MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

DETERMINANTS

Solved Examples

1. Find the value of the determinant $\begin{vmatrix} 3 & 4 & 7 \\ -1 & 6 & 5 \\ 2 & 8 & 10 \end{vmatrix}$

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2. Find the value of the determinant $\begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$

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3. Find the value of the determinant $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{26}+\sqrt{15} & 5 & \sqrt{10} \\ \sqrt{65}+3 & \sqrt{15} & 5 \end{vmatrix}$

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4. Without, prove that :

$$|(1+b, b+c, c+a), (p+q, q+r, r+p), (x+y, y+z, z+x),| = 2\begin{vmatrix} a \\ p \\ x \end{vmatrix}$$

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5. Prove that :

$$= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

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6. Prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

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7. Prove that : $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

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8. Solve the equation

$$\begin{vmatrix} x+2 & 1 & -3 \\ 1 & x-3 & -2 \\ -3 & -2 & 1 \end{vmatrix} = 0.$$

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9. Prove that

$$\begin{vmatrix} 1 & x+\alpha & y+z-\alpha \\ 1 & y+\beta & z+x-\beta \\ 1 & z+\gamma & x+y-\gamma \end{vmatrix} = 0$$

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10. if $\begin{vmatrix} 3 & 1 & -4 \\ 3 & 2 & 5 \\ 1 & -1 & 3 \end{vmatrix} = 49$, then evaluate $\begin{vmatrix} 6 & 3 & -\frac{8}{3} \\ 6 & 6 & \frac{10}{3} \\ 2 & -3 & 2 \end{vmatrix}$.

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11. Find the area of triangle whose vertices are (2,7), (1,1) and (10,8).

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12. Prove that the points (0,3), (4,6) and (-8, -3) are collinear.

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13. Find the equation of a line passing through the points (3,5) and (-2, 1).

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14. If $A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$, find A^{-1}.

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15. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & -1 & -2 \\ -3 & 4 & 2 \end{bmatrix}$, find A^{-1}

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16. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, shown that $A^2 - 4A + 5I = o$. Hence Find A^{-1}.

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17. Find a matrix B of order 2×2 such that :

$$\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$$

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18. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then show that $A^3 = A^{-1}$.

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19. Solve by matrix method :

$2x + y = 5$

$x - y = 1$

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20. Solve the following equations by matrix method.

$2x + y + z = 1, x - 2y - 3z = 1$ and $3x + 2y + 4z = 5$

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21. Use product $[1 - 1202 - 33 - 24]$ $[-20192 - 361 - 2]$ to solve the system of equation: $x - y + 2z = 1$ $2y - 3z = 1$ $3x - 2y + 4z = 2$

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22. Find whether the following system of equation has a solution or not ?

x+5y=10 2x+10y=9

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Exercise 4 A

1. Find the values of the following determinants

$$\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

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2. Find the values of the following determinants

$$\begin{vmatrix} -3 & -2 \\ 1 & 5 \end{vmatrix}$$

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3. Find the values of the following determinants

$$\begin{vmatrix} 4 & 0 & 2 \\ 1 & 5 & -6 \\ 3 & -2 & 8 \end{vmatrix}$$

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4. Find the values of the following determinants

$$\begin{vmatrix} 1 & 1 & 1 \\ 5 & -3 & 1 \\ 7 & 4 & -2 \end{vmatrix}$$

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5. Find the values of the following determinants

$$\begin{vmatrix} 13 & 15 & 17 \\ 14 & 16 & 18 \\ 15 & 17 & 19 \end{vmatrix}$$

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6. Find the values of the following determinants

$$\begin{vmatrix} 12 & -10 & 5 \\ 3 & 2 & -1 \\ -4 & 0 & 3 \end{vmatrix}$$

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Exercise 4 B

1. Prove that : $\begin{vmatrix} a+b & b+c & c+a \\ c & a & b \\ 1 & 1 & 1 \end{vmatrix}$

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2. Prove that : $\begin{vmatrix} x+a & x+2a & x+3a \\ x+2a & x+3a & x+4a \\ x+4a & x+5a & x+6a \end{vmatrix} = 0$

