MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

PRODUCT OF VECTORS

Exercise 5 A I

1. Find the angle between the vectors $\overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}$ and $3\overrightarrow{i} - \overrightarrow{j} + 2\overrightarrow{k}$.

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2. If the vectors $2\bar{i} + \lambda\bar{j} - \bar{k}$ and $4\bar{i} - 2\bar{j} + 2\bar{k}$ are perpendicular to each other than find λ.

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3. For what values of λ the vectors $\bar{i} - \lambda\bar{j} + 2\bar{k}, 8\bar{i} + 6\bar{j} - \bar{k}$ are at right angles.

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4. $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$. Find the vector $\vec{c}$ such that $\vec{a}, \vec{b}$ and $\vec{c}$ form the sides of a triangle.

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5. Find the angle between the planes $\vec{r}.\left(2\vec{i} - \vec{j} + 2\vec{k}\right) = 3$ an $\vec{r}.\left(3\vec{i} + 6\vec{j} + \vec{k}\right) = 4.$

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6. Let $\overrightarrow{e_1}$ and $\overrightarrow{e_2}$ be unit vectors making angle θ. If $\frac{1}{2}\left|\overrightarrow{e_1}-\overrightarrow{e_2}\right|=\sin\lambda\theta$, then find λ.

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7. Let $\overrightarrow{a}=\overrightarrow{i}+\overrightarrow{j}+\overrightarrow{k}$ and $\overrightarrow{b}=2\overrightarrow{i}+3\overrightarrow{j}+\overrightarrow{k}$. Find

(i) The projection vector of $\overrightarrow{b}$ on $\overrightarrow{a}$ and its magnitude.

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8. Find the equation of the plane passing through the point (3,-2,1) and perpendicular to the vector (4,7,-4)

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9. If $\overrightarrow{a}=2\overrightarrow{i}+2\overrightarrow{j}-3\overrightarrow{k}$, $\overrightarrow{b}=3\overrightarrow{i}-\overrightarrow{j}+2\overrightarrow{k}$, then find the angle between $2\overrightarrow{a}+\overrightarrow{b}$ and $\overrightarrow{a}+2\overrightarrow{b}$.

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Exercise 5 A Ii

1. Find unit vector parallel to the XOY-plane and perpendicular to the vector $4\bar{i} - 3\bar{j} + \bar{k}$.

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2. If $\vec{a} + \vec{b} + \vec{c} = 0, \left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5 \text{ and } \left|\vec{c}\right| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

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3. If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3 \text{ and } \left|\vec{c}\right| = 4$ and each of $\vec{a}, \vec{b}, \vec{c}$ is perpendicular to the sum of the other two vectors, then find the magnitude of $\vec{a} + \vec{b} + \vec{c}$.

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4. Find the equation of the plane passing through the point $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ and perpendicular to the vector $3\bar{i} - 2\bar{j} - 2\bar{k}$ and the distance of this plane from the oringin.

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5. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of four complanar points such that $\left(\vec{a} - \vec{d}\right).\left(\vec{b} - \vec{c}\right) = \left(\vec{b} - \vec{d}\right).\left(\vec{c} - \vec{a}\right) = 0$. Show that the point $\vec{d}$ represents the orthocentre of the triangle with $\vec{a}, \vec{b}$ and $\vec{c}$ as its vertices.

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Exercise 5 A Iii

1. Show that the points (5,-1,1),(7,-4,7),(1,-6,10) and (-1,-3,4) are the vertices of a rhombus.

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2. Let $\bar{a} = 4\bar{i} + 5\bar{j} - \bar{k}, \bar{b} = \bar{i} - 4\bar{j} + 5\bar{k}$ and $c = 3\bar{i} + \bar{j} - \bar{k}$. Find the vector which is perpendicular to both $\bar{a}$ and $\bar{b}$ whose magnitude is twenty one times the magnitude of $\bar{c}$.

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3. G is centroid of ΔABC and a,b,c are the lengths of the sides BC, CA and AB respectively. Prove that $a^2 + b^2 + c^2 = 3\left(\overline{OA}^2 + \overline{OB}^2 + \overline{OC}^2\right) - 9\left(\overline{OG}\right)^2$ where O is any point.

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4. If a line makes angles $\alpha, \beta, \lambda, \delta$ with the four diagonals of a cube, then show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \lambda + \cos^2 \delta = \frac{4}{3}$.

