MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

APPLICATIONS OF DERIVATIVES

Solved Examples

1. Find the rate of change of area of a circle with respect to its radius 'r' when r=7cm.

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2. The side of a cube is increasing at a rate of 4 cm/sec. Find the rate of change of the volume of the cube when its side is 5 cm.

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3. A stone is dropped in a quiet lake. If the produced circular waves increase at a rate of 4 cm/sec, then find the rate of increase in its area when the redius of circular wave is 7 cm.

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4. In a cylindrical tank, rice is increasing at a rate of $314m^3/hr$. Find the rate of increase of the height of the rice in cylindrical tank when the radius of the base of tank is 5 m.

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5. Find a point on the curve $y^2 = 2x$ at which the abscissa and ordinates are increasing at the same rate.

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6. A ladder 13 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 4 m/sec. Find the rate of decreasing at which the top of the ladder moving downwards on wall when the foot of the ladder is 5 m away from the wall.

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7. A man of height 1.7 m walks at a uniform speed of 6.6 m/min from a lamp post which is 5m high.Find the rate at which the length of his shadow increases.

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8. Show that the function f(x) = 2x+1 is strictly increasing on R.

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9. Prove that f (x) = e^{3x} is strictly increasing function on R.

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10. Show that the function f(x) = x -sin x is an increasing function $\forall x \in R$.

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11. Show that the function f(x) = 3 - 2x is strictly decreasing function on R.

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12. Show that the function f(x)= $x^3 - 3x^2 + 3x - 1$ is an increasing function on R.

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13. Find the intervals in which the function f(x) = $2x^3 - 15x^2 + 36x + 6$ is (i) increasing, (ii) decreasing.

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14. Find the intervals in which f(x) = sin 3x, $x \in \left[0, \frac{\pi}{2}\right]$ is (i) increasing, (ii) decreasing.

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15. Find the intervals in which function f(x) = $\sin x - \cos x, 0 < x < 2\pi$ is (i) increasing, (ii) decreasing.

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16. Using differentials, find the approximate value of $\sqrt[3]{127}$.

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17. Using differentials, find the approximate value of $\sqrt{0.037}$.

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18. Using differentials, find the approximate value of $\log_{10} 10.1$ when $\log_{10} e = 0.4343$.

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19. Find the approximate value of $f(3.01)$ when y = $f(x) = x^2 + 3x + 1$.

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20. Find the approximate percentage increase in volume of a cube of side x metre if the percentage increase in its side is 1%.

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21. The time period 'T' of a simple pendulum of length 'l' is given by T = $2\pi\sqrt{\frac{l}{g}}$.Find the percentage error in the value of 'T' if the percentage error in the value of 'l' is 2%.

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22. If in a triangle ABC, the side c and the angle C remain constant, while the remaining elements are changed slightly, show that $\frac{da}{\cos A}+\frac{db}{\cos B}=0.$

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23. Find the solpe of tangent at point (1, 1) of the curve $x^2=y$.

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24. Find the inclination from X - axis of the tangent at point (a, 2a) of the curve $y^2 = 4ax$.

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25. Find the equation of tangent of the curve $6y = 9 - 3x^2$ at point (1,1).

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26. Find the equation of tangent of tangent of the curve y = $b \cdot e^{-x/a}$ at that point at which the curve meets the Y-axis.

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27. Find the equation of tangent of the curve $x = at^2, y = 2at$ at point 't'.

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28. Find the point on the curve $y = x^2 + 1$ at which the tangent drawn makes an angle of 45° from X-axis.

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29. Find the equation of tangent of the curve $yx^2 + x^2 - 5x + 6 = 0$ at that point at which curve crosses the X-axis.

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30. Find a point on the curve $y = 3x^2 - 2x - 4$ at which the tangent is parallel to the line $10x - y + 7 = 0$.

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31. Find the angle of intersection of the curves $2y^2 = x^3$ and $y^2 = 32x$.

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32. Prove that the curves $y = x^3$ and $xy = k$ cut each other orthogonally, if 3k= 1.

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33. Find the equation of normal at point (a, a) of the curve xy = a^2.

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34. Find the equation of normal to the curve $x = \cos\theta, y = \sin\theta$ at point $\theta = \frac{\pi}{4}$.

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35. Prove that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $y\cos\theta - x\sin\theta = a\cos 2\theta$, where θ is the angle which the normal makes with the axis of x.

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36. Find the value of x for which the function $\left(x^3 - 3x + 4\right)$ is maximum or minimum.

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37. Find the maximum or minimum values of the function $y = x + \frac{1}{x} f \text{ or } x > 0.$

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38. Find the maximum value of the function $f(x) = \sin x + \cos x.$

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39. Find the minimum value of the function x^x.

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40. If x lies in first quadrant, find the maximum and minimum values of the function $4\sin x + 3\cos x$.

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41. If the function $x^2 - 2kx + 3$ is minimum at x=3 then find the value of k.

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42. Find two number whose sum is 8 and the sum of their cubes is minimum.

