MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

LIMITS AND CONTINUITY

Solved Problems

1. Evaluate the folllowing limits.

Evaluate $Lt_{x \to -3} \frac{1}{x+2}$

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2. Evaluate the folloiwng :

$$Lt_{x \to 2} \frac{x - 2}{x^3 - 8}$$

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3. Evaluate the folllowing limits.

Find $Lt_{x \to 1}(x + 2)(2x + 1)$

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4. Compute:

$$Lt_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$$

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5. Evaluate the folllowing limits.

$$Lt_{x \to 2} \frac{x^2 - 5}{4x + 10}$$

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6. Evaluate the folllowing limits.

$$Lt_{x \to 3} \frac{x^3 - 6x^2 + 9x}{x^2 - 9}$$

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7. Evaluate $Lt_{x \to 3} \frac{x^3 - 3x^2}{x^2 - 5x + 6}$

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8. Show that $Lt_{x \to 0^+} \frac{|x|}{x} = 1$ and $Lt_{x \to 0^-} \frac{|x|}{x} = -1 (x \neq 0)$.

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9. Let $f: R \to R$ defined by

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 3 \\ 5 & \text{if } x \geq 3 \end{cases}$$

Then show that $Lt_{x \to 3} f(x) = 5$

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10. Show that

$$Lt_{x \to -2} \sqrt{x^2 - 4} = 0 = Lt_{x \to 2} \sqrt{x^2 - 4}$$

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11. $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$ then

$Lt_{x \to 2+} f(x) =$

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12. Show that $Lt_{x \to 0} \frac{\tan x}{x} = 1.$

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13. Evaluate $Lt_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

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14. Evaluate $Lt_{x \to 0} \frac{e^x - 1}{\sqrt{1+x}-1}$

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15. Show that :

$$\lim_{x \to 3} \frac{x-3}{\sqrt{|x^2-9|}} = 0$$

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16. Compute $Lt_{x \to 0} \frac{a^x - 1}{b^x - 1} (a > 0, b > 0, b \neq 1).$

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17. Compute $Lt_{x \to 0} \frac{\sin ax}{\sin bx}, b \neq 0, a \neq b$

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18. Compute $Lt_{x \to 0} \frac{e^{3x} - 1}{x}.$

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19. Compute $\lim_{x \to 0} \frac{e^x - \sin x - 1}{x}$

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20. Evaluate $Lt_{x \to 1} \frac{\log_e x}{x - 1}$

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21. Using definition, Show that $Lt_{x \to \infty} \frac{1}{x^2} = 0$

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22. Show that $Lt_{x \to 0} \frac{e^x - 1}{x} = 1$

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23. Evaluate $Lt_{x \to 2} \frac{x^2 + 2x - 1}{x^2 - 4x + 4}$

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24. Compute the following limits.

$$\lim_{x \to \infty} \frac{3x^5 - 1}{4x^2 + 1}$$

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25. If $f(x) \frac{a_n x^n + .. + a_1 x + a_0}{b_m x^m + .. + b_1 x + b_0}$ When $a_n > 0, b_m > 0,$ then show that $Lt_{x \to \infty} \to f(x) = \infty$ if $n > m.$

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26. Compute $Lt_{x \to \infty} \frac{x^2 - \sin x}{x^2 + 2}$.

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27. Show that f(x) = [x] (x $\in$ R) is continuous at only those real numbers that are not integers.

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28. If f: R $\rightarrow$ R is a function such that f(x +y) =f(x) +f(y) $\forall$ x,y $\in$ R then f is continuous on R if it is continuous at a single point in R.

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29. Check the continuity of the function f defined by

$$f(x) = \begin{cases} x+1 & \text{if } x \leq 1 \\ 2x & \text{if } 1 < x < 2x \\ 1 + xA^2 & \text{if } x \geq 2 \end{cases}$$ at x=1 and 2

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30. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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31. Show that the function f defined on R by $f(x\} = |1 + 2x + |x||, x\varepsilon R$ is continuous fanction.