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Exercise 5 B I

1. If $\left|\overline{P}\right| = 2, |\bar{q}| = 3 \text{ and } (\bar{p}, \bar{q}) = \frac{\pi}{6}$, then find $|\bar{p} \times \bar{q}|^2$

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2. If $\vec{a} = 2\vec{i} - \vec{j} + \vec{k} \text{ and } \vec{b} = \vec{i} - 3\vec{j} - 5\vec{k}$, then find $\left|\vec{a} \times \vec{b}\right|$.

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3. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k} \text{ and } \vec{b} = \vec{i} + 4\vec{j} - 2\vec{k}$, then find $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)$.

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4. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p.

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5. Compute $\vec{a} \times \left(\vec{b} + \vec{c}\right) + \vec{b} \times \left(\vec{c} + \vec{a}\right) + \vec{c} \times \left(\vec{a} + \vec{b}\right)$.

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6. If $\vec{p} = x\vec{i} + y\vec{j} + z\vec{k}$, find the value of $\left|\vec{p} \times \vec{k}\right|^2$.

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7. Compute $2\vec{j} \times \left(3\vec{i} - 4\vec{k}\right) + \left(\vec{i} + 2\vec{j}\right) \times \vec{k}$.

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8. Find a unit vector perpendicular to both $\vec{i} + \vec{j} + \vec{k}$ and $2\vec{i} + \vec{j} + 3\vec{k}$.

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9. If θ is the angle between the vectors $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$, then find $\sin\theta$.

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10. Find the area of the parallelogram having $\vec{a} = 2\vec{j} - \vec{k}$ and $\vec{b} = -\vec{i} + \vec{k}$ as adjacent sides.

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11. Find the area of the parallelogram whose diagonals are $3\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{i} - 3\bar{j} + 4\bar{k}$

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12. Find the area of the triangle having $(3\bar{i}+4\bar{j}), (-5\bar{i}+7\bar{j})$ as adjacent sides.

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13. Find unit vector perpendicular to the plane determined by the vectors. $\vec{a}=4\vec{i}+3\vec{j}-\vec{k}$ and $\vec{b}=2\hat{i}-6\vec{j}-3\vec{k}$.

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14. Find the area of the triangle whose vertices are $A(1, 2, 3), B(2, 3, 1)$ and $C(3, 1, 2)$.

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Exercise 5 B Ii

1. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

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2.

$\vec{a} = 2\vec{i} + \vec{j} - \vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$,

then find $\left(\vec{a} \times \vec{b}\right).\left(\vec{b} \times \vec{c}\right)$.

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3. Find the vector area and area of the parallelogram having $\bar{a} = \bar{i} + 2\bar{j} - \bar{k}, \bar{b} = 2\bar{i} - \bar{j} + 2\bar{k}$ as adjacent sides.

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4. If $\bar{a} \times \bar{b} = \bar{b} \times \bar{c} \neq 0$, then show that $\bar{a} + \bar{c} = \bar{p}\bar{b}$, where p is some scalar.

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5. Let $\vec{a}$ and $\vec{b}$ be vectors, satisfying $\left|\vec{a}\right| = \left|\vec{b}\right| = 5$ and $\left(\vec{a}, \vec{b}\right) = 45^\circ$. Find the area of the triangle having $\vec{a} - 2\vec{b}$ and $3\vec{a} + 2\vec{b}$ as two of its sides.

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6. Find the vector having magnitude $\sqrt{6}$ units and perpendicular to both $2\bar{i} - \bar{k}, 3\bar{j} - \bar{i} - \bar{k}$.

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7. Find a unit vector perpendicular to the plane determined by the points $P(1, -1, 2), Q(2, 0, -1)$ and $R(0, 2, 1)$.

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8. If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\vec{0}$ then show that $\vec{b}=\vec{c}$

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9. Find a vector of magnitude and perpendicular to both the vectors $\vec{a}=2\vec{i}-2\vec{j}+\vec{k}$ and $\vec{b}=2\vec{i}+2\vec{j}+3\vec{k}$

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10. If $|\bar{a}|=13, |\bar{b}|=5, \text{ and } \bar{a}.\bar{b}=60$, then find $|\bar{a}\times\bar{b}|$.

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11. Find the unit vector perpendicular to the plane passing through the points $(1,2,3), (2,-1,1)$ and $(1,2,-4)$.

