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India's Number 1 Education App

MATHS

NCERT - FULL MARKS MATHEMATICS(TAMIL)

DIFFERENTIAL CALCULUS - LIMITS AND CONTINUITY

Example

1. Check if $\lim_{x \to -5} f(x)$ exists or not, where

$$f(x) = \begin{cases} \frac{|x+5|}{x+5}, \text{ for } & x \neq -5 \\ 0, \quad \text{for} & x = -5 \end{cases}$$

View Text Solution

2. Test the existence of the limit, $\lim_{x \to 1} \frac{4|x-1| + x - 1}{|x-1|}, x \neq 1.$

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3. Calculate $\lim_{x \to 3} \left(x^3 - 2x + 6\right)$.

View Text Solution

4. Calculate $\lim_{x \to x_0} (5)$ for any real number x_0.

View Text Solution

5. Compute : $\lim_{x \to 8} (5x)$

View Text Solution

6. Compute : $\lim_{x \to -2} \left(-\frac{3}{2}x\right)$.

View Text Solution

7. Compute $\lim_{x \to 0} \left[\frac{x^2 + x}{x} + 4x^3 + 3\right]$.

View Text Solution

8. Calculate $\lim_{x \to -1} \left(x^2 - 3\right)^{10}$.

View Text Solution

9. Calculate $\lim_{x \to -2} \left(x^3 - 3x + 6\right)\left(-x^2 + 15\right)$.

View Text Solution

10. Calculate $\lim_{x \to 3} \dfrac{\left(x^2 - 6x + 5\right)}{x^3 - 8x + 7}$.

View Text Solution

11. Calculate $\lim_{x \to 1} \dfrac{\sqrt{x} - 1}{x - 1}$.

View Text Solution

12. Find $\lim_{t \to 0} \dfrac{\sqrt{t^2 + 9} - 3}{t^2}$

View Text Solution

13. Calculate $\lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$.

View Text Solution

14. Calculate $\lim_{t \to 1} \dfrac{\sqrt{t} - 1}{t - 1}$.

View Text Solution

15. Find $\lim_{x \to 0} \dfrac{(2 + x)^5 - 2^5}{x}$.

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16. Find the positive integer n so that $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 27$.

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17. Calculate $\lim_{x \to \infty} \frac{1 - x^3}{3x + 2}$.

View Text Solution

18. Alcohol is removed from the body by the lungs, the kidneys, and by chemical processes in liver. At moderate concentration levels, the majority work of removing the alcohol is done by the liver, less than 5% of the alcohol is eliminated by the lungs and kidneys. The rate r at which the liver processes alcohol from the bloodstream is related to the blood alcohol concentration x by a rational function of the form $r(x) = \frac{\alpha x}{x + \beta}$ for some positive constants α and β. Find the maximum possible rate of removal.

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19. According to Einstein's theory of relativity, the mass m of a body moving with velocity v is $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where m_0 is the initial mass and c is the speed of light. What happens to m as $v \to c^-$. Why is a left hand limit necessary?

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20. The velocity in ft/sec of a falling object is modeled by $r(t) = -\sqrt{\dfrac{32}{k}}\dfrac{1 - e^{-2t\sqrt{32k}}}{1 + e^{-2t\sqrt{32k}}}$, where k is a constant that depends upon the size and shape of the object and the density of the air. Find the limiting velocity of the object, that is, find $\lim_{t \to \infty} r(t)$.

View Text Solution

21. Suppose that the diameter of an animal's pupils is given by $f(x) = \dfrac{160x^{-0.4} + 90}{4x^{-0.4} + 15}$, where x is the intensity of light and f(x) is in mm.

Find the diameter of the pupils with

minimum light

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22. Suppose that the diameter of an animal's pupils is given by $f(x) = \frac{160x^{-0.4} + 90}{4x^{-0.4} + 15}$, where x is the intensity of light and f(x) is in mm. Find the diameter of the pupils with

maximum light.