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43. The sum of two number is constant. Show that their product will be maximum if each number is half of their sum.

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44. The perimeter of a rectangle is 40 cm. Find the dimensions of the rectangle if its area is maximum.

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45. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

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46. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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47. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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Exercise 6 A

1. Find the rate of change of area of the circle with respect to its radius 'r' when $r = 3.5$ cm.

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2. (i) The radius of a circle is increasing at the rate of 5 cm/sec. Find the rate of increasing of its perimeter.

(ii) If the area of a circle increases at a constant rate, then show that the rate of increase of its circumference is inversely proportional to its radius.

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3. The side of a square is increasing at a rate of 3 cm/sec. Find the rate of increasing of its perimeter when the side of square is 5cm.

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4. The side of a square is increasing at a rate of 4cm/sec. Find the rate of increase of its area when the side of square is 10 cm.

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5. The rate of increase of the radius of an air bubble is 0.5 cm/sec. Find the rate of increase of its volume when the radius of air bubble is 2 cm.

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6. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

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7. The volume of cube is increasing at a rate of $9cm^3/\sec$. Find the rate of increase of its surface area when the side of the cube is 10 cm.

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8. The volume of a spherical balloon is increasing at a rate of $25cm^3/\sec$. Find the rate of increase of its curved surface when the radius of balloon is 5 cm.

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9. The surface of a spharical balloon is increasing at a rate of 2cm^2/sec. Find the rate of increase of its volume when its radius is 6 cm.

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10. The length of a rectangle is decreasing at a rate of 3 cm/sec and breadth is increasing at a rate of 4 cm/sec. Find the rate of change of its (a) perimeter (b) area, when the length and breadth of rectangle are 7 cm and 8 cm respectively.

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11. Find the point on the curve $y^2 = 8x \cdot$ for which the abscissa and ordinate change at the same rate.

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12. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which y-co-ordinate is changing 8 times as fast as the x-co-ordinate.

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13. The base of a cubical tank is $25m \times 40m$. The volume of water in the tank is increasing at the rate of $500m^3 / \min$. Find the rate at which the height of water is increasing.

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14. The oil is leaking from a drum at a rate of $16cm^3 / \sec$. If the radius and height of drum are 7 cm and 60 cm respectively, find the rate of change of the height of oil when height of oil in drum is 18 cm.

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15. The water is leaking from a conical funnel at a rate of $5cm^3 / \min$. If the redius and height of the funnel are 5cm.and 10 cm respectively, find the rate of change of the surface of water in the funnel when height of water surface from the vertex is $7.5cm$.

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16. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1 m/sec. How fast is the length of his shadow increasing when he is 1 m away from the pole?

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17. The total cost C(x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change

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18. The total revenue of selling of x units of a product is represented by $R(x) = 2x^2 + x + 5$.Find its marginal revenue for 5 units.

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19. A ladder is inclined to a wall making an angle of 30° with it. A man is ascending the ladder at the rate of 3 m/sec. How fast is he approaching the wall ?

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20. The one end of a 20 m long ladder is on the floor and the other end is in the contact of vertical wall. Its lower end slides on the floor. Show that the rate of sliding of the upper end is 4/3 times the rate of sliding the lower end when the lower end is 16 m away from the wall.

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Exercise 6 B

1. Show that the function f(x) = 3x + 2 is strictly increasing function on R.

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2. Show that the function $f(x) = x^2$ is strictly increasing function in the interval $]0, \infty[$.

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3. Show that the function $f(x) = (x - 1)e^x + 2$ is strictly increasing function $\forall x > 0$.

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4. Show that the function $f(x) = a^x, \ a > 1$ is strictly increasing on R.

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5. Show that the function $f(x) = -5x + 2$ is strictly decreasing function on R.

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6. Prove that the function given by $f(x) = \cos x$ is(a) strictly decreasing in $(0, \pi)$(b) strictly increasing in $(\pi, 2\pi)$, and(c) neither increasing nor decreasing in $(0, 2\pi)$

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7. Find the intervals in which the following functions are:

(i) increasing

.(ii)decreasing $f(x) = 2x^2 - 6x$

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8. Show that the function $f(x) = \frac{1}{1 + x^2}$ is increasing function in the interval $(-\infty, 0]$.

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9. Show that the function $f(x) = x^3 + \frac{1}{x^3}$ is decreasing function in the interval $[-1, 1] - \{0\}$.

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10. Show that y='log(1+x)-(2x)/(2+x)', is an increasing function of x throughout its domain.

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11. Find the intervals in which the function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing or decreasing.

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12. Let I be an interval disjointed from $[-1, 1]$. Prove that the function $f(x) = x + \frac{1}{x}$ is increasing on I.

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13. Show that the function $f(x) = \log(\sin x)$

(i) is strictly increasing in the interval $\left]0, \frac{\pi}{2}\right[$.

(ii) is strictly decreasing in the interval $\left]\frac{\pi}{2}, \pi\right[$.

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14. Show that the function $f(x) = \sin^4 x + \cos^4 x$

(i) is decreasing in the interval $\left[0, \frac{\pi}{4}\right]$.