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Exercise 5 B Iii

1. If $\bar{a}, \bar{b}, \bar{c}$ respresents the vertices A, B, and C respectively of ΔABC then prove that $\left|(\bar{a}\times\bar{b})+(\bar{b}\times\bar{c})+(\bar{c}\times\bar{a})\right|$ is twice the area of ΔABC.

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2. If $\vec{a}=2\vec{i}+3\vec{j}+4\vec{k}, \vec{b}=\vec{i}+\vec{j}-\vec{k}$ and $\vec{c}=\vec{i}-\vec{j}+\vec{k}$, then compute $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and verify that it is perpendicular to $\vec{a}$.

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3. If $\vec{a}=7\vec{i}-2\vec{j}+3\vec{k}, \vec{b}=2\vec{i}+8\vec{k}, \vec{c}=\vec{i}+\vec{j}+\vec{k}$, then verify that $\vec{a}\times\left(\vec{b}+\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)$ (or) prove that cross product is distributive over addition

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4. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}, \vec{c} = \vec{j} - \vec{k}$, then find vector $\vec{b}$ such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} . \vec{b} = 3$

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5. $\bar{a}, \bar{b}, \bar{c}$ are three vectors of equal magnitudes and each of them is inclined at an angle of 60° to the others. If $|\bar{a} + \bar{b} + \bar{c}| = \sqrt{6}$, then find $|\bar{a}|$.

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6. For any two vectors $\vec{a}$ and $\vec{b}$, show that $\left(1 + \left|\vec{a}\right|^2\right)\left(1 + \left|\vec{b}\right|^2\right) = \left|1 - \vec{a} . \vec{b}\right|^2 + \left|\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right|^2$

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7. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}$ is perpendicular to the plane of $\vec{b}, \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then find $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

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8.

$\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}, \vec{b} = -\vec{i} + 3\vec{j} + 2\vec{k}, \vec{c} = 4\vec{i} + 5\vec{j} - 2\vec{k}$ and

, then compute

$\left(\vec{a} \times \vec{b}\right).\vec{c} - \left(\vec{a} \times \vec{d}\right).\vec{b}$

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Exercise 5 C I

1. Compute $\left[\vec{i} - \vec{j}\,\vec{j} - \vec{k}\,\vec{k} - \vec{i}\right]$

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2. If $\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k}, \vec{b} = 2\vec{i} + \vec{j} - \vec{k}, \vec{c} = i + 3\vec{j} - 2\vec{k},$ then compute $\vec{a}.\left(\vec{b} \times \vec{c}\right)$.

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3. If $\vec{a} = (1, -1, -6), \vec{b} = (1, -3, 4)$ and $\vec{c} = (2, -5, 3),$ then compute the following (i) $\vec{a}.\left(\vec{b} \times \vec{c}\right)$, (ii) $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$

(iii) $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$.

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4. Simplify the following

(i) $(i - 2j + 3k) \times (2i + j - k).(j + k)$

(ii) $(2i - 3j + k).(i - j + 2k) \times (2i + j + k)$.

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5. Find the volume of the tetrahedron having the edges

$,\bar{i}+\bar{j}+\bar{k}, \bar{i}-\bar{j}, \bar{i}+2\bar{j}+\bar{k}.$

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6. Find t, for which the vectors $2\bar{i}-3\bar{j}+\bar{k}, \bar{i}+2\bar{j}-3\bar{k}, \bar{j}-t\bar{k}$ are coplanar.

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7. For non - coplanar vectors $\bar{a}, \bar{b}$ and $\bar{c}$ determine p for which the vectors $\bar{a}+\bar{b}+\bar{c}, \bar{a}+p\bar{b}+2\bar{c}$ and $-\bar{a}+\bar{b}+\bar{c}$ are coplaner.

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8. Determine λ, for which the volume of the parallelopiped having coterminus edges $\bar{i}+\bar{j}, 3\bar{i}-\bar{j}$ and $3\bar{j}+\lambda\bar{k}$ os 16 cubic units.

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9. Find the volume of the tetrahedron having the edges

$,\bar{i}+\bar{j}+\bar{k}, \bar{i}-\bar{j}, \bar{i}+2\bar{j}+\bar{k}.$

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10. Let a,b and $\vec{c}$ be non-coplanar vectors and $\vec{\alpha}=a+2b+3c, \vec{\beta}=2\vec{a}+\vec{b}-2\vec{c}$ and $\vec{\gamma}=3\vec{a}-7\vec{c}$, then find $\begin{bmatrix}\vec{\alpha} & \vec{\beta} & \vec{\gamma}\end{bmatrix}$:

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11. let a,b and c are non-coplanar vectors If 2vec(a)-vec(b)+3vec(c),vec(a)+vec)-2vec(c),vec(a)+vec(b)-3vec(c)=lamda[vecavecbvec(c)]

$, thenf \in dthevalueof$lamda`.