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Exercise 8 A

1. Evaluate the following limits.

$$Lt_{x \to a} \frac{x^2 - a^2}{x - a}$$

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2. Evaluate the following limits.

$$Lt_{x \to 1} \left(x^2 + 2x + 3\right)$$

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3. Evaluate the following limits.

$$Lt_{x \to 0} \frac{1}{x^2 - 3x + 2}$$

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4. Evaluate the following limits.

$$Lt_{x \to 3} \frac{1}{x + 1}$$

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5. Find $Lt_{x \to 1} \frac{2x - 1}{3x^2 - 4x + 5}$

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6. Evaluate the following limits.

$$Lt_{x \to 1}\left(\frac{x^2+2}{x^2-2}\right)$$

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7. Evaluate the following limits.

$$Lt_{x \to 2}\left(\frac{2}{x+1}-\frac{3}{x}\right)$$

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8. Evaluate the following limits.

$$Lt_{x \to 0} \frac{x - 1}{x^2 + 4}$$

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9. Evaluate :

$$Lt_{x \to 0} x^{3/2} (x > 0)$$

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10. Evaluate:

$Lt_{x \to 0} \sqrt{x} + x^{5/2} (x > 0)$

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11. Compute the following limits.

$Lt_{x \to 0} x^2 \cos\left(\frac{2}{x}\right)$

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12. Compute $Lt_{x \to 3} \frac{x^2 - 9}{x^3 - 6x^2 + 9x + 1}$

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13. Evaluate the following limits.

$$Lt_{x \to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$$

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14. Evaluate the following limits in

$$\lim_{x \to 3} \frac{x^4-81}{2x^2-5x-3}$$

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15. Evaluate :

$$Lt_{x \to 3} \frac{x^2 - 8x + 15}{x^2 - 9}$$

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16. Evaluate the following limits.

If $f(x) = -\sqrt{25 - x^2}$ then find

$$Lt_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

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Exercise 8 B

1. Find the left and right hand limits of the function at the point 'a' mentioned against them. Hence check whether the function have limits at those points.

$$f(x) = \begin{cases} 1 - x & \text{if} \quad x \leq 1 \\ 1 + x & \text{if} \quad x > 1 \end{cases}, a = 1$$

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2. Find the left and right hand limits of the function at the point 'a' mentioned against

them. Hence check whether the function have limits at those points.

$$f(x) = \begin{cases} x+2 & \text{if } -1 < x \leq 3 \\ x^2 & \text{if } 3 < x < 5 \end{cases}, a = 3$$

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3. Find the left and right hand limits of the function at the point 'a' mentioned against them. Hence check whether the function have limits at those points.

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x < 2 \\ \frac{x^2}{3} & \text{if } x \geq 2 \end{cases}, a = 2$$

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4. Find the left and right hand limits of the function at the point 'a' mentioned against them. Hence check whether the function have limits at those points.

$$f(x) = \begin{cases} 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } 0 \leq x < 1 \\ 3x & \text{if } x > 1 \end{cases}, a = 1$$

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5. Find the left and right hand limits of the function at the point 'a' mentioned against

them. Hence check whether the function have limits at those points.

$$f(x) = \begin{cases} x^2 \ , & x \leq 1 \\ x \ \text{ if } \ 1 < x \leq 2 \, , a = 2 \\ x - 3 \ \text{ if } \ x > 2 \end{cases}$$

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6. Show that

$$Lt_{x \to 2-} \frac{|x - 2|}{x - 2} = -1$$

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7. Show that $Lt_{x \to 0+}\left(\frac{2|x|}{x} + x + 1\right) = 3.$

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8. Compute $Lt_{x \to 2^+}(|x| + x)$ and $Lt_{x \to 2^-}(|x| + x).$

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9. Find $Lt_{x \to 0} f(x)$ where

$$f(x) = \begin{cases} x - 1 & \text{if } > 0 \\ 0 & \text{if } x = 0 \\ x + 1 & \text{if } x > 0 \end{cases}$$

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10. Define $f: \left[-\frac{1}{2}, \infty \right) \to R$ by $f(x) = \sqrt{1 + 2x}, x \varepsilon \left[-\frac{1}{2}, \infty \right)$. Then compute $Lt_{x \to \left(\frac{1}{2}\right)^+} \to f(x)$. Hence find $Lt_{x \to -\frac{1}{2}} f(x)$.