View Text Solution

23. Evaluate : $\lim_{x \to 0} (1 + \sin x)^{2 \text{ cosec x}}$

View Text Solution

24. Evaluate : $\lim_{x \to 0} \left(\frac{x+2}{x-2} \right)^{x}$.

View Text Solution

25. Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

View Text Solution

26. Do the limits of following functions exist as $x \to 0$? State reasons for your answer.

$\frac{\sin|x|}{x}$

View Text Solution

27. Do the limits of following functions exist as $x \to 0$? State reasons for your answer.

$\frac{\sin x}{|x|}$

View Text Solution

28. Do the limits of following functions exist as $x \to 0$? State reasons for your answer.

$$\frac{x\lfloor x \rfloor}{\sin|x|}$$

View Text Solution

29. Do the limits of following functions exist as $x \to 0$? State reasons for your answer.

$$\frac{\sin(x - \lfloor x \rfloor)}{x - \lfloor x \rfloor}$$

View Text Solution

30. Evaluate : $\lim_{x \to 0} \frac{3^x - 1}{\sqrt{1+x} - 1}$.

View Text Solution

31. Describe the interval(s) on which each function is continuous.

$$g(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

View Text Solution

32. Describe the interval(s) on which each function is continuous.

$$h(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

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33. A tomato wholesaler finds that the price of a newly harvested tomatoes is ₹ 0.16 per kg if he purchases fewer than 100 kgs each day. However, if he purchases at least 100 kgs daily, the price drops to ₹0.14 per kg. Find the total cost function and discuss the cost when the purchase is 100 kgs.

View Text Solution

34. Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, \text{ if } & x \neq 0 \\ 0, \text{ if } & x = 0 \end{cases}$ is continuous in $\mathbb{R}$.

View Text Solution

Illustration

1. Calculate $\lim\limits_{x \to \infty} \dfrac{2x^2 - 2x + 3}{x^2 + 4x + 4}$.

View Text Solution

2. $f(x) = x^2 + 3$

View Text Solution

3. $f(x) = \dfrac{16 - x^2}{4 + x}$

View Text Solution

View Text Solution

Exercise 9 1

1. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - x - 2}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

View Text Solution

2. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

View Text Solution

3. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to 0} \frac{(\sqrt{x} + 3) - \sqrt{3}}{x}$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

View Text Solution

4. Complete the table using calculator and use the result to estimate the limit.

$$\lim_{x \to -3} \frac{\sqrt{1 - x} - 2}{x + 3}$$

x	-3.1	-3.01	-3.00	-2.999	-2.99	-2.9
$f(x)$						

View Text Solution

5. Complete the table using calculator and use the result to estimate the limit.

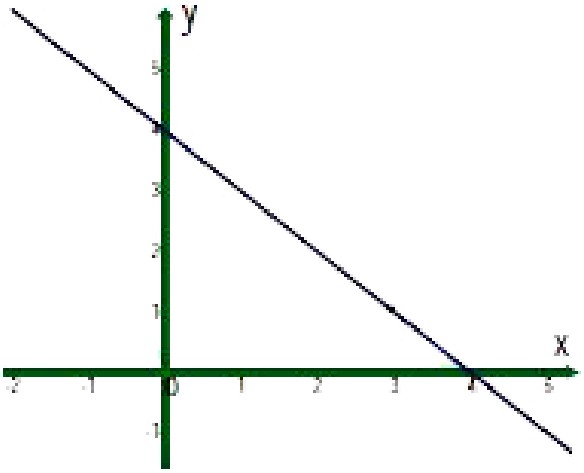
$$\lim_{x \to 0} \frac{\sin x}{x}$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

