MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

APPLICATION OF DERIVATIVES

Solved Problems

1. Find dy and $\triangle y$ of $y = x^2 + x$ at x=10 when $\triangle x = 0.1.$

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2. Find Δy and dy for the function $y = x^2 + x$, when x=10, $\Delta x = 0.1$

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3. The side of a square is increased from 3 cm to 3.01 cm. Find the approximate increase in the area of the square.

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4. If the radius of a sphere is increased from 7 cm to 7.02 cm. then find the approximate increase in the volume of the sphere.

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5. Show that the relative error in the n^{th} power of a number is 'n' times the relative error in that number

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6. If the increase in the side of a square is 2% then find the approximate percentage of increase in the area of the square.

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7. If an error of 0.01cm is made in measuring the perimeter of a circle and the perimeter is measured as 44cm then find the approximate error and relative error in its area.

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8. Find the approximate value of $\sqrt[3]{999}$

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9. Find the slope of the tangent to the following curves at the ponts as indicated.

(i) $y = 5x^2$ at $(- 1, 5)$

(ii) $y = \frac{1}{x - 1}(x \neq 1)$ at $\left[3, \frac{1}{2}\right]$

(iii) $x = a\sec\theta, y = a\tan\theta$ at $\theta = \frac{\pi}{6}$

(iv) $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at (a, b)

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10. Find the equation of the tangent and the normal to the curve $y = 5x^4$ at the point $(1, 5)$.

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11. Find the equations of the tangent and the normal to the curve $y^4 = ax^3$ at (a,a)

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12. Find the equations of the tangent to the curve $y = 3x^2 - x^3$, where it meets the X-axis.

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13. Find the points at which the curve $y = \sin x$ has horizontal tangents.

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14. The function $f(x) = x^{1/x}$ has stationary point at

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15. Find whether the curve $y = f(x) = x^{2/3}$ has a verical tangent at $x = 0$.

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16. S.T the tangent at any point θ on the curve $x = c\sec\theta, y = c\tan\theta$ is $y\sin\theta = x - \cos\theta$.

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17. Show that the area of the triangle formed by the tangent at any point on the curve xy=c, (c $\neq$ 0), with the coordinate axes is constant.

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18. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2(a \neq 0, b \neq 0)$ at the

point (a,b) is $\frac{x}{a} + \frac{y}{b} = 2$

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19. Show that the square of the lengh of the subtangent at any point on the curve $by^2 = (x+a)^3 (b \neq 0)$ varies witht the length of the subnormal at that point

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20. Find the value of k, so that the length of the subnormal at any point on the curve $y = a^{1-k} x^k$ is a constant

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21. Find the angle between the curves xy=2 and $x^2 + 4y = 0$

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22. Find the angle between the curve $2y = e^{-x/2}$ and y-axis.

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23. If $ax^2 + by^2 = 1, a_1x^2 + b_1y^2 = 1$, then show that the condition for orthogonality of above curves is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

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24. S.T the curves $y^2 = 4(x+1), y^2 = 36(9-x)$ intersect orthogonally.

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25. Find the average rate of change of $s = f(t) = 2t^2 + 3$ between $t = 2$ and $t = 4$.

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26. Find the rate of change of the area of a circle per second with respect to its radius r when r = 5 cm.

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27. The volume of a cube is increasing at a rate of 9 cubie centimeters per second. How fast is the surface area increasing when the length of edge is 10 cms?

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28. A particle is moving in a straight line so that after t seconds its distance is s (in cms) from a fixed point on the line is given by $s = f(t) = 8t + t^3$. Find the velocity at time t= 2sec (ii) the initial velocity can acceleration at t=2 sec

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29. A container in the shape of an inverted cone has height 12 cm and radius 6cm at the top. If it is filled with water at the rate of $12cm^3 / \text{sec}$, what is the rate of change in the rate of change in the height of water level when the tank is filled 8 cm?

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30. A particle is moving along a line according to s=f(t) $= 4t^3 - 3t^2 + 5t - 1$ where s is measured in meter and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero.