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Exercise 8 C

1. Compute the following limits

$$Lt_{x \to 1}\left[\frac{2x+1}{3x^2-4x+5}\right]$$

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2. Compute the following limits

$$Lt_{x \to \frac{\pi}{2}} \frac{\cos x}{\left[x-\frac{\pi}{2}\right]}$$

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3. Compute the following limits

$$Lt_{x \to 0} \frac{\sin ax}{x \cos x}$$

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4. Evaluate the following limits.

$$Lt_{x \to 1} \frac{\sin(x-1)}{x^2-1}$$

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5. Compute the following limits

$$Lt_{x \to 0} \frac{\sin(a + bx) - \sin(a - bx)}{x}$$

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6. Evaluate the following limits.

$$Lt_{x \to a} \frac{\tan(x - a)}{x^2 - a^2} (a \neq 0)$$

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7. Compute the following limits

$$Lt_{x \to 0} \frac{e^{7x} - 1}{x}$$

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8. Evaluate the following limits :

$$Lt_{x \to 0} \frac{e^{3+x} - e^{3}}{x}$$

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9. Compute the following limits

$$Lt_{x \to 3} \frac{e^x - e^3}{x - 3}$$

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10. Compute the following limits

$$Lt_{x \to 0} \frac{e^{\sin x} - 1}{x}$$

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11. Compute the following limits :

$$Lt_{x \to 1} \frac{(2x-1)\left(\sqrt{x}-1\right)}{2x^2+x-3}$$

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12. Find $Lt_{x \to a}\left(\frac{x \sin a - a \sin x}{x-a}\right)$

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13. Compute the following limits

$$Lt_{x \to 0}\left[\frac{\cos ax - \cos bx}{x^2}\right]$$

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14. Evaluate the following limits.

$$Lt_{x \to 2} \frac{2x^2 - 7x - 4}{(2x - 1)\left(\sqrt{x} - 2\right)}$$

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15. Evaluate the following limits :

$$Lt_{x \to 0} \frac{\log_e(1 + 5x)}{x}$$

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16. Evaluate the following limits.

$$Lt_{x \to 0} \frac{\sin\left(\pi \cos^2 x\right)}{x^2}$$

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17. Evulate $Lt_{x \to 0} \frac{(1+x)^{\frac{1}{8}} - (1-x)^{\frac{1}{8}}}{x}$

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18. Evaluate the following limits:

$$\lim_{x \to 0} \frac{3^x - 1}{\sqrt{1+x} - 1}$$

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19. Compute the following limits

$$Lt_{x \to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

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20. Evaluate $Lt_{x \to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

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21. Show that

$$Lt_{x \to a} \frac{\sin(x-a)\tan^2(x-a)}{(x^2-a^2)^3} = \frac{1}{8a^3}$$

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22. Compute the following limits

$$Lt_{x \to 0} \frac{1-\cos 2mx}{\sin^2 nx} (m, n\varepsilon - z)$$

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23. Compute the following limits :

$$Lt_{x \to 0} \frac{1-\cos x}{x^2}$$

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24. Compute the following limits

$$Lt_{x \to 0} \frac{\sec x - 1}{x^2}$$

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25. Compute the following limits

$$Lt_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}, N \neq 0$$

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26. Compute the following limits

$$Lt_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$$

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27. Compute the following limits

$$Lt_{x \to 0} \frac{\log(1 + x^3)}{\sin^3 x}$$

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28. Evaluate $Lt_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$

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Exercise 8 D

1. Evaluate $Lt_{x \to 3} \frac{x^2 + 3x + 2}{x^2 - 6x + 9}$

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2. Compute the following limits.

$Lt_{x \to \infty} \frac{1 + 4x^2}{1 - x^2}$

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3. Evaluate $Lt_{x \to \infty} \dfrac{3x^2 + 4x + 5}{2x^2 + 3x - 7}$

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4. Compute the following limits.

$$\lim_{x \to \infty} \frac{6x^2 - x + 7}{x + 3}$$

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5. Evaluate the following limits:

$$Lt_{x \to \infty} e^{-x^2}$$

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6. Compute the following limits.

$$Lt_{x \to \infty} \frac{\sqrt{x^2 + 6}}{2x^2 - 1}$$

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7. Compute the following limits.

$$Lt_{x \to \infty} \frac{8|x| + 3x}{3|x| - 2x}$$

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8. Compute $\lim_{x \to \infty} \frac{x^2 + 5x + 2}{2x^2 - 5x + 1}$

