

### India's Number 1 Education App

### **MATHS**

# **BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)**

### **ADDITION OF VECTORS**

### **Solved Problems**

**1.** Find unit vector in the direction of vector  $ar{a}=\left(2ar{i}+3ar{j}+ar{k}
ight)$ 



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**2.** Find a vector in the direction of vector  $ar{a}=ar{i}-2ar{j}$  has magnitude 7 units.



3. Find the unit vector in the direction of the sum of the vectors

$$\bar{a} = 2\bar{i} + 2\bar{j} - 5\bar{k} \text{ and } \bar{b} = 2\bar{i} + \bar{j} + 3\bar{k}.$$



**4.** Write direction ratios of the vector ar r=ar i+ar j-2ar k and hence calculate its direction cosines.



**5.** Consider two points P and Q with position vectors  $\overline{OP}=3\bar{a}-2\bar{b}$  and  $\overline{OQ}=\bar{a}+\bar{b}$ . Find the position vector of a point R which divides the line joining P and Q in the ratio 2:1

(i) internally.



**6.** Consider two points P and Q with position vectors  $\overline{OP}=3ar{a}-2ar{b}$  and  $\overline{OQ}=ar{a}+ar{b}$ . Find the position vector of a point R which divides the line joining P and Q in the ratio 2:1





**7.** Show that the points A (2i - j + k) B(i-3j-5k), C (3i-4j-4k) are the vertices of a right angled triangle



**8.** Let A, B, C, D be four points with position vectors ar a+2ar b,2ar a-ar b,ar a and 3ar a+ar b respectively. Express the vectors

 $\overline{AC}$ ,  $\overline{DA}$ ,  $\overline{BA}$  and  $\overline{BC}$  interms of  $\overline{a}$  and  $\overline{b}$ .



9. If ABCDEF is a regular hexagon with centre O , then P.T

$$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$$



**10.** In  $\Delta ABC$ , if  $\bar{a},\bar{b},\bar{c}$  are position vectors of the vertices A, B, and C respectively, then prove that the position vector of the centroid G is  $\frac{1}{3}(\bar{a}+\bar{b}+\bar{c})$ 



**11.** If O is the circumcentre, 'H' is the orthocentre of triangle ABC, then show that

$$\overline{OA} + \overline{OB} + \overline{OC} = \overline{OH}$$



**12.** If 'O' is circumcente and 'H' is orthocentre of  $\Delta ABC$ , then

#### List-II

A) 
$$\overline{OA} + \overline{OB} + \overline{OC}$$
 1)  $\frac{1}{2}\overline{HO}$ 

B) 
$$\overline{HA} + \overline{HB} + \overline{HC}$$
 2)  $2\overline{HO}$ 

$$\frac{1}{2}(1) = \frac{1}{2} \frac{1}{2}$$

C) 
$$\overline{AH} + \overline{HB} + \overline{HC}$$
 3)  $2\overline{AO}$   
D)  $\overline{OG}$  4)  $\frac{1}{2}\overline{OH}$ 

4) 
$$\frac{1}{3}\overline{OH}$$

5) 
$$\frac{3}{OH}$$

The correct matching is



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**13.** Let  $\bar{a}, \bar{b}, \bar{c}, \bar{d}$  be the position vectors of A, B, C and D respectively which are the vertices of a tetrahedron. Then prove that the lines joining the vertices to the centroids of the opposite faces are concurrent. (This point is called the centroid of the tetrahedron)



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14. Let OABC be a parallelogram and D the mid point of OA. Prove that segment CD trisects the diagonal OB and is trisected by the diagonal OB

**15.** Let 
$$ar a, ar b$$
 be non-collinear vectors. If  $lpha=(x+4y)ar a+(2x+y+1)ar b, eta=(y-2x+2)ar a+(2x-3y-1)ar b$  are such that  $3lpha=2eta$  then find x, y.



$$-2ar a+3ar b+5ar c,$$
  $ar a+2ar b+3ar c,$   $7ar a-ar c$  are collinear, where  $ar a,$   $ar b,$   $ar c$  are noncoplanar vectors.

P,V

are

Show that the points whose



**17.** If the points whose position vectors are 
$$3\bar{i}-2\bar{j}-\bar{k},\,2\bar{i}+3\bar{j}-4\bar{k},\,-\bar{i}+\bar{j}+2\bar{k},\,4\bar{i}+5\bar{j}+\lambda\bar{k}$$
 are coplanar, then show that  $\lambda=-\frac{146}{17}$ .