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31. The quantity (in mg) of a drug in the blood at time t(sec) is given by q=3 (0.4)^t. Find the instantaneous rate of change at t=2sec.

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32. Let a kind of bacteria grow by t^3 (t in sec). At what time the rate of growth of the bacteria is 300 bacteria per sec?

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33. The total cost C(x) in Rupees, associated with the production of x units of an item is given by

$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$

Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

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34. The total revenue in Rupees received from the sale of x units of a product is given by

R(x) = $3x^2$+ 36x + 5. The marginal revenue, when x = 15 is

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35. Verify Rolle's theorem for the function $y = f(x) = x^2 + 4$on [-3,3]

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36. Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-\frac{x}{2}}$ in [-3,0].

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37. Let $f(x) = (x-1)(x-2)(x-3)$ then prove that there is more than one 'c' in (1,3) such that $f'(c) = 0$

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38. On the curve $y = x^2$, find a point at which the tangent is parallel to the chord joining (0,0) and (1,1).

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39. Find the intervals on which $f(x) = x^2 - 3x + 8$ is increasing or decreasing ?

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40. Show that $f(x) = |x|$ is strictly decreasing on $(-\infty, 0)$ and strictly increasing on $(0, \infty)$.

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41. Find the intervals on which the function $f(x) = x^3 + 5x^2 - 8x + 1$ is a strictly increasing function.

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42. For the function $f(x) = x^x$, find the points at which it is (i) increasing and (ii) decreasing. Hence determine which of e^{π}, π^{e} is greater.

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43. Determine the intervals in which $f(x) = \frac{2}{(x-1)} + 18x, \forall x \in R - \{0\}$ is stricly increasing and decreasing.

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44. Let $f(x) = \sin x - \cos x$ be defined on $[0, 2\pi]$. Determine the intervals in which f(x) is strictly decreasing and strictly increasing.

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45. If $0 \leq x \leq \frac{\pi}{2}$ then show that $x \geq \sin x$.

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46. Let $f' : R \to R$ be defined by $f(x) = 4x^2 - 4x + 11$. Find the global minimum value and a point of global minimum.

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47. Let $f:[-2, 2] \rightarrow R$ be defined by $f(x) = \{x|$. Find the global maximum of f(x) and a point of global manimum.

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48. Find the global maximum and global minimumor of the function $f: R \rightarrow R$ defined by $f(x) = x^2$

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49. Find the extreme points of $f(x) = 3x^4 - 4x^3 + 1$ and state whether the function has local maxima and local minima at those points.

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50. Find all the points of local maxima and local minima of the function $f(x) = x^3 - 6x^2 + 12x - 8$

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51. find the points of local minimum and local maximum of the function $f(x) = \sin 2x \, \forall x \varepsilon [0, 2\pi]$

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52. Find the points of local extrema of the function $f(x) = x^3 - 9x^2 - 48x + 6 \forall x \in R$. Also find its local extrema.

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53. Find the points of loacal extrema of $f(x) = x^6 \forall x \varepsilon R$. Also find its local extrema.

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54. Find the points of local extrema for the function

f(x) = cos4x defined on $\left[0, \frac{\pi}{2}\right]$

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55. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum.

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56. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.

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57. Find the point on the graph $y^2 = x$ which is the nearest to the point (4,0)

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58. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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59. The profit function p(x) of a company, selling x items per day is given by p(x)=(150-x)x-1600. Find the number of items that the company should sell to get maximum profit. Also find the maximum profit.

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60. The profit function p(x) of a company, selling x items per day is given by p(x)=(150-x)x-1600. Find the number of items that the company should sell to get maximum profit. Also find the maximum profit.

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61. Find the absolute extremum of $f(x) = x^2$ is defined on [-2,2]

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62. Find the absolute maximum and absolute minimu values of $f(x) = x^{40} - x^{20}$ on [0, 1].