(ii) is increasing in the interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$.

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15. Find the intervals in which the function f given $f(x) = s \in x + \cos x, \ 0 \leq x \ \leq 2\pi,$ is strictly increasing or strictly decreasing.

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16. Show that the function $f(x) = \log \cos x$ is :

(i) Strictly decreasing in $\left]0, \frac{\pi}{2}\right[$

(ii) Strictly increasing in $\left]\frac{\pi}{2}, \pi\right[$

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17. prove that $\frac{x}{1+x} < \log(1+x) < x$, for all $x > 0$

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Exercise 6 C

1. Using differentials, find the approximate value of $\sqrt{26}$

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2. Using differentials, find the approximate value of $(0.007)^{1/3}$

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3. Use differentials and find approximate value of $(29)^{1/3}$

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4. Using the method of differentials, find the approximate value of $\sqrt{0.24}$.

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5. Using differentials, find the approximate value of $\sqrt{0.48}$

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6. Using differentials, find the approximate values of the following:

$(i)\sqrt[4]{15}(ii)(82)^{1/4}$

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7. Using differentials, find the approximate value of $\frac{1}{(2.002)^2}$

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8. Use differentials to approximate $\sqrt{25.2}$.

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9. Using differentials, find the approximate value of $(0.009)^{1/3}$

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10. Use differentials to find the approximate value of $(\log)_e(4.01)$, having given that $(\log)_e 4 = 1.3863$.

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11. Using differentials, find the approximate value of $(\log)_e 4.04$, it being given that $(\log)_{10} 4 = 0.6021$ and $(\log)_{10} e = 0.4343$.

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12. If $f(x) = 2x^2 + 5x + 2$, then find the approximate value of $f(2.01)$.

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13. If $f(x) = 3x^2 + 4x - 1$, then find the approximate value of $f(3.1)$.

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14. The radius of a circular plate increases by 2% on heating. If its radius is 10 cm before heating, find the approximate increase in its area.

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15. The radius of a sphere decreases from 10 cm to 9.9 cm. Find

(i) approximate decrease in its volume.

(ii) approximate decrease in its surface.

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16. The time t of a complete oscillation of a simple pendulum of length l is given by the equation $T = 2\pi\sqrt{\frac{1}{g}}$ where g is constant. What is the percentage error in T when l is increased by 1%?

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17. There is an error of $0.2\,\%$ in measurment of the redius of a sphere. Find the percentage error in its

(i) volume, (ii) surface.

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18. The radius of a sphere is 8 cm and 0.02 cm is the error in its measurement. Find the approximate error in its volume.

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19. The semi-vertical angle of a cone remains constant. If its height increases by 2%, then find the approximate increase in its volume.

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Exercise 6 D

1. Find the slope of tengents drawn of the following curves at the given points:

(i) Curve $y = x^3 + 1$ at point (0, 1)

(ii) Curve $x^2 - y^2$ = 20 at point (6, 4)

(iii) Curve $y^2 = 4x$ at point (1, 2)

(iv) Curve$y^2 = 4 \text{ ax at point } \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

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2. Find the inclination from X-axis of the tangent drawn of the following curves at the given points:

(i) Curve$x^2 - 2y^2 = 8$ at point (4, 2)

(ii) Curve $y = (x - 1)(x - 2)$ at point (2, 0)

(iii) Curve $y^2 = 2x^3$ at point (2, 4)

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3. Find the equation of tangent of the curve $x^2 + y^2 = 5$ at point (1, 2).

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4. Find the equation of tangent of the curve $y^2 = 4x + 5$ which is parallel to the line $2x - y = 5$.

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5. Find the equation of tangent of the curve $9x^2 + 16y^2 = 144$ at those points at which tangents are parallel to (i) X-axis, (ii) Y-axis.

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6. Find the co-ordinates of that point on the curve $x^3 + y^3 = a^3$ at which the tangent drawn is parallel to X-axis.

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7. Find the co-ordinates of that point on the curve$y^2 = x^2(1 - x)$ at which the tangent drawn is perpendicular to X-axis.

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8. Find the co-ordinates of that point on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at which the tangent drawn is parallel to Y-axis.

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9. Prove that the equation of tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at point $(x_1, y_1) is \frac{\times_1}{a^2} + \frac{yy_1}{b^2} = 1.$

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10. Find the value of $n \in N$ such that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straightl line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a,b).

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11. Show that the line $\frac{x}{a} + \frac{y}{b} = 1$, touches the curve $y = b.\,e^{-x/a}$ at the point, where curve intersects the axis of Y.

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12. Find the point on the curve$y^2 = x$ at which the tangent drawn makes an angle of 45° from X-axis.

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13. Find the coordinates of the points on the curve $y = x^2 + 3x + 4,$ the tangents at which pass through the origin.

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14. The tangent drawn at any point of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ meets the OX and OY axes at points P and Q respectively, prove that OP+OQ = a.