View Text Solution

6. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

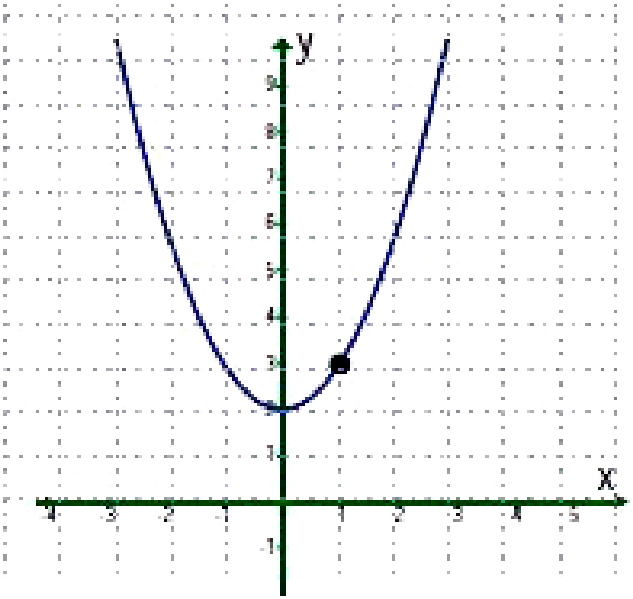
$$\lim_{x \to 3} (4 - x)$$



View Text Solution

7. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

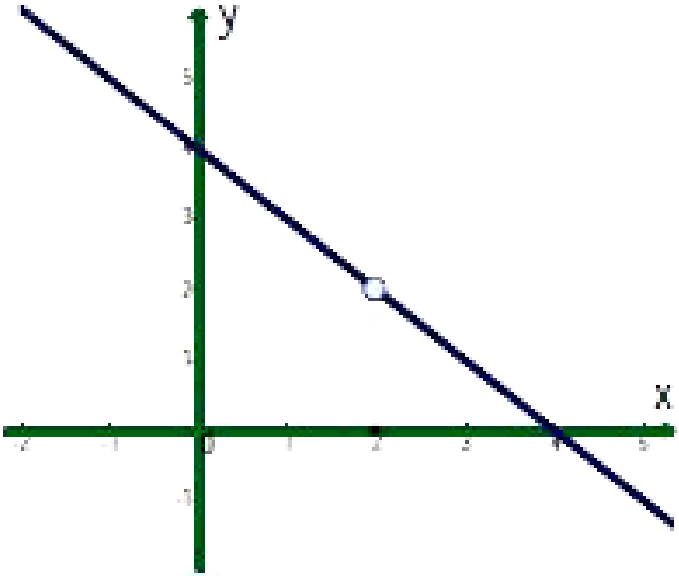
$\lim_{x \to 1} \left(x^2 + 2\right)$.



View Text Solution

8. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

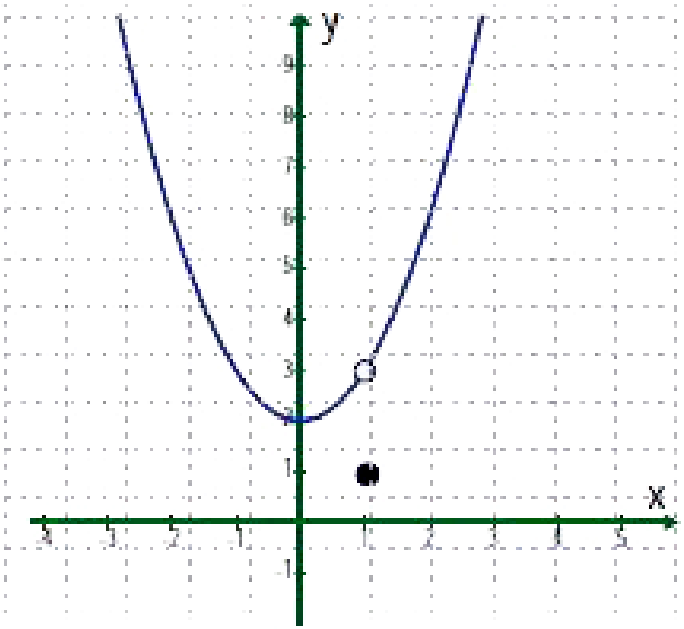
$\lim_{x \to 2} f(x)$ where $f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$



View Text Solution

9. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$



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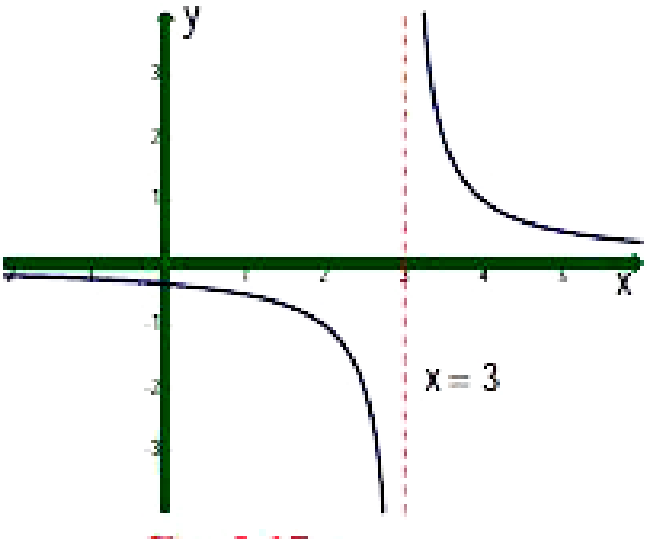
10. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