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12. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-coplanar vectors, if $\left[\vec{a}+2\vec{b} \quad 2\vec{b}+\vec{c} \quad 5\vec{c}+\vec{a}\right]=\lambda\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$, then find λ.

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13. If a,b,c are non-coplanar vectors, then find the value of $\dfrac{(a+2b-c).\,[(a-b)\times(a-b-c)]}{[a \ \ \mathrm{b\,c}]}$.

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14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, then find the value of $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$.

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15. $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors and $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$. If $|a|=2, \left|\vec{b}\right|=3, \left|\vec{c}\right|=4$ and $(b, c)=\dfrac{2\pi}{3}$, then find

$\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$.

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Exercise 5 C Ii

1. If $\left[\vec{b} \quad \vec{c} \quad \vec{d}\right] + \left[\vec{c} \quad \vec{a} \quad \vec{d}\right] + \left[\vec{a} \quad \vec{b} \quad \vec{d}\right] = [a \quad b \quad c]$ then show that the points with position vectors a,b,c and $\vec{d}$ are coplanar.

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2. It a,b and c are non-coplanar vectors and the four points with position vectors $2a + 3b - c, a - 2b + 3c, 3a + 4b + 2c$ and $ka - 6b + 6c$ are coplanar, then k=

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3. $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero and non-collinerr vectors and $\theta \neq 0$ is the angle between $\vec{b}$ and $\vec{c}$. If $(a \times b) \times c = \frac{1}{3}|b||c|a$, then find $\sin\theta$.

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4. Find the volume of the tetrahedron, whose vertices are $(1, 2, 1), (3, 2, 5), (2, -1, 0), (-1, 0, 1)$.

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5. Show that $\left(\vec{a} + \vec{b}\right).\left(\vec{b} + \vec{c}\right) \times \left(\vec{c} + \vec{a}\right) = 2[a \ \text{b c}]$.

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6. Show that the equation of plane passing through the points with position vectors $3\bar{i} - 5\bar{j} - \bar{k}, -\bar{i} + 5\bar{j} + 7\bar{k}$ and parallel to the vector $3\bar{i} - \bar{j} + 7\bar{k}$ is 3x+2y-z=0

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7. Prove that $\vec{a} \times \left[\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right] = \left(\vec{a} . \vec{a}\right)\left(\vec{b} \times \vec{a}\right)$.

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8. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplaner vectors, then $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) =$

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9. Show that $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} \times \vec{c}\right)\right] . \vec{d} = \left(\vec{a} . \vec{d}\right)\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.

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10. Show that $\vec{\alpha} . \left[\left(\vec{\beta} + \vec{\chi} \times \left(\vec{\alpha} + \vec{\beta} + \vec{\chi}\right)\right)\right] = 0.$

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11. Find λ in order that the four points $A(3, 2, 1), B(4, \lambda, 5), C(4, 2, -2)$ and $D(6, 5, -1)$ be coplanar.

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12. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.\left(2\bar{i}+2\bar{j}-3\bar{k}\right)=7, \bar{r}. =\left(2\bar{i}+5\bar{j}+3\bar{k}\right)=9$ and through the point (

.

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13. Find the equation of the plane passing through (a,b,c) and parallel to the plane $\bar{r}.\left(\bar{i}+\bar{j}+\bar{k}\right)=2.$

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14. Find the shortest distance between the lines and $\vec{r} = 6\vec{i} + 2\vec{j} + 2\vec{k} + \lambda\left(\vec{i} - 2\vec{j} + 2\vec{k}\right) \& \vec{r} = -4\vec{i} - \vec{k} + \mu\left(3\vec{i}\right.$

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15. Prove that the four points $4\bar{i} + 5\bar{j} + \bar{k}, -(\bar{j} + \bar{k}), 3\bar{i} + 9\bar{j} + 4\bar{k}, -4\bar{i} + 4\bar{j} + 4\bar{k}$ are coplanar.

