. The latus rectum of an ellipse is 10 and the minor axis Is equal to the distnace betweent the foci. The equation of the ellipse is

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33. Find the equation of an ellipse hose axes lie along the coordinate axes, which passes through the point (-3,1) and has eccentricity equal to $\sqrt{2/5}$.

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34. if in a hyperbola the eccentricity is $\sqrt{3}$ and the distance between the foci is 9 then the equation of hyperbola in the standard form is:

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35. Show that the equation $x^2 - 2y^2 - 2x + 8y - 1 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes, eccentricity, latusrectum,

coordinates of foci and vertices, equations of the directrices of the hyperbola.

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36. Find the equations of normal to the parabola $y^2 = 4ax$ at the ends of the latus rectum.

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37. Let P be a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci F_1 and F_2. If A is the area of the $\triangle PF_1F_2$, then the maximum value of A is

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38. The number of maximum normals that can be drawn from any point to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, is

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39. If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose centre is C, meets the major and the minor axes at P and Q respectively then $\frac{a^2}{CP^2}+\frac{b^2}{CQ^2}$ is equal to

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40. Find the length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose middle point is $\left(\frac{1}{2},\frac{2}{5}\right)$.

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41. If the line $lx + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Then

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42. Show that $3x^2 - 3y^2 - 18x + 12y + 2 = 0$ represents a rectangular hyperbola. Find its centre foci and eccentricity.

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43. The point P on the ellipse $4x^2 + 9y^2 = 36$ is such that the are of the $PF_1F_2 = \sqrt{10}$ where F_1, f_2 are foci. Then P has the coordinates $\left(\frac{3}{\sqrt{2}}, \sqrt{2}\right)$ (b) $\left(\frac{3}{2}, 2\right)$ $\left(-\frac{3}{2}, -2\right)$ (d) $\left(-\frac{3}{\sqrt{2}} - \sqrt{2}\right)$

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44. If the tangent and normal to a rectangular hyperbola cut off intercepts a_1 and a_2 on one axis and b_1 and b_2 on the other, then

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45. The equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represents

A. a)(2,3)

B. b)(2,-3)

C. c)(-2,3)

D. d)(-2,-3)

Answer:

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46. If e is eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$(where,a $<$ b), then

A. a) $\frac{2a^2}{b}$

B. b) $\frac{2b}{a^2}$

C. c) $2a\left(1-e^2\right)$

D. d) $2b\left(1-e^2\right)$

Answer:

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47. Find the eccentric angle of a point on the ellipse $x^2+3y^2=6$ at a distance 2 units from the centre of the ellipse.

A. a) $\frac{\pi}{4}$

B. b) $\frac{5\pi}{4}$

C. c) $\frac{3\pi}{4}$

D. d) $\frac{7\pi}{4}$

Answer:

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48. Which of the following is an exterior point of the ellipse $16x^2 + 9y^2 - 16x - 32 = 0$?

A. $\left(\frac{1}{2}, 2\right)$

B. $\left(\frac{1}{4}, 1\right)$

C. $(3, -2)$

D. d)none of these

Answer: C

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49. The line $3x + 5y = k$ touches the ellipse $16x^2 + 25y^2 = 400$,if k is

A. a)$\pm\sqrt{5}$

B. b)$\pm\sqrt{15}$

C. c)± 25

D. d)none of these

Answer:

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50. Find the equaiton of the tangents to the hyperbola $x^2 - 2y^2 = 18$ which are perpendicular to the line $x - y = 0$.

A. a)x+y=3

B. b)x+y+3=0

C. c)$x + y = 3\sqrt{2}$

D. d)$x + y + 3\sqrt{2} = 0$

Answer:

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51. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x - 5y = 2$

is $(5, -2)$ (b) $(5, 2)$ (c) $(-5, 2)$ (d) $(-5, -2)$

A. a)(5,-2)

B. b)(5,2)

C. c)(-5,2)

D. d)(-5,-2)

Answer:

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52. if the ordinate of the point of contact be 2 then the equation of the tangent to $x^2 + 4y^2 = 25$ is

A. $3x + 8y = 25$

B. $8x + 3y = 25$

C. $8y - 3x = 25$

D. $3x - 8y = 25$

Answer: A, C

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53. A tangent to the ellipse $16x^2 + 9y^2 = 144$ making equal intercepts on both the axes is

A. a)y=x+5

B. b)y=x-5

C. c)y=-x+5

D. d)y=-x-5

Answer:

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54. If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point 0 sanormal to the circle $x^2 + y^2 - 8x - 4y = 0$ then θ is equal to

A. a)$\pi / 2$

B. b)$\pi / 4$

C. c)0

D. d)$-\pi / 4$

Answer:

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55. An ellipse having foci $(3, 1)$ and $(1, 1)$ passes through the point $(1, 3)$ has the eccentricity

A. a) $\sqrt{2} - 1$

B. b) $\sqrt{3} - 1$

C. c) $\frac{\sqrt{2} - 1}{2}$

D. d) $\frac{\sqrt{3} - 1}{2}$

Answer:

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56. If P & Q are the ends of a pair of conjugate diameters & C is the centre of the ellipse $4x^2 + 9y^2 = 36$. Then the area of $\triangle CPQ$ is:

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57. If any point on a hyperbola is $(3\tan\theta, 2\sec\theta)$ then eccentricity of the hyperbola is

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58. If the normal to the rectangular hyperbola $xy = c^2$ at the point $'t'$ meets the curve again at t_1 then $t^3 t_1,$ has the value equal to

A. a)1

B. b)c

C. c)-c

D. d)-1

Answer:

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59. Equation

$$(2+\lambda)x^2 - 2\lambda xy + (\lambda - 1)y^2 - 4x - 2 = 0$$

represents a hyperbola if

A. a)$(-2, +\infty)$

B. b)$(-\infty, -2)$

C. c)$\left(-2, \frac{2}{7}\right) \cup \left(\frac{2}{7}, +\infty\right)$

D. d)none of these

Answer:

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60. Find the locus of the middle points of the normals chords of the rectangular hyperbola $x^2 - y^2 = a^2$.

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61. The line $y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2 m^2}}$ is normal to the ellise $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for all values of m belonging to $(0, 1)$ (b) $(0, \infty)$ (c) R (d) none of these

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62. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then, $CP^2 + CQ^2 + CR^2 + CS^2 =$ (A) r^2 (B) $2r^2$ (C) $3r^2$ (D) $4r^2$

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63. A normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the axes in M and N and lines MP and

NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = \left(a^2 + b^2\right)^2$

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64. Let P be a point such that the sum of the slopes of normals drawn from the point P to the rectangular hyperbola$xy = 16$is equal to the sum of ordinates of the feet of normals.Prove that the locus of P is a parabola.find its focus and latus rectum.

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65. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose pendiculars from A, B, C to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ meets the ellipse respectivelily at P, Q, R so that P, Q , R lies on same side of major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P Q nad R are concurrent.

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