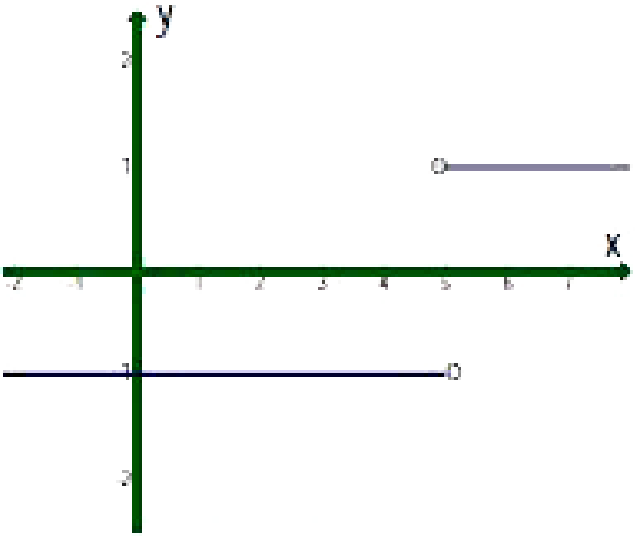
$$\lim_{x \to 3} \frac{1}{x - 3}$$



View Text Solution

11. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$$\lim_{x \to 5} \frac{|x-5|}{x-5}$$



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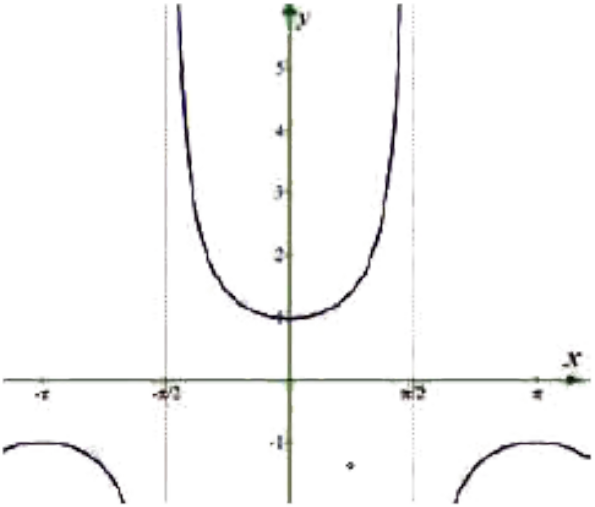
12. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

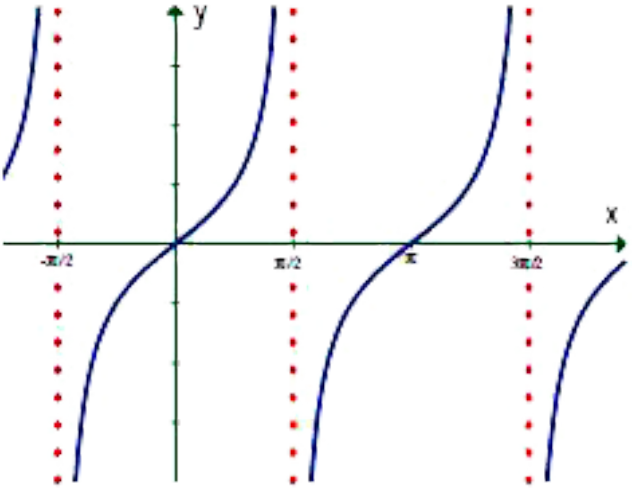
$\lim_{x \to 0} \sec x$



View Text Solution

13. Use the graph to find the limits (if it exists). If the limit does not exist, explain why?

$\lim_{x \to \frac{\pi}{2}} \tan x$



View Text Solution

14. Sketch the graph of f, then identify the values of x_0 for which $\lim_{x \to x_0} f(x)$ exists

$$f(x) = \begin{cases} x^2, & x < 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$$

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15. Sketch the graph of f, then identify the values of x_0 for which $\lim_{x \to x_0} f(x)$ exists

$$f(x) = \begin{cases} \sin x, & x < 0 \\ 1 - \cos x, & 0 \le x \le \pi \\ \cos x, & x > \pi \end{cases}$$

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16. Write a brief description of the meaning of the notation $\lim_{x \to 8} f(x) = 25$.