16.

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18. In the two dimensional plane, prove by using vector methods, the equation of the line whose intercepts on the axes are 'a' and 'b' is  $\frac{x}{a} + \frac{y}{b} = 1.$ 



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19. Using the vector equation of the straight line passing through two that points whose position points, prove the vectors are  $ar{a}, ar{b} \ ext{and} \ (3ar{a} - 2ar{b})$  are collinear.



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**20.** Find the equation of the line parallel to the vector  $2ar{i}-ar{j}+2ar{k}$ , and which passes through the point A whose position vector is  $3ar{i}+ar{j}-ar{k}.$  If P is a point on this line such that AP = 15, find the position vector of P.



**21.** Show that the line joining the pair of points  $6ar{a}-4ar{b}+4ar{c},\ -4ar{c}$  and the line joining the pair of points,  $-ar{a}-2ar{b}-3ar{c},$   $ar{a}+2ar{b}-5ar{c}$  intersect at the point  $-4\bar{c}$  when  $\bar{a}, \bar{b}, \bar{c}$  are non-coplanar vectors.



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22. Find the point of intersection of the line

$$ar{r}=2ar{a}+ar{b}+tig(ar{b}-ar{c}ig)$$
 and the plane

$$ar{r}=ar{a}+xig(ar{b}+ar{c}ig)+yig(ar{a}+2ar{b}-ar{c}ig)$$
 where

$$ar{a},\,ar{b},\,ar{c}$$
 are non coplanar vectors.



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# **Exercise 4 A**

1. ABCD is a parallelogram . If L arid M and the middle points of BC and CD respec tively, then find

AL and AM interms of AB and AD



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2. ABCD is a parallelogram . If L arid M and the middle points of BC and CD respec tively, then find

$$\lambda$$
, if AM =  $\lambda AD - LM$ 



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**3.** In  $\triangle ABC$ , P, Q and R are mid points of the sides AB, BC, and CA respectively. If D is any point

- i) then express  $\overline{DA} + \overline{DB} + \overline{DC}$  in terms of  $\overline{DP}$ ,  $\overline{DQ}$  and  $\overline{DR}$
- ii) If  $\overline{PA} + \overline{QB} + \overline{RC} = \bar{a}$  then find  $\bar{a}$ .



- **4.** In  $\Delta ABC$ , P, Q and R are mid points of the sides AB, BC, and CA
- respectively. If D is any point
- i) then express  $\overline{DA}+\overline{DB}+\overline{DC}$  in terms of  $\overline{DP},\overline{DQ}$  and  $\overline{DR}$
- ii) If  $\overline{PA}+\overline{QB}+\overline{RC}=ar{a}$  then find  $ar{a}.$ 
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- **5.** Let ar a=ar i+2ar j+3ar k and ar b=3ar i+ar j. Find the unit vector in the direction of ar a+ar b.
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- **6.** If the vectors  $-3\bar{i}+4\bar{j}+\lambda\bar{k}$  and  $\mu\bar{i}+8\bar{j}+6\bar{k}$  are collinear vectors, then find  $\lambda$  and  $\mu$ 
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**7.** ABCDE is a pentagon. If the sum of the vectors

 $\overline{AB}$ ,  $\overline{AE}$ ,  $\overline{BC}$ ,  $\overline{DC}$ ,  $\overline{ED}$ ,  $\overline{AC}$  is  $\lambda \overline{AC}$  then find the value of  $\lambda$ .



- **8.** If the position vectors of the points A,B,C are
- $-2ar{i}+ar{j}-ar{k},\ -4ar{i}+2ar{j}+2ar{k},6ar{i}-3ar{j}-13ar{k}$  respectively



 $\overline{AB} = \lambda \overline{AC}$  then find the value of  $\lambda$ .

9.

 $\overline{OA}=ar{i}+ar{j}+ar{k}, \overline{AB}=3ar{i}-2ar{j}+ar{k}, \overline{BC}=ar{i}+2ar{j}-2ar{k}, \overline{CD}=2ar{i}+ar{j}+$  then find the vector  $\overline{OD}$ .

and

If

**10.** If  $ar{a}=2ar{i}+5ar{j}+ar{k}$  and  $ar{b}=4ar{i}+mar{j}+nar{k}$  are collinear vectors then find m,n.



**11.** Let ar a=2ar i+4ar j-5ar k, ar b=ar i+ar j+ar k, ar c=ar j+2ar k . Find the unit vector in the opposite direction of a + b + c

the triangle formed by

 $3ar{i}+5ar{j}+2ar{k}, 2ar{i}-3ar{j}-5ar{k} \ ext{and} \ -5ar{i}-2ar{j}+3ar{k}$  equilateral ?

the

vectors



If

12.