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15. If p and q are the intercept on the axis cut by the tangent of $\sqrt{\left(\frac{x}{a}\right)} + \sqrt{\left(\frac{y}{b}\right)} = 1,$ prove that $\frac{p}{a} + \frac{q}{b} = 1$

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16. If tangents are drawn from the origin to the curve y=sin x , then their points of contact lie on the curve

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17. Find the angle of intersection of the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$.

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18. Prove that the curves $x^2 - y^2 = 16$ and $xy = 15$ intersect each other at 90° angle.

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19. Show that the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ intersect orthogonally is

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{a} - \frac{1}{b},$$

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20. Prove that the curves $\mathrm{x} = \mathrm{y}^2$ and $\mathrm{x\,y{=}k}$ intersect at right angles if $8\mathrm{k}^2 = 1.$

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21. Find the equations of the tangent and the normal at the point $'t'$ on the curve $x = a\sin^3 t\,, y = b\cos^3 t\,.$

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22. Prove that all the points of the curve $y^2 = 4\alpha\left(x + a\frac{\sin(x)}{a}\right),$ where tangents are parallel to the axis of x lie on the curve $y^2 4ax.$

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23. Prove that the tangents drawn on the parabola $y^2 = 4ax$at points x = a intersect at right angle.

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24. Prove that the curve $y^2 = 4x$ and $x^2 + y^2 - 6x + 1 = 0$ touches each other at thepoint $(1, 2)$, find the equation of the common tangents.

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Exercise 6 E

1. Find the equation of normals of the following curves at the given points:

(i) Curve$y^2 = 4ax$ at point $(at^2, 2at)$.

(ii) Curve $y = e^x$ at point $(0, 1)$

(iii) Curve $y = x^3$ at point $(1, 1)$.

(iv) Curve $2y = 3 - x^2$ at point $(1, 1)$.

(v) Curve $16x^2 - 9y^2 = 432$ at point (6, 4).

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2. Find the equation of the normal to the curve $y = 5x + x^2$ which makes an angle 45° with x axis.

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3. Find the equation of normal to the curve $y(x-2)(x-3) - x + 7 = 0$ at that point at which the curve meets X-axis.

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4. Find the equation of normal to the curve $x = at^2, y = 2at$ at point 't'.

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5. Find the equation of the normal to the curve $x = a\cos\theta$ and $y = b\sin\theta$ at θ

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6. Find the equation of normal to the curve $x = a\cos^3\theta, y = b\sin^3\theta$ at point $'\theta'$.

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7. Find the equations of tangent and normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x_1, y_1).

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8. (i) Find the co-ordinates of the points on the curve xy = 16 at which the normal drawn meet at origin.

(ii) Find the points on the curve $4x^2 + 9y^2 = 36$ at which the normal drawn is parallel to X-axis.

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9. Find the equation of normal at point (4, 3) for the hyperbola $3x^2 - 4y^2 = 14.$

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10. Find the equation of normal to the curves $x = t^2, y = 2t + 1$ at point

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11. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

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12. If the normal at point 't' of the curve xy = c^2 meets the curve again at point $'t'_1$, then prove that

$t^3 \cdot t_1 = -1.$

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13. Find the area erea angle which made by the tangent to the curve $y(2a - x) = x^2$ at point (a, a) its normal and x-axis.

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14. Find the co-ordinates of the point on the curve $y = x^3 + 4x^2 + 4x - 12$ at which the normal's inclination is $-\frac{1}{7}$.

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15. If the normal drawn on the curve $y = x^2 + 4$ at point (1, 5) makes an angle θ from X-axis then show that:

$\tan\theta = -\frac{1}{2}$

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16. Find the equation of normal of the curve $2y = 7x - 5x^2$ at those points at which the curve intersects the line x = y.

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Exercise 6 F

1. Find the values of x for which the following functions are maximum or minimum:

(i) $x^3 - 3x^2 - 9x$

(ii) $4x^3 - 15x^2 + 12x + 1$

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2. Find the maximum or minimum values of the following functions:

(i)$x^3 - 2x^2 + x + 3$

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3. For what values of x, the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ is maximum or minimum? Prove that at x = 0, the function is neither maximum nor minimum.

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4. Find the maximum and minimum values of the function $f(x) = \sin x + \cos 2x$.

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5. Find the maximum and minimum values of the function $f(x) = x + \sin 2x, (0 < x < \pi)$.

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6. Find the maximum and minimum values of the function $f(x) = \frac{\sin x}{1 + \tan x}, (0 < x < 2\pi)$.

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7. Show that $s \in^p \; \theta \cos^q \theta$ attains a maximum, when $\theta = \tan^{-1} \sqrt{\frac{p}{q}}$.

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8. Find the maximum value of the function $\frac{\log x}{x}$ when $x > 0$.

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9. Find the maximum value of the function $x^{1/x}$.

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10. Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$.

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11. Show that the function $x^3 - 3x^2 + 3x + 1$ has neither a maxima nor a minima.

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12. Show that $f(x) = \sin x(1 + \cos x)$ is maximum at $x = \frac{\pi}{3}$ in the interval $[0, \pi]$.