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17. If f(2) = 4, can you conclude anything about the limit of f(x) as x approaches 2?

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18. If the limit of f(x) as x approaches 2 is 4, can you conclude anything about f(2)? Explain reasoning

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19. Evaluate : $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$ if it exists by finding $f(3^-)$ and $f(3^+)$.

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20. Verify the existence of $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} \frac{|x-1|}{x-1}, & \text{for } x \neq 1 \\ 0, & \text{for } x = 1 \end{cases}$.

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Exercise 9 2

1. Evaluate the following limits :

$$\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$$

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2. Evaluate the following limits :

$$\lim_{x \to 1} \frac{x^m - 1}{x^n - 1}$$, m and n are integers.

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3. Evaluate the following limits :

$$\lim_{\sqrt{x} \to 3} \frac{x^2 - 81}{\sqrt{x} - 3}$$

View Text Solution

4. Evaluate the following limits :

$$\lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}, x > 0$$

View Text Solution

5. Evaluate the following limits :

$$\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$$

View Text Solution

6. Evaluate the following limits :

$$\lim_{x \to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2}$$

View Text Solution

7. Evaluate the following limits :

$$\lim_{x \to 1} \frac{\sqrt{x}-x^2}{1-\sqrt{x}}$$

View Text Solution

8. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$$

View Text Solution

9. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$$

View Text Solution

10. Evaluate the following limits :

$$\lim_{x \to 1} \frac{3\sqrt{7+x^3}-\sqrt{3+x^2}}{x-1}$$

View Text Solution

11. Evaluate the following limits :

$$\lim_{x \to 2} \frac{2-\sqrt{x+2}}{3\sqrt{2}-3\sqrt{4-x}}$$

View Text Solution

12. Evaluate the following limits :

$$\lim_{x \to 0} \frac{\sqrt{1-x}-1}{x^2}$$

View Text Solution

13. Evaluate the following limits :

$$\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$$

View Text Solution

14. Evaluate the following limits :

$$\lim_{x \to a} \frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2} (a > b)$$

View Text Solution

Exercise 9 3

1. Find the left and right limits of $f(x) = \dfrac{x^2 - 4}{(x^2 + 4x + 4)(x + 3)}$ at x = -2.

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2. $f(x) = \tan x$ at $x = \dfrac{\pi}{2}$.

View Text Solution

3. Evaluate the following limits

$$\lim_{x \to 3} \frac{x^2 - 9}{x^2(x^2 - 6x + 9)}$$

View Text Solution

4. Evaluate the following limits

$$\lim_{x \to \infty} \frac{3}{x - 2} - \frac{2x + 11}{x^2 + x - 6}$$

View Text Solution

5. Evaluate the following limits

$$\lim_{x \to \infty} \frac{x^4 - 5x}{x^2 - 3x + 1}$$

View Text Solution

6. Evaluate the following limits

$$\lim_{x \to \infty} \frac{1 + x - 3x^3}{1 + x^2 + 3x^3}$$

View Text Solution

7. Evaluate the following limits

$$\lim_{x \to \infty} \left(\frac{x^3}{2x^2 - 1} - \frac{x^2}{2x + 1} \right)$$

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8. Show that

$$\lim_{n \to \infty} \frac{1 + 2 + 3 + \ldots + n}{3n^2 + 7n + 2} = \frac{1}{6}$$

View Text Solution

9. Show that

$$\lim_{n \to \infty} \frac{1^2 + 2^2 + \ldots + (3n)^2}{1 + 2 + \ldots + 5n)(2n + 3)} = \frac{9}{25}$$

View Text Solution

10. Show that

$$\lim_{n \to \infty} \frac{1}{1.2} + \frac{1}{2.3} = \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = 1$$

View Text Solution

11. An important problem in fishery science is to estimate the number of fish presently spawning in streams and use this information to predict

the number of mature fish or "recruits" that will return to the rivers during the reproductive period. If S is the number of spawners and R the number of recruits, "Beverton-Holt spawner recruit function" is $R(S) = \dfrac{S}{(\alpha S + \beta)}$ where α and β are positive constants. Show that this function predicts approximately constant recruitment when the number of spawners is sufficiently large.