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16. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then show that the vectors $\vec{a} - \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are planar.

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17. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points A,B and C respectively, then prove that the vector $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is perpendicular to the plane of ΔABC.

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Exercise 5 C Iii

1. If A=(1, -2, -1), B= (4, 0, -3), C = (1, 2, -1), D=(2, -4, -5)`, then distance between AB and CD is

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2. if $\vec{a}=\vec{i}-2\vec{j}+\vec{k}, \vec{b}=2\vec{i}+\vec{j}+\vec{k}, \vec{c}=\vec{i}+2\vec{j}-\vec{k}$, then find $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$ and $\left|\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right|$.

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3. If $\vec{a}=\vec{i}-2\vec{j}-3\vec{k}, \vec{b}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{c}=\vec{i}+3\vec{j}-2\vec{k}$, verify that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)\neq\left(\vec{a}\times\vec{b}\right)\times\vec{c}$.

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4. If

$\vec{a} = 2\vec{i} + \vec{j} - 3\vec{k}, \vec{b} = \vec{i} - 2\vec{j} + \vec{k}$ and $\vec{c} = -\vec{i} + \vec{j} - 4\vec{k}$, &

compute $\left|\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{\vec{d}}\right)\right|$

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5. If $\overline{A} = (1, a, a^2), \overline{B} = (1, b, b^2), \overline{C} = (1, c, c^2)$ are non-coplanar vectors and $\left|(a, a^2, 1 + a^3), (b, b^2, 1 + b^3), (c, c^2, 1 + c^3)\}\rangle\right| = 0$ then abc=

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6. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

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7. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}, \vec{b} = 2\vec{i} + \vec{j} + \vec{k}, \vec{c} = \vec{i} + \vec{j} + 2\vec{k}$ then find $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ and $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right|$.

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8. If $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 2$ and $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = 0$ then find the angle between $\vec{a}$ and $\vec{c}$.

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9. Let $\vec{a} = \vec{i} - \vec{k}, \vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k}$ and $\vec{c} = y\vec{i} + x\vec{j} + (1+x$, prove that the scalar triple product $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$ is independent of both x and y.

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10. Let $\vec{b} = 2\vec{i} + \vec{j} - \vec{k}, \vec{c} = \vec{i} + 3\vec{k}$. If $\vec{a}$ is a unit vector then find the maximum value of $\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$.

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11. Let $\vec{\alpha} = \vec{\iota} - \vec{\varphi}, \vec{\beta} = \vec{\varphi} - \vec{\kappa}, \vec{\chi} = \vec{\kappa} - \vec{\iota}$. Find unit vector $\vec{d}$ such that.

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Solved Problems

1. If $\vec{a} = 6\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} - 9\vec{j} + 6\vec{k}$ then find $\vec{a} . \vec{b}$ and the angle between $\vec{a}$ and $\vec{b}$.

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2. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$are and perpendicular to each other .

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3. IF $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$, then what is the angle between $\vec{a}$ and $\vec{b}$?

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4. If $\left|\vec{a}\right| = 11, \left|\vec{b}\right| = 23$ and $\left|\vec{a} - \vec{b}\right| = 30,$ then find the angle between the vectors $\vec{a}$, $\vec{b}$ and also find $\left|\vec{a} + \vec{b}\right|$.

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5. If $\vec{a} = \vec{i} - \vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - 3\vec{j} + \vec{k}$, then find the projection vector of $\vec{b}$ on $\vec{a}$ and its magnitude.

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6. If P, Q, R and S are points whose position vectors are $\bar{i}-\bar{k}, -\bar{i}+2\bar{j}, 2\bar{j}-3\bar{k}$ and $3\bar{i}-2\bar{j}-\bar{k}$ respectively, then find the component of $\overline{RS}$ on $\overline{PQ}$.

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7. If vectors $\lambda\bar{i}-3\bar{j}+5\bar{k}, 2\lambda\bar{i}-\lambda\bar{j}-\bar{k}$ are perpendicular to each other find λ.

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8. Let $\vec{a}=2\vec{i}+3\vec{j}+\vec{k}, \vec{b}=4\vec{i}+\vec{j}$ and $\vec{c}=\vec{i}-3\vec{j}-7\vec{k}$. Find the vector $\vec{r}$ such that $\vec{r}.\vec{a}=9, \vec{r}.\vec{b}=7$ and $\vec{r}.\vec{c}=6$.