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13. If the function $f(x) = x^3 - 24x^2 + 6kx - 8$ is maximum at x = 2 then find value of k.

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14. Prove that value of the function $xy(y - x) = 2a^3$ is minimum at $x = a$.

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15. Show that the function $\left(-x^2 \log x\right)$ is maximum at $x = \frac{1}{\sqrt{e}}$.

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16. Find the maximum value of the function $x \cdot e^{-x}$.

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17. Show that the maximum value of the function $\sqrt{2}(\sin x + \cos x)$ is 2.

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18. Show that the maximum and minimum values of the function $\frac{(x+1)^2}{(x+3)^3}$ are respectively given by $\frac{2}{27}$ and 0.

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19. If the function $y = ae^x + bx^2 + 3x$ is maximum at x = 0 and minimum at x = - 3, then find the values of a and b.

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Exercise 6 G

1. Divide 16 into two parts such that the sum of their cubes is minimum.

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2. If $x + y = 1$ then find the maximum value of the function xy^2

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3. Find two number whose sum is 100 and the sum of twice of first part and square of the second is minimum.

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4. Find two numbers whose sum is 12 and the product of the square of one part and the 4th power of other is maximum.

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5. Divide 15 into two parts such that product of square of one part and cube of other is maximum

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6. (i) The two sides of a rectangle are x units and (10 - x) units. For what value of x, the area of rectangle will be maximum?

(ii) Prove that a rectangle, whose area is constant has minimum perimeter if it is a square.

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8. If the surface area of an open cylinder is $100cm^2$, prove that its maximum volume will be $\frac{1000}{3\sqrt{3\pi}cm^3}$.

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9. An open tank with a square base and vertical sides is to be constructed form a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of tank is half its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in the question

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10. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

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11. The base of a cuboid is square and its volume is given. Show that its total surface will be minimum, if it is a cube.

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12. Show that the least cloth is required to construct a conical tent of given volume if the ratio of height to the base radius is $\sqrt{2}:1$.

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13. Show that the height of a cone of maximum volume inscribed in a sphere has the ration with the radius of sphere as $4:3$.

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14. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ having its vertex coincident with one extremity of the major axis.

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15. The volume of a closed square based rectangular box is 1000 cubic metre. The cost of constructing the base is 15 paise per square metre and the cost of constructing the top is 25 paise per square metre. The cost of constructing its sides is 20 paise per square metre and the cost of constructing the box is Rs. 3.Find the dimensions of box for minimum cost of construction.

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16. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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17. The sum of the perimeters of a square and a circle is given. Show that the sum of their areas is minimum when the length of a side of the square is equal to the diameter of the circle.

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18. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

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19. The stiffness of a beam of rectangular cross-section varies as the product of the breadth and square of the depth. What must be the breadth of the stiffest beam that can be cut from a leg of diameter 'd'?

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20. The fuel charges for running a train are proportional to the square of the speed generated in km/h, and the cost is Rs. 48 at 16 km/h. If the fixed charges amount to Rs. 300/h, the most economical speed is 60 km/h (b) 40 km/h 48 km/h (d) 36 km/h

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21. The conbined resistance R of two resistors R,& $R_2 (R_1, R_2 > 0)$ is given by . $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1 + R_2 =$ constant Prove that the maximum resistance R is obtained by choosing $R_1 = R_2$

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22. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

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23. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

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Exercise 6 H Multiple Choice Questions

1. Using differentials, find the approximate value of $(82)^{1/4}$

A. 3.008

B. 3.009

C. 3.010

D. None of these

Answer: B

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2. Find the equation of the tangent to the curve $x = \theta + \sin\theta, y = 1 + \cos\theta \ at \ \theta = \frac{\pi}{4}$.

A. $y = (1 + \sqrt{2})x + \frac{(\sqrt{2} - 1)\pi}{4} + 2$

B. $y = (1 - \sqrt{2})x + \frac{(\sqrt{2} - 1)\pi}{4} + 2$

C. $y = (1 + \sqrt{2})x + (\sqrt{2} - 1)\pi$

D. None of the above

Answer: B

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3. The equation of normal to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2(n \in N)$ at the point with abscissa equal to 'a can be

A. $xa + yb = a^2 - b^2$

B. $xa + yb = a^2 + b^2$

C. $xa - yb = a^2 - b^2$

D. $bx - ay = a^2 - b^2$

Answer: A::C

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4. Find the point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.

A. $\left(3, \pm \frac{3}{8}\right)$

B. $\left(4, \pm \frac{8}{3}\right)$

C. $\left(3, \pm \frac{8}{3}\right)$

D. $\left(4, \pm \frac{3}{8}\right)$

Answer: D

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5. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$.Find the rate of change of its volume with respect to x.