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9. Compute the following limits.

$$\lim_{x \to -\infty} \frac{2x^2 - x + 3}{x^2 - 2x + 3}$$

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10. Evaluate $Lt_{x \to \infty} \frac{11x^3 - 3x + 4}{13x^3 - 5x^2 - 7}$

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11. Evaluate the following limits.

$$\lim_{x \to 2} \frac{1}{x - 2} - \frac{4}{x^2 - 4}$$

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12. Find $Lt_{x \to -\infty} \dfrac{5x^3 + 4}{\sqrt{2x^4 + 1}}$

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13. Evaluate the following limits.

$$\lim_{x \to \infty} \left(\sqrt{x+1} - \sqrt{x}\right)$$

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14. Evaluate the following limits.

$$Lt_{x \to \infty} \left(\sqrt{x^2 + x} - x\right)$$

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15. Compute the following limits.

$$\lim_{x \to \infty} \frac{2x + 3}{\sqrt{x^2 - 1}}$$

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16. Compute the following limits.

$$Lt_{x \to \infty} \frac{2 + \sin x}{x^2 + 3}$$

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17. Compute the following limits.

$$\lim_{x \to \infty} \frac{2 + \cos^2 x}{x + 2007}$$

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18. Compute the following limits.

$$\lim_{x \to \infty} \frac{6x^2 - \cos 3x}{x^2 + 5}$$

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19. Compute the following limits.

$$Lt_{x \to \infty} \frac{cosx + \sin^2 x}{x + 1}$$

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Exercise 8 E

1. Is the function f, defined by $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ continuous on R?

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2. Is f defined by $f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

continuous at 0?

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3. Show that the function $f(x) = \left[\cos\left(x^{10} + 1\right)\right]^{1/3}, x \in R$ is

continuous function.

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4. Check the continity of the following function

at 2 . $f(x) = \begin{cases} \frac{1}{2}\left(x^2 - 4\right) & \text{if } 0 < x < 2 \\ 0 & \text{if } x = 2 \\ 2 - 8x^{-3} & \text{if } x > 2 \end{cases}$

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5. Check the continuity of the functionn 'f' defined by

$fx = \begin{cases} \frac{x^2 - 9}{x^2 - 2x - 3} & \text{if } 0 < x < 5 \text{ and } x \neq 3 \\ 1.5 & \text{if } x = 3 \end{cases}$

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6. Show that f ,given by $f(x) = \frac{x - |x|}{x}(x \neq 0)$ is continuous on R - {0}

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7. If f is a function defined by

$$f(x) = \begin{cases} \frac{x-1}{\sqrt{x}-1} & \text{if } x > 1 \\ 5 - 3x & \text{if } -2 < x \leq 1 \\ \frac{6}{1-10} & \text{if } x < -2 \end{cases}$$ then

disuss the continuity of f .

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8. If f is given by $f(x) = \begin{cases} k^2x - k & \text{if } x \geq 1 \\ 2 & \text{if } x < 1 \end{cases}$ is a continuous function on R, then find k .

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9. Prove that the functions sin x, cos x are continuous on R

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10. Check the continuity of function defined by

$$f(x) = \begin{cases} 4 - x^2 & \text{if} \quad x \leq 0 \\ x - 5 & \text{if} \quad 0 < x \leq 1 \\ 4x^2 - 9 & \text{if} \quad 1 < x < 2 \\ 3x + 4 & \text{if} \quad x \geq 2 \end{cases}$$ at the

points x=0, and 2.

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11. Find real contans, a b so that the function

'f' given by

$$f(x) = \begin{cases} \sin x & \text{if} \quad x \leq 0 \\ x^2 + a & \text{if} \quad 0 < x < 1 \\ bx + 3 & \text{if} \quad 1 \leq x \leq 3 \\ -3 & \text{if} \quad x > 3 \end{cases}$$ is

continuous on R.

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12. Show that

$$f(x) = \begin{cases} \dfrac{\cos ax - \cos bx}{x^2} & \text{if} \quad x \neq 0 \\ \dfrac{1}{2}\left(b^2 - a^2\right) & \text{if} \quad x = 0 \end{cases}$$

where a and b are real constants is continuous at x = 0.

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