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Exercise 10 A

1. Find Δy and dy for the following functions for the values of x and Δx which are shown against each of the functions.

$y = x^2 + 3x + 6, \ x = 10$ and $\Delta x = 0.01$

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2. Find δy and dy for the following functions

$y = e^x + x, \ x = 5$ and $\delta x = 0.002$

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3. Find Δy and dy for the following functions for the values of x and Δx which are shown against each of the functions.

$y = 5x^2 + 6x + 6, x = 2$ and $\Delta x = 0.001$

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4. Find δy and dy for the following functions

$$y = \frac{1}{x+2}, x = 8 \text{ and } \delta x = 0.02$$

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5. Find Δy and dy for the following functions for the values of x and Δx which are shown against each of the functions.

$y = \cos(x), x = 60^\circ \text{ and } \Delta x = 1^\circ$

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6. Find the approximations of the following

$\sqrt{82}$

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7. Find the approximations of the following

$\sqrt[3]{65}$

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8. Find the approximations of the following

$\sqrt{25001}$

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9. Find the approximations of the following

$\sqrt[3]{7.8}$

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10. Find the approximations of the following

$\sin(62^\circ)$

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11. Find the approximations of the following

cos(60^(@)5')`

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12. Find the approximations of the following

$\sqrt[4]{17}$

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13. If the increase in the side of a square is 4% Then find the approximate percentage of increase in the area of the square.

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14. The radius of a sphere is measured as 14 cm. Later it was found that there is an error 0.02 cm in measuring the radius. Find the approximate error in surface of the sphere.

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15. The diameter of a sphere is measured to be 40 cm. If an error of 0.02 cm is made in it, then find approximate errors in volume and surface area of the sphere.

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16. The time t of a complete oscillation of a simple pendulum of length l is given by $t2\pi\sqrt{\frac{l}{g}}$ where g is gravitational constant. Find the approximate percentage of error in t when the percentage of error in l is 1%.

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Exercise 10 B

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4.

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2. Find the slopr of the tangent to the curve $y = \frac{x-1}{x-2}$ at $x \neq 2$ and $x = 10.$

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3. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x co-ordinate is 2.

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4. Find the slope of the tangent to the curve, $y = x^3 - 3x + 2$ at the point whose x co-ordinate is 3.

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5. Find the slope of the normal to the curve $x = a\cos^3\theta,\ y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

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6. Find the slope of the normal to the curve x=1 -a sin θ , y = b $\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

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7. Find the points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

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8. Find a point on the curve y =$(x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

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9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$

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10. Find the equations of all lines having slope 0 which are tangent to the curve

$$y = \frac{1}{x^2 - 2x + 3}.$$

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11. Find the equations of tangent and normal to the following curves at the points indicated againts:

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0,5)

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12. Find the equations of tangent and normal to the following curves at the points indicated against:

$y = x^3 at(1, 1)$

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13. Find the equations of tangent and normal to the following curves at the points indicated againts:

$y = x^2 at(0, 0)$

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14. Find the equations of tangent and normal to the following curves at the points indicated againts:

$x = \cos t, y = \sin t$ at $t = \frac{\pi}{4}$

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15. Find the equations of tangent and normal to the following curves at the points indicated againts:

$y = x^2 - 4x + 2at(4, 2)$

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16. Find the equations of tangent and normal to the following curves at the points indicated againts:

$y = \frac{1}{1+x^2} at(0, 1)$

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17. Find the equations of tangent and normal to the curve xy = 10 at (2 , 5)

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18. Find the equation of tangent and normal to the curve $y = x^3 + 4x$ at (-1,3)

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19. If the slope of the tangent to the curve $x^2 - 2xy + 4y = 0$ at the point it is $\frac{-3}{2}$ then find the equation of that point.

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20. If the slope of the tangent to the curve $y = x \log x$ at a point on it is $\frac{3}{2}$ then find the equation of tangent and normal at the point

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21. Find the equation of tangent and normal to the curve $y = 2.e^{\frac{-x}{3}}$ at the point where the curve meets the Y - axis

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22. Show that the tangent at $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $xx_1^{\frac{-1}{2}} + yy_1^{\frac{-1}{2}} = a^{\frac{1}{2}}$

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23. At what points on the curve $x^2 - y^2 = 2$. The slops of tangents are equal to 2 ?