A. $3\pi(2x+1)^2$

B. $\frac{8}{9}\pi(2x+1)^2$

C. $\frac{8}{27}\pi(2x+1)^2$

D. $\frac{8}{3}\pi(2x+1)^2$

Answer: C

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6. In the interval (-1, 1), the function $f(x) = x^2 - x + 4$ is :

A. increasing

B. decreasing

C. neither increasing nor decreasing

D. None of the above

Answer: C

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7. If an error of $k\%$ is made in measuring the radius of a sphere, then percentage error in its volume. k% (b) 3k% (c) 3k% (d) $\frac{k}{3}\%$

A. 2%

B. 4%

C. 6%

D. 8%

Answer: C

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8. Minimum value of the function $f(x) = \left(\frac{1}{x}\right)^{1/x}$ is:

A. e

B. $\frac{1}{e}$

C. e^e

D. $e^{-1/e}$

Answer: D

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9. If the sum of two +ve numbers is 18 then the maximum value of their product is

A. 8,10

B. 9,9

C. 6,12

D. None of these

Answer: B

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10. For minimum curved surface area and given volume, the ration of the height and radius of base of a cone is :

A. $1:\sqrt{2}$

B. $\sqrt{2}:1$

C. $1:2$

D. None of these

Answer: B

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Exercise 6 I Multiple Choice Questions

1. 26. The points of contact of the tangents drawn from the origin to the curve y=sinx, lie on the curve

A. $x^2 - y^2 = x^2y^2$

B. $x^2 + y^2 = x^2y^2$

C. $y^2 - x^2 = x^2y^2$

D. None of these

Answer: A

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2. Equation of tangent of the curve $y = 1 - e^{x/2}$ at that point at which the curve crosses the y-axis, is :

A. $x + y = 1$

B. $2x + y = 1$

C. $x = -2y$

D. None of these

Answer: C

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3. Curve $b^2x^2 + a^2y^2 = a^2b^2$ and $m^2x^2 - y^2l^2 = l^2m^2$ intersect each other at right-angle if:

A. $a^2 + b^2 = l^2 + m^2$

B. $a^2 - b^2 = l^2 - m^2$

C. $a^2 - b^2 = l^2 + m^2$

D. $a^2 + b^2 = l^2 - m^2$

Answer: C

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4. The function $x^{100} + \sin x - 1$ is decreasing in the interval:

A. $\left(0, \frac{\pi}{2}\right)$

B. $(0, 1)$

C. $\left(\frac{\pi}{2}, \pi\right)$

D. None of these

Answer: D

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5. Find the intervals in which the function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing or decreasing.

A. $(-\infty, 0]$

B. $[-1\infty)$

C. $(-\infty, 1]$

D. None of these

Answer: B

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6. If the function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ is maximum at $x = \frac{\pi}{3}$ a=?

A. 1

B. 2

C. 3

D. 4

Answer: B

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7. If $x + y = 8$, then the maximum value of xy is (a) 8 (b) 16 (c) 20 (d) 24

A. 8

B. 12

C. 16

D. 7

Answer: C

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8. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

A. $\pi 3$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. None of these.

Answer: C

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9. Find minimum value of $px + qy$ where $p > 0, q > 0, x > 0, y > 0$ when $xy = r,^2$ without using derivatives.

A. $pq\sqrt{r}$

B. $2pq\sqrt{r}$

C. $2r\sqrt{pq}$

D. None of these

Answer: C

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10. Maximum area of a reactangle which can be inscribed in a circle of a given radius R is

A. πr^2

B. r^2

C. $2r^2$

D. $\frac{1}{4}\pi r^2$

Answer: C

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Exercise 6 1

1. Find the rate of change of the area of a circle with respect to its radius r when(a) $r = 3cm$ (b) $r = 4cm$

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2. The volume of a cube is increasing at the rate of $8cm^3 / s$. How fast is the surface area increasing when the length of an edge is 12 cm?

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3. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

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4. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

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5. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

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6. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference?

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7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

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9. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

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10. A ladder of length 5 m is leaning against a wall. The bottom of ladder is being pulled along the ground away from wall at rate of $2cm/\sec$. How fast is the top part of ladder sliding on the wall when foot of ladder is 4m away form wall.

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11. A particle is moving along a curve $6y = x^3 + 2$. Find the points on the curve at which y-coordinate is changing 8 times as fast as the x-coordinate.

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12. The radius of an air bubble is increasing at the rate of 0.5 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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13. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x + 1)$.Find the rate of change of its volume with respect to x.

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14. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm ?

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15. The total cost C (x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.

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16. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.

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17. The rate of change of the area of a circle with respect to its radius r at r = 6 cm is:

A. 10π

B. 12π

C. 8π

D. 11π

Answer: B

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18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

A. 116

B. 96

C. 90

D. 126

Answer: D

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Exercise 6 2

1. Show that the function given by $f(x) = 3x + 17$is strictly increasing on R.

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2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

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3. Show that $f(x) = \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ and neither increasing nor decreasing in $(0, \pi)$.

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4. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is(a) strictly increasing (b) strictly decreasing

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5. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$is (a) strictly increasing (b) strictly decreasing

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6. Find the intervals in which the following functions are strictly increasing or decreasing:(a $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$ (c) $6 - 9x - x^2$(d) $(x + 1)^3(x - 3)^3$

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7. Show that $y = \log(1 + x) - \frac{2x}{2 + x}, x \succ 1$ is an increasing function of x throughout its domain.