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3. Prove that : $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix} = (x-y)(y-z)(z-x)$

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4. Prove that : $Det \begin{bmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{bmatrix} = xyz(x-y)(y-z)(z-x)$

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5. Prove that : $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$=(x-y)(y-z)(z-x)(x+y+z)

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6. Prove that : $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = a^2(3x+a)$

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7. Prove that : $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

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8. Prove that : $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$

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9. Prove that : $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & x+z+2y \end{vmatrix} = 2(x+y+z)^3$

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10. Using the properties of determinants, prove that

$$\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

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11. Prove that : $\begin{vmatrix} a+b & b & c \\ b+c & c & a \\ c+a & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

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12. Prove that :

$$\begin{vmatrix} a^2 & b^2+c^2 & bc \\ b^2 & c^2+a^2 & ca \\ c^2 & a^2+b^2 & ab \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

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13. Prove that :

(i) $\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} = 2abc$

(ii) Prove that : $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

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14. Prove that : $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

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15. Prove that : $\begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix} = 0$

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16. $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix}$=0

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17. Solve the equation $\begin{vmatrix} x+a & x+b & x+c \\ x+b & x+c & x+a \\ x+c & x+a & x+b \end{vmatrix} = 0$

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18. One root of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0 is (A)$8/3

(B)2/3(C)1/3 (D)16/3`

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19. If $2s = a + b + c$ and $A = \begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix}$ is equal to

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20. If the sides of a ΔABC are a, b, c and $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$ then prove that ΔABC is an isosceles triangle.

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21. If the pth, qth and rth terms of a G.P, are x,y and z repectively, then prove that $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = 0$

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22. Prove that : $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$

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23. Prove that : $\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2$

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Exercise 4 C

1. Find the area of the triangle whose vertices are given below :

(i) (-3,-4), (-2,-7), (-1,-9)

(ii) (3,8),(-4,2), (5,1)

(iii) (2,1), (3,2), (-1, 4)

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2. Using determinants, show that the following points are collinear :

(i) (a+b,c),(b+c,a),(c+a,b)

(ii) (5, 5),(10,7), (-5,1)

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3. If area of triangle is 35 sq units with vertices $(2, \ -6), (5, 4) and (k, 4)$. Then k is(A) 12 (B) -2 (C) $12, 2$ (D) $12, \ -2$

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4. If $(x, y), (a, 0), (0, b)$ are collinear, then using determinants prove that $\frac{x}{a} + \frac{y}{b} = 1$.

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5. If the value of K, is the points (k, 2-2k), (1-k, 2k) and (-4-k, 6-2 k) are collinear.

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6. Find the equation fo a line passing through the points (2,0) and (0,4).

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7. If the points $(a_1, b_1)m(a_2, b_2)$ and $(a_1 - a_2, b_2 - b_2)$ are collinear, then prove that $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

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Exercise 4 D

1. Are the following matrices invertible ?

(i) $\begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$

(ii) $\begin{vmatrix} 7 & 0 \\ 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 1 & -2 & -3 \\ 1 & -3 & -4 \\ 1 & -4 & -5 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 1 & 4 \\ -5 & 0 & -2 \end{vmatrix}$

(v) $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$

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2. Find the value of 'k' for which the following matrices are invertible ?

(i) $\begin{vmatrix} 6 & k \\ -2 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 0 & k & 3 \\ 1 & -2 & 2 \\ 4 & 3 & -1 \end{vmatrix}$

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3. Find the inverse of the following matrices if exist :

(i) $\begin{vmatrix} 5 & -3 \\ 2 & 2 \end{vmatrix}$

(ii) $\begin{vmatrix} 1 & -3 \\ -1 & 2 \end{vmatrix}$

(iii) $\begin{vmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix}$

(iv) $\begin{vmatrix} 1 & -3 & 3 \\ 2 & 2 & -4 \\ 2 & 0 & 2 \end{vmatrix}$

(v) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & -2 \\ 1 & 2 & -1 \end{vmatrix}$

(vi) $\begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -1 \\ -1 & 2 & 4 \end{vmatrix}$

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4. If $A = \begin{bmatrix} 3 & -1 \\ 1 & -3 \end{bmatrix}$ and B $= \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix}$, then show that:

$(AB)^{-1} = B^{-1}A^{-1}$

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5. Find the inverse matrix of the matrix $A = \begin{vmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{vmatrix}$

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6. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I_3 = 0$. Hemce find A^{-1}.