View Text Solution

12. A tank contains 5000 litres of pure water. Brine (very salty water) that contains 30 grams of salt per litre of water is pumped into the tank at a rate of 25 litres per minute. The concentration of salt water after t minutes (in grams per litre) is $C(t) = \dfrac{30t}{200 + t}$. What happens to the concentration as $t \to \infty$?

View Text Solution

Exercise 9 4

1. Evaluate the following limits:

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{7x}$$

View Text Solution

2. Evaluate the following limits:

$$\lim_{x \to 0} (1 + x)^{1/3x}$$

View Text Solution

3. Evaluate the following limits:

$$\lim_{x \to \infty} \left(1 + \frac{k}{x}\right)^{\frac{m}{x}}$$

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4. Evaluate the following limits:

$$\lim_{x \to \infty} \left(\frac{2x^2 + 3}{2x^2 + 5}\right)^{8x^2 + 3}$$

View Text Solution

5. Evaluate the following limits:

$$\lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{x+2}$$

View Text Solution

6. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin^3\left(\frac{x}{2}\right)}{x^3}$$

View Text Solution

7. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin \alpha x}{\sin \beta x}$$

View Text Solution

8. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan 2x}{\sin 5x}$$

View Text Solution

9. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin(a+x) - \sin(a-x)}{x}$$

View Text Solution

10. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{x^2 + a^2} - a}{\sqrt{x^2 + b^2} - b}$$

View Text Solution

11. Evaluate the following limits:

$$\lim_{x \to 0} \frac{2\text{arc sin x}}{3x}$$

View Text Solution

12. Evaluate the following limits:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

View Text Solution

13. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan 2x}{x}$$

View Text Solution

14. Evaluate the following limits:

$$\lim_{x \to 0} \frac{2^x - 3^x}{x}$$

View Text Solution

15. Evaluate the following limits:

$$\lim_{x \to 0} \frac{3^x - 1}{\sqrt{x+1} - 1}$$

View Text Solution

16. Evaluate the following limits:

$$\lim_{x \to 0} \frac{1 - \cos^2 x}{x \sin 2x}$$

View Text Solution

17. Evaluate the following limits:

$$\lim_{x \to \infty} x\left[3^{\frac{1}{x}} + 1 - \cos\left(\frac{1}{x}\right) - e^{\frac{1}{x}}\right]$$

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18. Evaluate the following limits:

$$\lim_{x \to \infty} \{x[\log(x+a) - \log(x)]\}$$

View Text Solution

19. Evaluate the following limits:

$$\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$$

View Text Solution

20. Evaluate the following limits:

$$\lim_{x \to \frac{\pi}{2}} (1 + \sin x)^{2 \text{ cosec x}}$$

View Text Solution

21. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$$

View Text Solution

22. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\tan x}$$

View Text Solution

23. Evaluate the following limits:

$$\lim_{x \to \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x$$

View Text Solution

24. Evaluate the following limits:

$$\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$$

View Text Solution

25. Evaluate the following limits:

$$\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$$

View Text Solution

26. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\sin x(1 - \cos x)}{x^3}$$

View Text Solution

27. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

View Text Solution

Exercise 9 5

1. Examine the continuity of the following :

$x + \sin x$

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2. Examine the continuity of the following :