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9. Show that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{k}$ form the vertices of a right angled triangle.

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10. P.T the smaller angle θ between any two diagonals of a cube is given by $\cos\theta = 1/3$

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11. Let $\bar{a}, \bar{b}, \bar{c}$ be non-zero mutually orthogonal vectors. If $x\bar{a}+y\bar{b}+z\bar{c}=\bar{0}, \quad \text{then show that} \quad x=y=z=0.$

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12. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be mutually orthogonal vectors of equal magnitudes. Prove that the vector $\bar{a}+\bar{b}+\bar{c}$ is equally inclined to each of $\bar{a}, \bar{b}$ and $\bar{c}$, the angle of inclination being $\cos^{-1}\frac{1}{\sqrt{3}}$

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13. The vectors $\overline{AB} = 3\overline{i} - 2\overline{j} + 2\overline{k}$ and $\overline{BC} = -\overline{i} - 2\overline{k}$ represent adjacent sides of a parallelogram ABCD. Find the angle between the diagonals.

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14. Find the Cartesian equation of the plane passing through the point $(-2, 1, 3)$ and perpendicular to the vector $3\overline{i} + \overline{j} + 5\overline{k}$.

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15. Find the cartesian equation of the plane through the point $A(2, -1, -4)$ and parallel to the plane $4x - 12y - 3z - 7 = 0$.

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16. Find the angle between the planes $2x-3y-6z=5$ and $6x+2y-9z=4.$

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17. Find unit vector orthogonal to the vector $3\bar{i}+2\bar{j}+6\bar{k}$ and coplanar with the vectors $2\bar{i}+\bar{j}+\bar{k}$ and $\bar{i}-\bar{j}+\bar{k}.$

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18. If $\vec{a}=2\vec{i}-3\vec{j}+5\vec{k}, \vec{b}=-\vec{i}+4\vec{j}+2\vec{k}$, then find $\vec{a}\times\vec{b}$ and unit vector perpendicular to both a and b.

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19. If $\bar{a} = 2\bar{i} - 3\bar{j} + 5\bar{k}, \bar{b} = -\bar{i} + 4\bar{j} + 2\bar{k},$ then find $(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})$ and unit vector perpendicular to both $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$

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20. Find the area of the parallelogram for which the vectors $\vec{a} = 2\vec{i} - 3\vec{j}$ and $\vec{b} = 3\vec{i} - \vec{k}$ are adjacent sides.

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21. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ then show that $\vec{a} - \vec{d}$ and $\vec{b} - \vec{c}$ are parallel vectors.

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22. If $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}, \bar{b} = 3\bar{i} + 5\bar{j} - \bar{k}$ are 2 sides of a triangle, find its area.

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23. Let $\vec{a} = 2\vec{i} - j + \vec{k}$ and $\vec{b} = 3i + 4j - \vec{k}$. If θ is the angle between $\vec{a}$ and $\vec{b}$, then find $\sin\theta$.

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24. Let $\vec{a}, \vec{b}, \vec{c}$ be such that $\vec{c} \neq \vec{0}, \vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then show that $\vec{a}, \vec{b}$ and $\vec{c}$ are pairwise perpendicular, $\left|\vec{b}\right| = 1$ and $\left|\vec{c}\right| = \left|\vec{a}\right|$.

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25. Let $\overrightarrow{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\overrightarrow{b} = \hat{i} + \hat{j}$. If $\overrightarrow{c}$ is a vector such that $\overrightarrow{a} \cdot \overrightarrow{c} = \overrightarrow{c} - \overrightarrow{a} = 2\sqrt{2}$ and the angle between $\left|\left(\overrightarrow{a} \times \overrightarrow{b}\right) \times \overrightarrow{c}\right|$ equals:

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26. Let $\bar{a}, \bar{b}$ be two non-collinear unit vectors, if $\overline{\alpha} = \bar{a} - (\bar{a}. \bar{b})\bar{b}$ and $\bar{\beta} = \bar{a} \times \bar{b}$, then show that $|\bar{\beta}| = |\overline{\alpha}|$.