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8. Find the values of x for which $f(x) = [x(x - 2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

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9. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

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10. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

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11. Prove that the function $f(x) = x^2 - x + 1$ is neither increasing nor decreasing on $(-1,\ 1)$.

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12. Which of the following functions are strictly decreasing on $\left[0, \frac{\pi}{2}\right]$(A) $\cos x$ (B)$\cos 2x$ (C) $\cos 3x$ (D) $tanx$

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13. On which of the following intervals is the function $x^{100} + \sin x - 1$ decreasing? $\left(, \frac{\pi}{2}\right)$ (b) $(0, 1)$ $\left(\frac{\pi}{2}, \pi\right)$ (d) none of these

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14. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$is strictly increasing on $(1, 2)$.

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15. Let I be any interval disjoint from $(1, 1)$. Prove that the function f given by $f(x) = x + \frac{1}{x}$is strictly increasing on 1.

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16. Prove that the function f given by $f(x) = \log \sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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17. Prove that the function f given by $f(x) = \log|\cos x|$ is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.

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18. Prove that the function $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing on R.

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19. The interval in which $y = x^2 e^{-x}$ is increasing is (A) $(-\infty, \infty)$ (B) (2, 0) (C) $(2, \infty)$ (D) (0, 2)

A. $(-\infty, \infty)$

B. $(-2, 0)$

C. $(2, \infty)$

D. $(0, 2)$

Answer: D

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Exercise 6 3

1. Find the slope of the tangent to the curve $y = x^3 - xatx = 2.$

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2. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$at $x = 10$

.

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3. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

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4. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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5. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

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6. Find the slope of the normal to the curve $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

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7. The points at which the tangents to the curve $y = x^3 - 12x + 18$ are parallel to the X-axis are

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8. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

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9. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x \quad 11$.

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10. Find the equation of all lines having slope -1that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1.$

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11. Find the equations of all lines having slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, \quad x \neq 3\,.$

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12. Find the equations of all lines having slope 0 which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}.$

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13. Find pints on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

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14. Find the equations of the tangent and normal to the given curves at the indicated points: (i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, \quad 5)$ (ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, \quad 3)$

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15. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is(a) parallel to the line $2xy + 9 = 0$(b) perpendicular to the line $5y15x = 13$.

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16. Show that the tangents to the curve $y = 7x^3 + 11$at the points where $x = 2$and $x = 2$are parallel.

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17. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y coordinate of the point.

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18. For the curve $y = 4x^3 - 2x^5$ find all points at which the tangent passes through the origin.

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19. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$at which the tangents are parallel to the x-axis.

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20. Find the equation of the normal at the point $\left(am^2, am^3\right)$ for the curve $ay^2 = x^3$.

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21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

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22. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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23. Show that the curves $x = y^2$ and $xy = k$ cut at right angles, if $8k^2 = 1$.

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24. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. at the point (x_0, y_0)

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25. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$which is parallel to the line $4x - 2y + 5 = 0$.

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26. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$is(A) 3 (B) $\frac{1}{3}$ (C)-3 (D) $-\frac{1}{3}$

A. 3

B. $\frac{1}{3}$

C. -3

D. $-\frac{1}{3}$

Answer: D

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27. The line $y = x + 1$is a tangent to the curve $y^2 = 4x$at the point(A) $(1, 2)$ (B)$(2, 1)$ (C) $(1, 2)$ (D) $(1, 2)$

A. $(1, 2)$

B. $(2, 1)$

C. $(1, -2)$

D. $(-1, 2)$

Answer: A

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Exercise 6 4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

$(i)\sqrt{25.3}$ $(ii)\sqrt{49.5}$

`

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2. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

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3. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

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4. Find the approximate change in the volume 'V' of a cube of side x metres caused by decreasing the side by 1%.

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5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

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6. If the radius of a sphere is measured as 7 m with an error or 0.02 m, find the approximate error in calculating its volume.

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7. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find the approximate error in calculating its surface area.

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8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$is(A) 47.66 (B) 57.66 (C) 67.66 (D) 77.66

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer: D

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9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is(A) 0.06 x^3m^3 (B) 0.6 x^3m^3 (C) 0.09 x^3m^3 (D) 0.9 x^3m^3

A. $0.06x^3m^3$

B. $0.6x^3m^3$

C. $0.09x^3m^3$

D. $0.9x^3m^3$

Answer: C

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Exercise 6 5

1. Find the maximum and minimum values, if any, of the following functions given by(i) $f(x) = (2x-1)^2 + 3$ (ii) $f(x) = 9x^2 + 12x + 2$(iii) $f(x) = -(x-1)^2 + 10$ (iv) $g(x) = x^3 + 1$

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2. Find the maximum and minimum values, if any, of the following functions given by (i) $f(x) = |x + 2| 1$ (ii) $g(x) = |x + 1| + 3$ (iii) $h(x) = s \in (2x) + 5$