**13.** If  $lpha,\,eta\,$  and  $\,\gamma$  be the angle made by the vector  $3ar{i}\,-6ar{j}\,+2ar{k}$  with the positive direction of the coordinate axes, then find  $\cos \alpha$ ,  $\cos \beta$ ,  $\cos \gamma$ .

14. Find the angles made by the straight line passing through the points

(1, -3, 2) and (3, -5, 1) with the corrdinate axes.



**15.** i)  $\bar{a}, \bar{b}, \bar{c}$  are pairwise non zero and non collinear vectors. If  $\bar{a}+\bar{b}$  is collinear with  $\bar{c}$  and  $\bar{b}+\bar{c}$  is collinear with  $\bar{a}$  then find the vector  $\bar{a}+\bar{b}+\bar{c}.$ 

ii) If  $\bar{a}+\bar{b}+\bar{c}=\alpha\bar{d}$ ,  $\bar{b}+\bar{c}+\bar{d}=\beta\bar{a}$  and  $\bar{a},\bar{b},\bar{c}$  are non coplanar vectors, then show that  $\bar{a}+\bar{b}+\bar{c}+\bar{d}=\bar{0}$ .



**16.**  $ar{a},\,ar{b},\,ar{c}$  are non-coplanar vectors. Prove thate the following four points are coplanar

$$egin{aligned} -ar{a} + 4ar{b} - 3ar{c}, \, 3ar{a} + 2ar{b} - 5ar{c} \ -3ar{a} + 8ar{b} - 5ar{c}, \, -3ar{a} + 2ar{b} + ar{c} \end{aligned}$$



17.  $\bar{a}, \bar{b}, \bar{c}$ , are non-coplanar vectors, Prove that the following four points are coplanar.

$$6\bar{a}+2\bar{b}-\bar{c}, 2\bar{a}-\bar{b}+3\bar{c}, -\bar{a}+2\bar{b}-4\bar{c}, -12\bar{a}-\bar{b}-3\bar{c}.$$



coordinate that the then shown four points axes,  $4ar{i}+5ar{j}+ar{k},\ -ar{j}-ar{k},3ar{i}+9ar{j}+4ar{k}\ ext{and}\ -4ar{i}+4ar{j}+4ar{k}$  are coplanar

**18.** If  $\bar{i}, \bar{j}, \bar{k}$  are unit vectors along the positive directions of the



**19.** If  $\bar{a}, \bar{b}, \bar{c}$  are non coplanar vectors, then test for the collinearity of the following points whose position vectors are given.

- i)  $\bar{a} 2\bar{b} + 3\bar{c}$ ,  $2\bar{a} + 3\bar{b} 4\bar{c}$ ,  $-7\bar{b} + 10\bar{c}$
- ii)  $3ar{a}-4ar{b}+3ar{c},\ -4ar{a}+5ar{b}-6ar{c}, 4ar{a}-7ar{b}+6ar{c}$
- iii)  $2ar{a}+5ar{b}-4ar{c},$   $ar{a}+4ar{b}-3ar{c},$   $4ar{a}+7ar{b}-6ar{c}$



**20.** If  $\bar{a}, \bar{b}, \bar{c}$  are non coplanar vectors, then test for the collinearity of the following points whose position vectors are given.

- i)  $ar{a}-2ar{b}+3ar{c},$   $2ar{a}+3ar{b}-4ar{c},$   $-7ar{b}+10ar{c}$
- ii)  $3ar{a}-4ar{b}+3ar{c},\;-4ar{a}+5ar{b}-6ar{c},\,4ar{a}-7ar{b}+6ar{c}$
- iii)  $2ar{a}+5ar{b}-4ar{c},$   $ar{a}+4ar{b}-3ar{c},$   $4ar{a}+7ar{b}-6ar{c}$ 
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**21.** If  $\bar{a}, \bar{b}, \bar{c}$  are non coplanar vectors, then test for the collinearity of the following points whose position vectors are given.