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24. Show that the curves $x^2 + y^2 = 2, 3x^2 + y^2 = 4x$ have a common tangent at the point (1,1)

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25. At the point (x_1, y_1) on the curve $x^3 + y^3 = 3axy$ show that the tangent is $(x_1^2 - ay_1)x + (y_1^2 - ax_1)y = ax_1y_1$

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26. Show that the tangent at the point P(2,-2) on the curve y(1-x)=x makes intercepts of equal length on the coordinates axex and the normal at P passes through the origin.

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27. IF the tangent at a point on the curve $x^{2/3}+y^{2/3}=a^{2/3}$ intersects the coordinate axes in A and B then show that the length AB is a constant.

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28. IF the tangent at any point P on the curve $x^m y^n = a^{m+n}, mn \neq 0$ meets the coordinate axes in A.B then show that $AP : BP$ is a constant.

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Exercise 10 C

1. Find the lengths of subtangent and subnormal at a point on the curve $y = b\sin\left(\frac{x}{a}\right)$

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2. Show that the length of subnormal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point

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3. Show that at any point (x,y) on the curve $y = b^{\frac{x}{a}}$, the length of the subtangent is a constant and the length of the subnormal is $\frac{y^2}{a}$.

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4. Find the value of k, so that the length of the subnormal at any point on the curve $xy^k = a^{k+1}$ is a constant.

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5. At any point t on the curve x=a(t+sint), y=a(1-cost), find the lengths of tangent, normal, subtangent and subnormal.

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6. Find lengths of normal and subnormal at a point on the curve $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$

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7. Find the length of subtangent, subnormal at a point on the curve

x=a(cost+sint), y=a(sint-tcost)

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Exercise 10 D

1. Find the angle between the curves $x + y + 2 = 0$ and $x^2 + y^2 - 10y = 0$

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2. Find the angle between the curves given below.

$y^2 = 4x, x^2 + y^2 = 5.$

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3. Find the angle between the cures given below :

$x^2 + 3y = 3, x^2 - y^2 + 25 = 0$

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4. Find the angle between the cures given below : $x^2 = 2(x+1), y = \frac{8}{x^2+4}$

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5. Find the angle between the curve and line given below $: 2y - 9x = 0, 3x^2 + 4y = 0$ (in the 4^{th} quadrant)

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6. Find the angle between the cures given below :

$y^2 = 8x, 4x^2 + y^2 = 32$

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7. Find the angle between the curves given below.

$x^2y = 4, y(x^2 + 4) = 8.$

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8. S.T the curves $6x^2 - 5x + 2y = 0, 4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

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Exercise 10 E

1. At time t, the distance s of a particle moving in a straight line is given by $s = 4t^2 + 2t$. Find the average velocity between t= 2sec and t= 8 sec.

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2. If $y = x^4$ then find the average rate of change of y between x= 2 and x= 4.

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3. A particle moving along a straight line has the relation $s = t^3 + 2t + 3$, connecting the distance s describe by the particle in time t. Find the velocity and acceleration of the particle at t=4 sec.

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4. The distance-time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$. Find when and where the velocity is zero.

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5. The displacement s of a particle travelling in a straight line in t seconds is given by $s = 45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

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6. The volume of a cube is increasing at the rate of $8cm^3 / \sec.$ How fast is the surface area increasing when the length of an edge is 12 cm ?

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7. A stone is dropped into a quiet lake and ripples move in circles at the speed of 5 cm/sec. At the instant when the radius of circular ripple is 8cm, how fast is the enclosed area increases?

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8. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

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9. A balloon which always remains spherical on inflation is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius in 15 cm.