$x^2 \cos x$

View Text Solution

3. Examine the continuity of the following :

$e^x \tan x$

View Text Solution

4. Examine the continuity of the following :

$e^{2x} + x^2$

View Text Solution

5. Examine the continuity of the following :

x. ln x

View Text Solution

6. Examine the continuity of the following :

$$\frac{\sin x}{x^2}$$

View Text Solution

7. Examine the continuity of the following :

$$\frac{x^2 - 16}{x + 4}$$

View Text Solution

8. Examine the continuity of the following :

$$|x + 2| + |x - 1|$$

View Text Solution

9. Examine the continuity of the following :

$$\frac{|x-2|}{|x+1|}$$

View Text Solution

10. Examine the continuity of the following :

$\cot x + \tan x$

View Text Solution

11. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} 4x+5, \text{ if } & x \leq 3 \\ 4x-5, \text{ if } & x > 3 \end{cases}$$

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12. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} x + 2, \text{ if } & x \geq 2 \\ x^2, \text{ if } & x < 2 \end{cases}$$

View Text Solution

13. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} x^3 - 3, \text{ if } & x \leq 2 \\ x^2 + 1, \text{ if } & x > 2 \end{cases}$$

View Text Solution

14. Find the points of discontinuity of the function f, where

$$f(x) = \begin{cases} \sin x, & 0 \leq x \leq \frac{\pi}{4} \\ \cos x, & \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$$

View Text Solution

15. At the given point x_0 discover whether the given function is continuous or discontinuous citing the reasons for your answer :

$x_0 = 1, f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

View Text Solution

16. At the given point x_0 discover whether the given function is continuous or discontinuous citing the reasons for your answer :

$x_0 = 3, f(x) = \begin{cases} \frac{x^2-9}{x-3}, \text{ if } & x \neq 3 \\ 5, \text{ if } & x = 3 \end{cases}$

View Text Solution

17. Show that the function $\begin{cases} \frac{x^3-1}{x-1}, \text{ if } & x \neq 1 \\ 3, \text{ if } & x = 1 \end{cases}$ is continuous on $(-\infty, \infty)$

View Text Solution

18. Let $f(x) = \begin{cases} 0, \text{ if } & x < 0 \\ x^2, \text{ if } & 0 \leq x < 2 \\ 4, \text{ if } & x \geq 2 \end{cases}$. Graph the function. Show that f(x) continuous on $(-\infty, \infty)$.

View Text Solution

19. If f and g are continuous functions with f(3) = 5 and $\lim_{x \to 3} [2f(x) - g(x)] = 4$, find g(3).

View Text Solution

20. Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x < 1 \\ 2x - 1, & \text{if } x \geq 1 \end{cases}$$

View Text Solution

21. Find the points at which f is discontinuous. At which of these points f is continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} (x-1)^3, & \text{if } x < 0 \\ (x+1)^3, & \text{if } x \geq 0 \end{cases}$$

View Text Solution

22. A function f is defined as follows :

$$f(x) = \{(0, \text{for}, x < 0,), (x, \text{for}, 0 \leq x < 1,), (-x^2 + 4x - 2, \text{for}, 1 \leq x$$

Is the function continuous?

View Text Solution

23. Find the constant b that makes g continuous on $(-\infty, \infty)$.

$$g(x) = \begin{cases} x^2 - b^2 & \text{if } x < 4 \\ bx + 20 & \text{if } x \geq 4 \end{cases}$$

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24. Consider the function $f(x) = x \sin \frac{\pi}{x}$. What value must we give f(0) in order to make the function continuous everywhere?

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25. The function $f(x) = \frac{x^2 - 1}{x^3 - 1}$ is not defined at x = 1. What value must we give f(1) inorder to make f(x) continuous at x =1 ?

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Exercise 9 6 Choose The Correct Or The Most Suitable Answer From The Given Four Alternatives

1. $\lim_{x \to \infty} \frac{\sin x}{x}$

A. 1

B. 0

C. ∞

D. $-\infty$

Answer: B

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2. $\lim_{x \to \pi/2} \frac{2x - \pi}{\cos x}$

A. 2

B. 1

C. -2

D. 0

Answer: C

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3. $\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{x}$

A. 0

B. 1

C. $\sqrt{2}$

D. does not exist

Answer: D

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4. $\lim_{\theta \to 0} \frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}}$

A. 1

B. -1

C. 0

D. 2

Answer: A

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5. $\lim_{x \to \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 3} \right)^x$ is

A. e^4

B. e^2

C. e^3

D. 1

Answer: A

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6. $\lim_{x \to \infty} \frac{\sqrt{x^2 - 1}}{2x + 1} =$