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27. A non-zero vector $\bar{a}$, is parallel to the line of intersection of the plane determined by the vectors $\bar{i}, \bar{i} + \bar{j}$ and the plane determined by the vectors $\bar{i} - \bar{j}, \bar{i} + \bar{k}$ Find the angle between $\bar{a}$ and the vector $\bar{i} - 2\bar{j} + 2\bar{k}$

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28. If $\overrightarrow{a} = 4\overrightarrow{i} + 5\overrightarrow{j} - \overrightarrow{k}, \overrightarrow{b} = \overrightarrow{i} - 4\overrightarrow{j} + 5\overrightarrow{k}$ and $\overrightarrow{c} = 3\overrightarrow{i} + \overrightarrow{j} - \overrightarrow{k}$, the find vector $\overrightarrow{\alpha}$ which is perpendicular to both $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{\alpha}.\overrightarrow{c} = 21$

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29. For any vector $\bar{a}$ show that $|\bar{a} \times \bar{i}|^2 + |\bar{a} \times \bar{j}|^2 + |\bar{a} \times \bar{k}|^2 = 2|\bar{a}|^2$

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30. If $\overrightarrow{a}$ is non-zero vector and $b, \overrightarrow{c}$ are two vector such that $\overrightarrow{a} \times b = a \times \overrightarrow{c}$ and $\overrightarrow{a}.b = a.\overrightarrow{c}$, then prove that $\overrightarrow{b} = \overrightarrow{c}$.

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31. Prove that the vectors $\overrightarrow{a} = 2\overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}, \overrightarrow{b} = \overrightarrow{i} - 3\overrightarrow{j} - 5\overrightarrow{k}$ and $\overrightarrow{c} = 3\overrightarrow{i} - 4\overrightarrow{j} - 4\overrightarrow{k}$ are coplanar.

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32. Find the volume of the parallelopiped having co-tenninus edges are represented by the vectors $2\bar{i}-3\bar{j}+\bar{k}, \bar{i}-\bar{j}+2\bar{k}, 2\bar{i}+\bar{j}-\bar{k}$.

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33. If the vectors $\vec{a}=2\vec{i}-\vec{j}+\vec{k}, \vec{b}=\vec{i}+2\vec{j}-3\vec{k}$ and $\vec{c}=3\vec{i}+p\vec{j}+5\vec{k}$ are coplanar, then find p.

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34. Show that $\vec{i}\times\left(\vec{a}\times\vec{i}\right)+\vec{j}\times\left(\vec{a}\times\vec{j}\right)+k\times\left(\vec{a}\times\vec{k}\right)=2\vec{a}$ for any vector a.

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35. Prove that for any three vectors $\vec{a}, \vec{b}, \vec{c}, [b+c \quad c+a \quad a+b] = 2[a \quad \mathrm{b} \; \mathrm{c}]$

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36. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ prove that $\left[\vec{b} \times \vec{c} \quad \vec{c} \times \vec{a} \quad \vec{a} \times \vec{b}\right] = \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2$

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37. If a,b,c are three unit vectors such that $a \times (b \times c) = \frac{1}{2}b$ then the angles between a,b and a,c are

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38. Find the equation of the plane passing through the point A=(2,3,-1),B=(4,5,2),C=(3,6,5).

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39. Find the equation of the plane passing through the point $A=(3,\ -2,\ -1)$ and parallel to the vector $\vec{b}=\vec{i}-2\vec{j}+4\vec{k}$ and $\vec{c}=3\vec{i}+2\vec{j}-5\vec{k}$.

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40. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.\left(\bar{i}+\bar{j}+\bar{k}\right)=6$ and $\bar{r}.\left(2\bar{i}+3\bar{j}+\bar{k}\right)=-5$ and the point $(1,1,1)$.

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41. Find the distance of a point (2,5,-3) from the planer r.(6i-3j+2k)=4

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42. Find the angle between line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

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43. For any four vectors a,b,c and d, $(a \times b) \times (c \times d) = [\text{a c d}]b - [\text{b c d}]a$ and $(a \times b) \times (c \times d) = [\text{a b d}]c -$

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44. Find the shortest distance between the skew lines .

$\bar{r} = (6\bar{i} + 2\bar{j} + 2\bar{k}) + t(\bar{i} - 2\bar{j} + 2\bar{k})$ and $\bar{r} = (-4\bar{i} - \bar{k}) + s(3\bar{i} - 2\bar{j}$

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45. Prove that angle in a semi circle is a rightangle by using Vector method.

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46. Show that the vector area of the quadrilateral ABCD having diagonals $\overrightarrow{AC}, \overrightarrow{BD}$ is $\frac{1}{2}\left(\overrightarrow{AC} \times \overrightarrow{BD}\right)$.

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