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7. If $A = \begin{vmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{vmatrix}$, then show that $A = A^{-1}$

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8. If $A = \begin{vmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{vmatrix}$, then evaluate A. (adj. A).

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9. If $A = \begin{vmatrix} -3 & -1 & 2 \\ 2 & 2 & -3 \\ 1 & 3 & -1 \end{vmatrix}$, then show that :

A.(adj.A)=(adj.A). A.

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10. If $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}$, then show that A. (ajd.A)= (adj.A)A.

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11. $A = \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}$, then show that $\left(A^{-1}\right)^{-1} = A.$

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12. If $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ and $\text{A} = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 2 & 3 \\ 0 & 5 & 5 \end{bmatrix}$, then find $(I - A)^{-1}$

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13. If $A^{-1} = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$ and B $= \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, then find$(AB)^{-1}$

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Exercise 4 E

1. Solve the following cquations by matrix method :

(i) 3x+y=10

x+2y=5

(ii) x+3y=11

3x-y=3.

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2. Solve the equations by matrix method: x+y+z=6 x+y-z=0 2x+y+2z=10

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3. Solve the equations by matrix method : (i) x+2y+z=7 x+2y+z=7 x+3z=11

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4. Test the consistency of the following system of equations : (i) 3x-y=2 ,6x-2y=4 (ii) x+5y=1 , 2x+2y=4 (iii) 2x-z=-1, 6x-6y-2z=5 , 3x-y-2z=2

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Exercise 4 F

1. Choose the correct answer from the following :

The value of $\begin{vmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{vmatrix}$ is:

A. -1

B. 1

C. 0

D. None of these

Answer: B

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2. Choose the correct answer from the following :

$\begin{vmatrix} x & 2 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 8 \\ 9 & 12 \end{vmatrix}$, then x:

A. ± 6

B. ± 4

C. ± 2

D. None of these

Answer: B

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3. Choose the correct answer from the following :

The value of $\begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ -1 & 0 & 4 \end{vmatrix} is:$

A. -6

B. 18

C. -18

D. None of these

Answer: C

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4. Choose the correct answer from the following :

The value of $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} is:$

A. $(a-b)(b-c)(c-a)$

B. $-(a-b)(b-c)(c-a)$

C. $(a+b+c)(a-b)(b-c)(c-a)$

D. None of these

Answer: A

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5. Choose the correct answer from the following : The value of $\begin{vmatrix} bc & -c^2 & ca \\ ab & ac & -a^2 \\ -b^2 & bc & ab \end{vmatrix} is:$

A. 4abc

B. $4a^2b^2c^2$

C. $4a^3b^3c^3$

D. None of these

Answer: B

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6. Choose the correct answer from the following :

If A is a matrix of order $3 \times 3 \text{ and } |A| = 6$, then |adj.A|

A. 36

B. 216

C. 729

D. None of these

Answer: A

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7. Choose the correct answer from the following :

If a, b, c are in arithmetic progression, then $\begin{vmatrix} x+1 & x+4 & x+a \\ x+2 & x+5 & x+b \\ x+3 & x+6 & x+c \end{vmatrix} =$

A. 2x

B. 1

C. 0

D. None of these

Answer: C

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8. Choose the correct answer from the following :

The verices of a triangle (2,-4), (-6,3) and (3,5). The area of triangle is :

A. $\frac{79}{2}$sq.units

B. $\frac{81}{2}$sq.units

C. $\frac{75}{5}$sq.units

D. $\frac{85}{2}$sq.units

Answer: A

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9. Choose the correct answer from the following :