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3. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

$(i) f(x) = x^2$

$(ii) g(x) = x^3 - 3x$

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4. Prove that the following functions do not have maxima or minima:(i) $f(x) = ex$ (ii) $g(x) = \log x$(iii) $h(x) = x^3 + x^2 + x + 1$

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5. Find the absolute maximum value and the absolute minimum value of the followingfunctions in the given intervals:(i) $f(x) = x^2, x \in [-2, 2]$

(ii) `f(x)=sinx+cosx ,x in [0,pi]

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6. Find the maximum profit that a company can make, if the profit function isgiven by $p(x) = 41 - 24x - 18x^2$

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7. Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

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8. At what points in the interval $[0, 2\pi]$, does the function $s \in \quad 2x$ attain its maximum value?

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9. What is the maximum value of the function $s \in \quad x \quad + \quad \cos \quad x$?

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10. Find the maximum value of $2x^3 - 24x + 107$in the interval [1, 3]. Find the maximum value of the same function in $[-3, -1]$.

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11. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of a .

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12. Find the maximum and minimum values of $f(x) = x + \sin 2x$ in the interval $[0, 2\pi]$

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13. Find two numbers whose sum is 24 and whose product is as large as possible.

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14. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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15. Find two positive numbers x and y such that their sum is 35 and the product $x^2 y^5$ is maximum.

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16. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

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17. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the slops to form a box. What should be the side of the square to be cut off such that volume of the box is maximum possible .

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18. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top by cutting off squares from the corners and folding up the flaps. What should be the side of the square in order the volume of the box is maximum.

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19. Show that of all the rectangles inscribed in a given circle, the square has the maximum area.

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20. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

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21. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic centimetres, find the dimensions of the can which has the minimum surface area?

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22. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

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23. Prove that the volume of the largest cone, that can be inscribed in a sphere of radius $R\cdot$ is $\frac{8}{27}$ of the volume of the sphere.

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24. Show that right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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25. Show that semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

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26. Show that the semi vertical angle of a right circular cone of given total surface area and max volume is $\sin^{-1}\left(\frac{1}{3}\right)$

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27. The point on the curve $x^2 = 2y$which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

A. $(2\sqrt{2}, 4)$

B. $(2\sqrt{2}, 0)$

C. (0, 0)`

D. $(2, 2)$

Answer: A

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28. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is(A) 0 (B) 1 (C) 3 (D) $\frac{1}{3}$

A. 0

B. 1

C. 3

D. $\frac{1}{3}$

Answer: D

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29. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:

A. $\left(\frac{1}{3}\right)^{\frac{1}{3}}$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C

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Miscellaneous Exercise

1. Using differentials, find the approximate value of each of the following:

(a) $\left(\frac{17}{81}\right)^{\frac{1}{4}}$ $(b)(33)^{-\frac{1}{5}}$

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2. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

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3. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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4. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

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5. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta,\ y = a\sin\theta - a\,\theta\cos\theta$ is at a constant distance from the origin.

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6. Find the intervals in which the function f given by $f(x) = \dfrac{4\sin x - 2x - xc\otimes}{2 + \cos x}$ is (i) increasing (ii) decreasing.

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7. Find the intervals in which the function f given by $f(x) = x^3 + \dfrac{1}{x^3},\quad x \neq 0$ is increasing and decreasing.

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8. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ having its vertex coincident with one extremity of the major axis.

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9. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is $2m$ and volume is $8m^3$. If building of tank costs $Rs. 70$ per sq. metre for the base and $Rs. 45$ per sq. metre for sides, what is the cost of least expensive tank?

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10. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

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11. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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12. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

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13. Find the points at which the function f given by $f(x)=(x-2)^4(x+1)^3$ has local maxima local minima point of inflexion

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14. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x$, $x \in [0, \ \pi]$.

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15. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

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16. Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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17. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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18. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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19. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

A. 1 m/h

B. $0.1m/h$

C. $1.1m/h$

D. $0.5m/h$

Answer: A

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20. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$at the point $(2, -1)$is(A) $\frac{22}{7}$ (B) $\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $\frac{-6}{7}$

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $\frac{-6}{7}$

Answer: B

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21. Use differential to approximate $(25)^{\frac{1}{3}}$.

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer: A

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22. The normal at the point (1,1) on the curve $2y + x^2 = 3$is

(A) $x - y = 0$

(B) $xy = 0$

(C) $x + y + 1 = 0$

(D) $xy = 0$

A. x+y=0

B. x-y= 0

C. x+y+1=0

D. x-y=1

Answer: B

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23. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

A. x+y = 3

B. x - y = 3

C. x+ y = 1

D. x- y = 1

Answer: A

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24. Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.

A. $\left(4, \pm \frac{8}{3}\right)$

B. $4, \frac{-8}{3}\right)$

C. $\left(4, \pm \frac{3}{8}\right)$

D. $\left(\pm 4, \frac{3}{8}\right)$

Answer: A

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