MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

PAIR OF STRAIGHT LINES

Solved Problems

1. Does the equation $x^2 + xy + y^2 = 0$represent a pair of lines?

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2. Find the nature of the triangle of formed by the lines $x^2 - 3y^2 = 0$ and $x = 2$.

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3. Find the centroid and the area of the triangle formed by the lines $12x^2 - 20xy + 7y^2 = 0, 2x - 3y + 4 = 0$

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4. Prove that the lines represented by the equations $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle.

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5. Show that the product of the perpendicular from (alpha,beta) to the pair of lines $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{\left|a\alpha^2 + 2h\alpha\beta + 2g\alpha + 2f\beta + c\right|}{\sqrt{(a-b)^2 + 4h^2}}$ Hence or otherwise find the product of the perpendicular from the origin

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6. Let $ax^2 + 2hxy + by^2 = 0$ represent a pair of straight lines. Then show that the equation of the pair of straight lines.

Passing through (x_0, y_0) and parallel to the given pair of lines is

$a(x - x_0)^2 + 2h(x - x_0)(y - y_0) + b(y - y_0)^2 = 0$

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7. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $\mathrm{lx} + \mathrm{my} + n = 0$ is

$$\left| \frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2} \right|$$

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8. Two equal sides of an isoceles triangle are given by 7x-y+3=0 and x+y-3=0 and the third side passes through the point (1,10) then slope m of the third side is given by

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9. Find the angle between the straight lines represented by $2x^2 + 5xy + 2y^2 - 5x - 7y + 3 = 0$

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10. Find the equation of the pair of lines passing through the origin and parallel to the pair of lines $2x^2 + 3xy - 2y^2 - 5x + 5y - 3 = 0$

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11. Find the equation of the pair of lines passing through the origin and perpendicular to the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

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12. If $x^2 + xy - 2y^2 + 4x - y + k = 0$ represents a pair of straight lines, find k.

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13. Prove that the equation $2x^2 + xy - 6y^2 + 7y - 2 = 0$ represents a pair of straight lines.

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14. Prove that the equation $2x^2 + 3xy - 2y^2 - x + 3y - 1 = 0$ represents a pair of perpendicular straight lines.

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15. S.T the equation $2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ represents a pair of straight lines. Also find the angle between them and the coordinates of the point of intersection of the lines.

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16. Find the value of λ for which the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of striaght lines.

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17. Show that two pairs of two straight lines $6x^2 - 5xy - 6y^2 = 0$ and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ form a square

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18. Show that the following equations represents a pair of parallel lines and also find the distance between them.

Show that the equation $8x^2 - 24xy + 18y^2 - 6x + 9y - 5 = 0$ represents a pair of parallel lines and find the distance between them.

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19. If the pair of lines $ax^2 + 2hxy + by^2 = 0, ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ form a rhombus then prove that $(a - b)fg + h(f^2 - g^2) = 0.$

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20. If two of the sides of a parallelogram are represented by $ax^2 + 2hxy + by^2 = 0$ and $pq + qy = 1$ is one of its diagonals, prove that the other diagonal is $y(bp - hq) = x(aq - hp)$.

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Exercise 4 A I

1. Find the acute angle between pair of lines represented by following equations.

$x^2 - 7xy + 12y^2 = 0$

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2. Find the acute angle between pair of lines represented by following equations.

$y^2 - xy - 6x^2 = 0$

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3. Find the acute angle between pair of lines represented by following equations.

$x^2 + 2xy \cot \alpha - y^2 = 0$

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4. Find the acute angle between pair of lines represented by following equations.

$x^2 + 2xy \cot \alpha - y^2 = 0$

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Exercise 4 A Ii

1. Show that the following pairs of lines are equally inclined to each other.

$2x^2 + 6xy + y^2 = 0, 4x^2 + 18xy + by^2 = 0$, b=

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2. Show that the following pairs of lines are equally inclined to each other.

$a^2x^2 + 2h(a + b)xy + b^2y^2 = 0, ax^2 + 2hxy + by^2 = 0(a + b \neq 0)$

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3. Show that the following pairs of lines are equally inclined to each other.

$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0, (\lambda \in R) ax^2 + 2hxy + by^2 = 0$

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4. Find the value of h if the slopes of the lines represented by $6x^2 + 2hxy + y^2 = 0$ are in the ratio 1:2.

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5. If $ax^2 + 2hxy + by^2 = 0$ represents two straight lines such that slope of one line is twice the slope of the other, prove that $8h^2 = 9ab$.

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6. Show that equation of pair of lines passing through origin and making an angle of 30° with the line $3x - y - 1 = 0 is 13x^2 - 12xy - 3y^2 = 0$

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7. Find the equation to the pair of lines passing through origin and making an acute angle a' with the straight line x + y + 5 =0

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8. Show that the lines $(x + 2a)^2 - 3y^2 = 0, x = a$ form an equilateral triangle.

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9. Show that pair of bisectors of angles between the lines $(ax+by)^2 = c(bx-ay)^2 (c>0)$ are parallel and perpendicular to the line $ax+by+k=0$

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10. The adjacent sides of a parallelogram are $2x^2 - 5xy + 3y^2 = 0$ and one diagonal is x + y + 2 = 0. Find the vertices and the other diagonal.