If the points (p,7), (2,-5) and (6,3) are collinear, then p =

A. 0

B. 1

C. 2

D. None of these

Answer: D

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10. Choose the correct answer from the following :

The inverse of matrix $\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} is:$

A. $\frac{1}{7}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$

B. $\frac{1}{7}\begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}$

C. $\frac{1}{7}\begin{vmatrix} 1 & -3 \\ -2 & -1 \end{vmatrix}$

D. $\frac{1}{7}\begin{vmatrix} -1 & -1 \\ 2 & 1 \end{vmatrix}$

Answer: A

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Exercise 4 G

1. If x+y+z=0=a+b+c, then $\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} =$

A. 0

B. $xa + yb + zc$

C. 1

D. None of these

Answer: A

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2. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ .^{m}C_1 & .^{m+1}C_1 & .^{m+2}C_1 \\ .^{m}C_2 & .^{m+1}C_1 & .^{m+2}C_2 \end{vmatrix}$ is equal to

A. 0

B. -1

C. -2

D. -3

Answer: B

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3. For $x \neq y \neq z$, $\begin{vmatrix} 1+x^3 & x^2 & 1 \\ 1+y^3 & y^2 & 1 \\ 1+z^3 & z^2 & 1 \end{vmatrix} = 0$ if xyz is

A. 0

B. -1

C. -2

D. -3

Answer: B

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4. If $\begin{vmatrix} a-b & b-c & c-a \\ x-y & y-z & z-x \\ p-q & q-r & r-p \end{vmatrix} = m\begin{vmatrix} c & a & b \\ z & x & y \\ r & p & q \end{vmatrix}, \quad \text{then m} =$

A. 2

B. 1

C. 0

D. None of these

Answer: C

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5. For what value of 'K', the system of equations $kx + y + z = 1, x + ky + z = k$ and $x + y + kz = K^2$ has no solution ?

A. 0

B. -1

C. 1

D. None of these

Answer: D

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6. If $a \ne b \ne c$, then solution of equation

$$\begin{vmatrix} x-a & x-b & x-c \\ x-b & x-c & x-a \\ x-c & x-a & x-b \end{vmatrix} = 0 \quad is:$$

A. $x = 0$

B. $x = a + b + c$

C. $x = \frac{1}{2}(a + b + c)$

D. $x = \frac{1}{3}(a + b + c)$

Answer: D

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7. One root of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0 \quad is:$

A. 6

B. 3

C. 0

D. None of these

Answer: C

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8. $\begin{vmatrix} \sin^2 x & \cos x^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix} = 0$

A. 0

B. $12\cos^2 x - 10\sin^2 x$

C. $12\sin^2 x - 10\cos^2 x - 2$

D. $10\sin 2x$

Answer: A

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9. $\begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ then } \sin\ 4\theta \text{ equal to :}$

A. $\frac{\pi}{24}, \frac{5\pi}{24}$

B. $\frac{5\pi}{24}, \frac{7\pi}{24}$

C. $\frac{7\pi}{24}, \frac{11\pi}{24}$

D. None of these

Answer: C

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10. Let a, b, c be the real numbers. The following system of equations in $x, y, andz$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{a^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{a^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{a^2}=1$ has a. no solution b. unique solution c. infinitely many solutions d. finitely many solutions

A. no solution

B. exactly one solution

C. infinite solutions

D. None of these

Answer: B

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Exercise 4 1

1. Evalute the determinant: :

$$\begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$$

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2. Evalute the determinants in queations 1 and 2 :

(i) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

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3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that |2A|=4|A|.

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4. Evalute the determinants : $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

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5. Evalute the determinants :

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & 1 \\ 3 & -5 & 0 \end{vmatrix}$

(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

(iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

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6. If $A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$, find |A|.

