MATHS

BOOKS - UNITED BOOK HOUSE

SETS,FUNCTIONS AND CALCULUS

Exercise

1. If $A = \{a, b, c\}$, $B = \{b, c, d\}$, $P = \{a, c, d)$, $Q = \{b, d, e\}$, show that $(A \times B) \cap (P \times Q) = (A \cap P) \times (B \cap Q)$

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2. Find the inverse of the function $f(x) = \frac{x^4 + x^2 + 1}{x^2}$

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3. If $f(x) = \cos\left[\pi^2\right]x + \cos\left[-\pi^2\right]x$where [x] denotes the greatest integer function, then show that $f(\pi/4) = \dfrac{1}{\sqrt{2}}$

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4. Evaluate the following limits : $\lim_{x \to \frac{\pi}{2}} \left(x \tan x - \dfrac{\pi}{2}\sec x\right)$

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5. Evaluate $\lim_{x \to \pi/4} \dfrac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

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6. Evaluate : $\lim_{x \to 0} \dfrac{(a+x)^2 \sin(a+x) - a^2 \sin a}{\pi}$

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7. Evaluate the following limits : $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$

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8. Let $f(x) = \frac{4^x}{4^x + 2}$ prove.that f(x) + f(1- x)= 1. Hence prove that $f\left(\frac{1}{1997}\right) + f\left(\frac{2}{1997}\right) + ... + f\left(\frac{1996}{1997}\right) = 998$

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9. If $f(a) = 2, f'(a) = 1, g(a) = -1$ and $g'(a) = 2$, find the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$

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10. If f(x + y) = f(x)f(y) for all x, y and and f(x) = 1 + x g(x), where $\lim_{x \to 0} g(x) = 1)$, show that f'(x) = f(x).

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11. Find the domain of definition of the function y(x) given by the equation $2^x + 2^y = 2$.

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12. If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$,find f(x).

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13. If $\phi(x)$is the inverse of g(x)and g'(x)=$\frac{1}{1+x^3}$,show that $\phi'(x) = 1 + [\phi(x)]^3$

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14. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x$

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15. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$,show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

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16. If $y=\sqrt{(a-x)(x-b)}-(a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}$,show that $\frac{dy}{dx}=\sqrt{\frac{a-x}{x-b}}$

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17. If $y = y = \sqrt{x}^{\sqrt{x}^{\sqrt{x}^{\cdots\infty}}}$,show that $x\frac{dy}{dx} = \frac{y^2}{2 - y\log x}$

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18. The value of

$\tan\alpha + 2\tan(2\alpha) + 4\tan(4\alpha) + \ldots + 2^{n-1}\tan\left(2^{n-1}\alpha\right) + 2^n\cot(2^n\alpha)$

is

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