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11. Find the centriod and hence find the area of the triangle formed by the following lines

$2y^2 - xy - 6x^2 = 0x + y + 4 = 0$

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12. Find the centriod and hence find the area of the triangle formed by the following lines

$3x^2 - 4xy + y^2 = 0, 2x - y = 6$

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13. Find the equation of pair of lines intersecting at (2, -1) and Perpendicular to pair $6x^2 - 13xy - 5y^2 = 0$

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14. Find the equation of pair of lines intersecting at (2, -1) and Perpendicular to pair $6x^2 - 13xy - 5y^2 = 0$

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15. Find the equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0, 12x + 5y - 2 = 0$

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16. Find the equation of bisector of the obtuse angle between the lines $x + y - 5 = 0$ and $x - 7y + 7 = 0$

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Exercise 4 A Iii

1. Show that the lines represented by $(lx + \mathrm{my})^2 - 3(mx - ly)^2 = 0$ and $lx + \mathrm{my} + n = 0$ form an equilateral triangle with area $\frac{n^2}{\sqrt{3}(l^2 + m^2)}$.

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2. Show that the straight lines represented by $3x^2 + 48xy + 23y^2 = 0, 3x - 2y + 13 = 0$ form an equilateral triangle of area $\frac{13}{\sqrt{3}}$ sq. units

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3. Show that the equation of the pair of lines bisecting the angles between the pair of bisectors of the angles between the pair of lines $ax^2 + 2hxy + by^2 = 0$ is $(a-b)(x^2 - y^2) + 4hxy = 0$

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4. If one jine of the pair of lines $ax^2 + 2hxy + by^2 = 0$ bisects the angle between the coordinate axes, then prove that $(a+b)^2 = 4h^2$.

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5. If (α, β) is the centroid of the triangle, whose sides are $ax^2 + 2hxy + by^2 = 0$ and $lx + my = 1$, then show that $\frac{\alpha}{bl - hm} = \frac{\beta}{am - hl} = \frac{2}{3(bl^2 - 2hlm + am^2)}$

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6. Prove that the distance from the origin to the orthocentre of the triangle formed by the lines $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ and $ax^2 + 2hxy + by^2 = 0$ is $\left(\alpha^2 + \beta^2\right)^{1/2}$ $\left|\frac{(a+b)\alpha\beta}{a\alpha^2 - 2h\alpha\beta + b\beta}\right|$.

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7. The straight line $lx + my + n = 0$ bisects and angle between the pair of lines of which one is $px + qy + r = 0$. Show that the other line is $(l^2m^2)(px + qy + r) = 2(lp + mq)(lx + my + n) = 0$

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Exercise 4 B I

1. Find the angle between the lines represented by $2x^2 + xy - 6y^2 + 7y - 2 = 0$.

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2. Prove that the equation $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines and find the lines.

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Exercise 4 B Ii

1. Prove that the equation $3x^2 + 7xy + 2y^2 + 5x + 5y + 2 = 0$ represents a pair of straight lines and find the co-ordinates of the point of intersection.

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2. Find k, if the equation $2x^2 + kxy - 6y^2 + 3x + y + 1 = 0$ represents a pair of lines. Find the point of intersection of the lines and angle between the lines for this value of k.

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3. Show that the equation $x^2 - y^2 - x + 3y - 2 = 0$ represents a pair of perpendicular lines and find their equations.

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4. Show that the line $x^2 + 2xy - 35y^2 - 4x + 44y - 12 = 0$ and $5x + 2y - 8 = 0$ are concurrent.

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5. Show that the following equations represents a pair of parallel lines and also find the distance between them.

$9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$

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6. Show that the following equations represents a pair of parallel lines and also find the distance between them.

$x^2 + 2\sqrt{3}xy + 3y^2 - 3x - 3\sqrt{3}y - 4 = 0$

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7. Show that two pairs of lines $3x^2 + 8xy - 3y^2 = 0$ and $3x^2 + 8xy - 3y^2 + 2x - 4y - 1 = 0$ forms a square.

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Exercise 4 B Iii

1. Find the product of lengths of perpendiculars drawn from (2, 1) upon the lines $12x^2 + 25xy + 12y^2 + 10x + 11y + 2 = 0$

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2. Show that the straight line $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 + 5x + 10y + 4 = 0$ form a parallelogram and find the length of its sides.

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3. Show that product of the perpendicular distances from origin to pair of lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\frac{|c|}{\sqrt{(a-b)^2 + 4h^2}}$

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4. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines, then show that the square of the

distance of their point of intersection from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}$. Also show that the square of this distance is $\frac{f^2+g^2}{h^2+b^2}$ if the given lines are perpendicular.

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Exercise 4 C I

1. Find the equation of the lines joining the origin to the points of intersection of $x^2+y^2=1$ and x + y = 1.

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2. Find the angle between the lines joining the origin to the points of intersection of $y^2=x$ and $x+y=1$.

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Exercise 4 C Ii

1. Show that the lines joining the origin to the points of intersection of the curve $x^2 + xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$are mutually perpendicular .

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2. Find the value if k , if the lines joining the origin with the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the x + 2y = k are mutually perpendicular .

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3. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2 + 2xy + y^2 + 2x + 2y - 5 = 0$ and

the line 3x-y+1=0.

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Exercise 4 C Iii

1. Find the condition for the chord lx + my=1 of the circle $x^2 + y^2 = a^2$ to subtend a right angle at the origin.

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2. Find the condition for the lines joining the origin to the points of intersection of the circle $x^2 + y^2 = a^2$ and the line lx+my=1 to coincide.

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3. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the $6x - y + 8 = 0$ with the pair of straight lines $3x^2 + 4xy - 4y^2 - 11x + 2y + 6 = 0$. Show that the lines so obtained make equal angles with the coordinates axes.

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