BOOKS - UNITED BOOK HOUSE

TRIGONOMETRY

Exercise

1. If $x \sin \alpha = y \cos \alpha = \dfrac{2z \tan \alpha}{1 - \tan^2 \alpha}$, show that $\left(x^2 - y^2\right)^2 = 4z^2\left(x^2 + y^2\right)$

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2. If $\left(a^2 - b^2\right)\sin\theta + 2ab\cos\theta - \left(a^2 + b^2\right) = 0$, then prove that $\tan\theta = \dfrac{1}{2}\left(\dfrac{a}{b} - \dfrac{b}{a}\right)$.

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3. If $\dfrac{ax}{\cos\theta} + \dfrac{by}{\sin\theta} = a^2 - b^2$ and $\dfrac{ax\sin\theta}{\cos^2\theta} - \dfrac{by\cos\theta}{\sin^2\theta} = 0$

„Prove that $(ax)^{2/3} + (by)^{2/3} = \left(a^2 - b^2\right)^{2/3}$

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4. If $\dfrac{\cos^4 x}{\cos^2 y} + \dfrac{\sin^4 x}{\sin^2 y} = 1$, show that $\dfrac{\cos^4 y}{\cos^2 x} + \dfrac{\sin^4 y}{\sin^2 x} = 1$

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5. Show that $\dfrac{1}{3} \leq \dfrac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta} \leq 3$

6. If $\sec\theta = \sec\alpha s \sec\beta + \tan\alpha\tan\beta$,show that $\tan\theta = \pm\left(\sec\alpha\tan\beta + \tan\alpha\sec\beta\right)$

7. If $n^2\sin^2(\alpha+\beta) = \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta \times \cos(\alpha-\beta)$,show that $\tan\alpha = \dfrac{1\pm n}{1\pm n}\tan\beta$

8. If $(a+b)\tan(\theta-\phi) = (a-b)\tan(\theta-\phi)$ and $a\cos 2\phi + b\cos 2\theta = c$ show that $a^2 - b^2 + c^2 = 2a\cos 2\phi$

9. If $\dfrac{\tan 3\alpha}{\tan \alpha} = \lambda$, show that $\dfrac{\sin 3\alpha}{\sin \alpha} = \dfrac{2\lambda}{\lambda - 1}$ and hence prove that the value of λ does not lie between 1/3 and 3

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10. Prove that

$$\cos^8\left(\dfrac{\pi}{8}\right) + \cos^8\left(\dfrac{3\pi}{8}\right) + \cos^8\left(\dfrac{5\pi}{8}\right) + \cos^8\left(\dfrac{7\pi}{8}\right) = \dfrac{17}{16}$$

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11. Solve : $\sqrt{\tan x + \sin x} + \sqrt{\tan x - \sin x} = 2\cos x \sqrt{\tan x}$

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12. Solve : $3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$

13. Solve   :

$$\tan\left(x - \frac{\pi}{4}\right)\tan x \tan\left(x + \frac{\pi}{4}\right) = \frac{4\cos^2 x}{\tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right)}$$

14. Solve : $\sqrt{2\cos^2 x + 1} + \sqrt{2\sin^2 x + 1} = 2\sqrt{2}$

15. Show   that

$$\cos\left(\frac{\pi}{11}\right) + \cos\left(\frac{3\pi}{11}\right) + \cos\left(\frac{5\pi}{11}\right) + \cos\left(\frac{7\pi}{11}\right) + \cos\left(\frac{9\pi}{11}\right) = \frac{1}{2}$$

16. In an acute angled triangle ABC, show that

$$\tan^2 A + \tan^2 B + \tan^2 C \geq 9$$

17. If the medians of a triangle ABC makes angles α, β, γ with each other then show that

$$\cot \alpha + \cot \beta + \cot \gamma + \cot A + \cot B + \cot C = 0.$$

18. If $\cos^3 \theta = k \cos(\alpha - 3\theta)$ and $\sin^3 \theta = k \sin(\alpha - 3\theta)$, show that $2k^2 - k \cos \alpha - 1 = 0$

19. show that $\cos^2\left(\dfrac{\pi}{9}\right) + \cos^2\left(\dfrac{2\pi}{9}\right) + \cos^2\left(\dfrac{4\pi}{9}\right) = \dfrac{3}{2}$

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20. Show that $\tan 70° + \tan 10° - \tan 50° = \sqrt{3}$

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21. In $\triangle ABC$, show that the minimum value of $\tan^2\left(\dfrac{A}{2}\right) + \tan^2\left(\dfrac{B}{2}\right) + \tan^2\left(\dfrac{C}{2}\right)$ is 1.

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