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10. The radius of an air bubble is increasing at the rate of 1/2 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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11. Assume that an object is launched upward at 980m/seclis position would be given by $s = -4.9t^2 + 980$. Find the maximum height attained by the object

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12. Let a kind of bacteria grow by t^3 (t in sec). At what time the rate of growth of the bacteria is 300 bacteria per sec?

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13. Suppose we have a rectangular aquarium with dimensions of length 8m, width 4m and height 3m. Suppose we are filling the tank with water at the rate of $0.4m^3/\text{sec}$. How fast is the height of water changing when the water level is 2.5m?

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14. A container is in the shape of an inverted cone has height 8m and radius 6m at the top. If it is filled with water at the rate of $2m^3$/minute, how fast is the height of water changing when the level is 4m?

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15. The total cost C(x) in Rupees associated with the production of x units of an item is given by

$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$

Find the marginal cost when 17 units are produced.

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16. The total revenue in rupees received from the sale os x units of a produce is given by $R(x) = 13x^2 + 26x + 15.$ Find the marginal revenue when x =7.

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17. A point P is moving on the curve $y = 2x^2$. The x coordinate of P is increasing at the rate of 4 units per second . Find the rate at which y coordinate is incerasing when the point is at (2,8).

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Exercise 10 F

1. Verify Rolle's theorem for the functions $(x^2 - 1)(x - 2)$ on [-1,2]. Find the point in the interval where the derivative vanishes.

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2. Verify Rolle's theorem for the function sinx-sin2x on $[0, \pi]$

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3. It is given that Rolle's theorem holds for the function $f(x) = x^3 + bx^2 + ax$ on [1,3] with C = $2 + \frac{1}{\sqrt{3}}$. Find the values a and b

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4. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

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5. Find a point on the graph of the curve $y = (x - 3)^2$, where the tangent is parallel to the chord joining (3,0) and (4,1)

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6. Find a point on the curve $y = x^3$, when the tangent is parallel to the chord joining (1,1), (3,27).

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7. Find c, so that $f(c) = \frac{f(b) - f(a)}{b - a}$ in the following cases.

$f(x) = x^2 - 3x - 1, a = \frac{-11}{7}, b = \frac{13}{7}$.

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8. Find c so that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ where $f(x) = e^x, a = 0, b = 1$

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9. Verify Rolle's theorem for the functions $(x^2 - 1)(x - 2)$ on [-1,2]. Find the point in the interval where the derivative vanishes.

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10. Verify the conditions of Lagrange's mean value theorem for the function $x^2 - 1$ on [2,3]

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11. Verify Rolle's theorem for the function sinx-sin2x on $[0, \pi]$

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12. The value of 'c' in Lagrange's mean value theorem for f(x) = $\log x$ on [1,e] is

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Exercise 10 G

1. Without using the derivative, show that

The function $f(x) = 3x + 7$ is strictly increasing on R

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2. Without using the derivative, show that

The function $f(x) = e^{3x}$ is tricktly increasing on R

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3. Show that the function $f(x) = \sin x$ defined on R is neither increasing nor decreasing on $(0, \pi)$.

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4. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$x^2 + 2x - 5$

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5. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$6 - 9x - x^2$

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6. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$(x+1)^3(x-1)^3$

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7. Find the intervals in which the following functions are stricly increasing and in which they are stricly decreasing.

$x^3(x-2)^2$

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8. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

xe^x

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9. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$\sqrt{(25-4x^2)}$

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10. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$\ln(\ln(x)), x > 1$

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11. Find the intervals in which the following functions are strictly increasing or strictly decreasing.

$x^3 + 3x^2 - 6x + 12$

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12. Show that $f(x) = \cos^2 x$ is strictly decreasing on $(0, \pi/2)$.

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13. Show that $x + \dfrac{1}{x}$ is increasing on $[1, \infty)$.

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14. At what points the slopes of the tangents to the curve $y = \frac{x^3}{6} - \frac{3x^2}{2} + \frac{11x}{2} + 12$ increase ?