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

Answer: D

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7. $\lim_{x \to \infty} \frac{a^x - b^x}{x} =$

A. log ab

B. $\log\left(\frac{a}{b}\right)$

C. $\log\left(\frac{b}{a}\right)$

D. $\frac{a}{b}$

Answer: B

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8. $\lim_{x \to 0} \frac{8^x - 4x - 2^x + 1^x}{x^2} =$

A. 2 log 2

B. $2(\log 2)^2$

C. log 2

D. 3 log 2

Answer: B

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9. If $f(x) = x(-1)^{\left\lfloor \frac{1}{x} \right\rfloor}, x \leq 0$, then the value of $\lim_{x \to 0} f(x)$ is equal to

A. -1

B. 0

C. 2

D. 4

Answer: B

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10. $\lim_{x \to 3} \left\lfloor x \right\rfloor$ =

A. 2

B. 3

C. does not exist

D. 0

Answer: C

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11. Let the function f be defined by $f(x) = \begin{cases} 3x & 0 \leq x \leq 1 \\ -3x + 5 & 1 < x \leq 2 \end{cases}$, then

A. $\lim_{x \to 1} f(x) = 1$

B. $\lim_{x \to 1} f(x) = 3$

C. $\lim_{x \to 1} f(x) = 2$

D. $\lim_{x \to 1} f(x)$ does not exist

Answer: D

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12. If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \lfloor x - 3 \rfloor + |x - 4|$ for $x \in \mathbb{R}$, then $\lim_{x \to 3^-} f(x)$ is equal to

A. -2

B. -1

C. 0

D. 1

Answer: C

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13. $\lim_{x \to 0} \frac{xe^x - \sin x}{x}$ is

A. 1

B. 2

C. 3

D. 0

Answer: D

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14. If $\lim_{x \to 0} \frac{\sin px}{\tan x}$, then the value of p is

A. 6

B. 9

C. 12

D. 4

Answer: C

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15. $\lim_{\alpha \to \pi/4} \frac{\sin\alpha - \cos\alpha}{\alpha - \frac{\pi}{4}}$ is

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. 2

Answer: A

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16. $\lim_{n \to \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n}{n^2} \right)$ is

A. $\frac{1}{2}$

B. 0

C. 1

D. ∞

Answer: A

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17. $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$ =

A. 1

B. e

C. $\frac{1}{e}$

D. 0

Answer: A

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18. $\lim_{x \to 0} \frac{e^{\tan x} - e^{x}}{\tan x - x}$ =

A. 1

B. e

C. $\frac{1}{2}$

D. 0

Answer: A

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19. The value of $\lim_{x \to 0} \frac{\sin x}{\sqrt{x^2}}$ is

A. 1

B. -1

C. 0

D. ∞

Answer: D

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20. The value of lim , where $\lim_{x \to k^-} x - \left\lfloor x \right\rfloor$, where k is an integer is

A. -1

B. 1

C. 0

D. 2

Answer: B

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21. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

A. continuous

B. discontinuous

C. differentiable

D. non-zero

Answer: B

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22. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} x & \text{x is irrational} \\ 1-x & \text{x is irrational} \end{cases}$ then f is

A. discontinuous at $x = \frac{1}{2}$

B. continuous at $x = \frac{1}{2}$

C. continuous everywhere

D. discontinuous everywhere

Answer: B

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23. The function $f(x) = \begin{cases} \frac{x^2-1}{x^3+1} & x \neq -1 \\ P & x = -1 \end{cases}$ is not defined for x = −1. The value of f (- 1) so that the function extended by this value is continuous is

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. 1

D. 0

Answer: B

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24. Let f be a continuous function on [2, 5]. If f takes only rational values for all x and f (3) = 12, then f (4.5) is equal to

A. $\frac{f(3)+f(4.5)}{7.5}$

B. 12

C. 17.5

D. $\frac{f(4.5)-f(3)}{1.5}$

Answer: B

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25. Let a function f be defined by $f(x)=\frac{x-|x|}{x}$ for $x \neq 0$ and $f(0)=2$. Then f is

A. continuous nowhere

B. continuous everywhere

C. continuous for all x except x = 1

D. continuous for all x except x = 0

Answer: D

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