i)  $\bar{a} - 2\bar{b} + 3\bar{c}$ ,  $2\bar{a} + 3\bar{b} - 4\bar{c}$ ,  $-7\bar{b} + 10\bar{c}$ 

ii) 
$$3ar{a}-4ar{b}+3ar{c},\; -4ar{a}+5ar{b}-6ar{c},\, 4ar{a}-7ar{b}+6ar{c}$$

iii) 
$$2ar{a}+5ar{b}-4ar{c},$$
  $ar{a}+4ar{b}-3ar{c},$   $4ar{a}+7ar{b}-6ar{c}$ 



**22.** In the cartesian plane, O is the origin of the coordinate axes. A person starts at O and walks a distance of 3 units in the NORTH - EAST direction and reaches the point P. From P he walks 4 units distance parallel to NORTH - WEST direction and reaches the point Q. Express the vector  $\overline{OQ}$ 

interms of  $\bar{i}$  and  $\bar{j}$  (Observe  $\angle XOP = 45^{\circ}$ )

Ο,

Α,

points

23.

The

 $\overline{OA}=ar{a}, \overline{OB}=ar{b}, \overline{OX}=3ar{a} \ \ ext{and} \ \ \overline{OY}=3ar{b}, \ \ ext{find} \ \ \overline{BX} \ \ ext{and} \ \ \overline{AY} \ \ \ ext{in}$  terms of  $ar{a}$  and  $ar{b}$ . Further if the point p divides  $\overline{AY}$  in the ratio 1 : 3 then express  $\overline{BP}$  interms of  $ar{a}$  and  $ar{b}$ .

В,

X and

such

are

that

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**24.** In  $\triangle OAB$ , E is the midpoint of AB and F is a point on OA such that OF = 2FA. If C is the point of intersection of  $\overline{OE}$  and  $\overline{BF}$ , then find the ratios

OC : CE and BC : CF.



**25.** The point 'E' divides the segment PQ internally in the ratio 1 : 2 and R is any point not on the line PQ. If F is a point on QR such that QF : FR = 2 :

1 then show that EF is parallel to PR.



# Exercise 4 B

- 1. Find the vector equation of the line passing through the point
- $2ar{i}+3ar{j}+ar{k}$  and parallel to the vector  $4ar{i}-2ar{j}+3ar{k}$

**2.** OABC is a parallelogram. If  $\overline{OA}=\bar{a},$   $\overline{OC}=\bar{c},$  find the vector equation of the side BC.



**3.** If  $\bar{a},\bar{b},\bar{c}$  are the position vectors of the vertices A, B, C respectively of  $\Delta ABC$  then find the vector equation of the median through the vertex



A.

4. Find the vector equation of the line passing through the points

$$2\overline{i} + \overline{j} + 3\overline{k}$$
 and  $-4\overline{i} + 3\overline{j} - \overline{k}$ .



5. Find the vector equation of the plane passing through the points

$$ar{i} - 2ar{j} + 5ar{k}, \ -5ar{j} - ar{k}, \ -3ar{i} + 5ar{j}.$$



**6.** Find the vector equation of plane passing through Points (0,0,0) , (0,5,0) and (2,0,1)



7. If  $\bar{a},\bar{b},\bar{c}$  are noncoplanar, find the point of intersection of the line passing through the points  $2\bar{a}+3\bar{b}-\bar{c}, 3\bar{a}+4\bar{b}-2\bar{c}$  with the line joining the points  $\bar{a}-2\bar{b}+3\bar{c}$  and  $\bar{a}-6\bar{b}+6\bar{c}$ .



**8.** ABCD is trapezium in which AB and CD are parallel. Prove by vector methods that the mind points of the sides AB, CD and the intersection of

the diagonals are collinear.



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**9.** In a quadrilateral ABCD, if the mid points of one pair of opposite sides and the point of intersection of the diagonals are collinear, using vector methods, prove that the quadrilateral ABCD is a trapezium



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**10.** Find the vector equation of the plane which passes through the points  $2\bar{i}+4\bar{j}+2\bar{k},\,2\bar{i}+3\bar{j}+5\bar{k}$  and parallel to the vector  $3\bar{i}-2\bar{j}+\bar{k}$ . Also find the point where this plane meets the line joining the points  $2\bar{i}+\bar{j}+3\bar{k}$  and  $4\bar{i}-2\bar{j}+3\bar{k}$ .



11. Find the vector equation of the line passing through the points

$$2ar{i}+ar{j}+3ar{k},\;-\;ar{i}+3ar{j}-ar{k}.$$