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15. Find the intervals in which the function $f(x) = x^3 - 3x^2 + 4$ is strictly increasing for all $x \in R$

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16. Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x \, \forall x \in [0, \pi/2]$ is increasing and decreasing.

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Exercise 10 H

1. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = x^2, \, \forall x \in R$

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2. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$(x) = \sin x, [0, 4\pi]$

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3. Find the points at which the functions

$f(x) = x^3 - 6x^2 + 9x + 15$

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4. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = x\sqrt{(1-x)}\,\forall x \in (0, 1)$

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5. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = 1/\left(x^2 + 2\right)\forall x \in R$

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6. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = x^3 - 3x \, \forall x \in R$

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7. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = (x - 1)(x + 2)^2 \, \forall x \in R$

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8. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = \frac{x}{2} + \frac{2}{x} \forall x \in R(0, \infty)$

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9. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = -(x-1)^3(x+1)^2 \forall x \in R$

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10. Find the points of local extrema (if any) and local extrema of the following functions each of whose domain is shown against the function.

$f(x) = x^2 e^{3x} \forall x \in R$

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11. Prove that the functions do not have maxima or minima:

f(x) $= e^x$

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12. Prove that the functions do not have maxima or minima:

g(x) = log x

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13. Prove that the following functions do not have absolute maximum and absolute minimum.

$x^3 + x^2 + x + 1$ in R

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14. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) $= x^3, x \in$ [– 2, 2]

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15. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f(x) =$(x - 1)^2 + 3, x \in$ [-3 ,1]

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16. Find the absolute maximum value and absolute minimum value of the following functions on the domain specified against the function.

$f(x) = 2|x|$ on $[-1, 6]$

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17. Find the absolute maximum value and the absolute minimum value of the functions in the given intervals:

f (x) = sin x + cos x , $x \in [0, \pi]$

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18. Find the absolute maximum value and absolute minimum value of the following functions on the domain specified against the function.

$f(x) = x + \sin 2x$ on $[0, \pi]$

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19. Use the first derivative test to find the local extrema of $f(x) = x^3 - 12x$ on R.

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20. Use the first derivative test to find local extrema of $f(x) = x^2 - 6x + 8$ on R.

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21. Use the second derivative test to find local extrema of the function $f(x) = x^3 - 9x^2 - 48x + 72$ on R

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22. use the second derivative test to find local extrema of the function. $f(x) = -x^3 + 12x^5 - 5$

on R.

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23. Find local maximum or local minimum of $f(x) = -\sin 2x - x$ defined o $[-\pi/2, \pi/2]$.

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24. Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 3x^2 - 36x + 2$ on the interval [0, 5].

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25. Find the absoloute extremum of $f(x) = 4x - \frac{x^2}{2}$ on $\left[-2, \frac{9}{2}\right]$.

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26. Find the maximum profit that a company can make, if the profit function is given by p(x) = 41 – 24x – $18x^2$

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27. The profit function P (x) of a company selling x items is given by $P(x) = -x^3 + 9x^2 - 15x - 13$ where x represents units in thousands. Find the absolute maximum profit if the company can manufacture a maximum of 6000 units.

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28. The profit function P(x) of a company selling x items per day is given by $P(x) = (150 - x)x - 1000$. Find the number of items that the company should manufacture to get maximum profit. Also find the maximum profit.

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29. Find the absolute maximum and absolute minimum of $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-8, 2]$

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30. Find two positive integers whose sum is 16 and the sum of squares is minimum.

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31. Find the positive integers x and y such that $x + y = 60$ and xy^3 is maximum.

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32. From a rectangular sheet of dimension $30cm \times 80cm$, four equal squares of side x cm. are removed at the corners, and the sieds are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.

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33. A window is in the shape of a rectangle surmounted by a semi-circle. If the perimeter of the window be 20 feet then find the maximum area.

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34. Show that when the curved surface of a is right circular cylinder inscribed in a sphere of radius R is maximum , then the height of the cylinder is $\sqrt{2R}$.

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35. A wire of length l is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least.

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