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7. Find the values of x, if

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

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8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to :

(a) 6

(b) ± 6

(c) -6

0

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Exercise 4 2

1. Using the property of determinants and without expanding in questions 1 to 7 prove that ,

$$\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

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2. Using the property of determinants and without expanding in questions 1 to 7 prove that ,

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

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3. Using the property of determinants and without expanding, prove that

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

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4. Using the property of determinants and without expanding, prove that:

$|1bca(b+c)1cab(c+a)1abx(a+b)| = 0$

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5. Using the property of determinants and without expanding, prove that:

$|b+cq+ry+zc+ar+pz+xa+bp+qx+y| = 2|apxbqycrz|$

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6. Using the property of determinants and without expanding, prove that:

$|0a-b-a0-cbc0| = 0$

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7. Using the property of determinants and without expanding, prove that:

$|-a^2abacbab^2bacb-c^2| = 4a^2b^2c^2$

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8. By using properties of determinants. Show that:(i)

$\left|1aa^21\hat{\ }21\hat{\ }2\right| = (a-b)(b-c)(c-a)$(ii)

$\left|111abca^3b^3c^3\right| = (a-b)(b-c)(c-a)(a+b+c)$

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9. Using the properties of determinants, show that:

$$\begin{bmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{bmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

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10. By using properties of determinants. Show that: (i)

$|x+42x2x2\times+42x2x2\times+4| = (5x-4)(4-x)^2$ (ii)

$|y+kyyyy+kyyyy+k| = k^2\left(2y\dot{k}\right)^2$

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11. By using properties of determinants. Show that:(i) $|a-b-c2a2a2-c-a2b2c2-a-b|=(a+b+c)^3$ (ii) $|x+y+2zxyzy+z+2xyzxz+x+2y|=2(x+y+z)^3$

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12. Using properties of determinants, prove the following: $\left|1\times^2 x^2 1\times x^2 1\right|=\left(1-x^3\right)^2$

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13. By using properties of determinants. Show that: $\left|1+a^2-b^2 2ab-2b2ab1-a^2+b^2 2a2b-2a1-a^2-b^2\right|=\left(1+a^2+b^2\right)$

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14. Using properties of determinants, prove the following:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

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15. Let A be a square matrix of order 3×3, then $|kA|$is equal to(A) $k|A|$ (B) $k^2|A|$ (C) $k^3|A|$ (D) $3k|A|$

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16. Which of the following is correct (A) Determinant is a square matrix. (B) Determinant is a number associated to a matrix. (C) Determinant is a number associated to a square matrix. (D) None of these

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Exercise 4 3

1. Find area of the triangle with vertices at the point given in each of the following : (i) (1,0), (6,0), (4,3) (ii) (2,7), (1,1), (10,8)

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2. Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear.

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3. Find the values of k if area of tringle is 4 sq. units and dvertices are : (i) (k,0), (4,0), (0,2)

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4. (i) Find the equation of line joining (1,2) and (3,4) using determinants,

(ii) Find the equation of the line joining (3,1) and (9,3) using determinants.

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5. If area of triangle is 35 sq units with vertices $(2, \ -6), (5, 4) and (k, 4)$.

Then k is(A) 12 (B) -2 (C) $12, 2$ (D) $12, \ -2$

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Exercise 4 4

1. Write Minors and Cofactors of the elements of following determinants :

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

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2. Write minros and cofactros of the elements of the determinants: (i)

$$\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$$

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3. Using Cofactors of elements of second row, evaluate $\Delta = |538201123|$

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4. Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

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5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is cofactors of a_{ij} , then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

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Exercise 4 5

1. Find the adjoint of each of the matrices $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

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2. Find adjoint of the matrice in $[1 - 12235 - 201]$

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3. Verify $A(adjA) = (adjA)A = |A|If$ or $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

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4. Verify $A \quad (adj \quad A) \quad = \quad (adj \quad A) \quad A \quad = \quad |A|\mathrm{I}$

$|1 - 1230 - 2103|$

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5. Find the inverse the matrix (if it exists)given in $[2 - 243]$

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6. Find the inverse the matrix (if it exists)given in $[-15 - 32]$

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7. Find the inverse the matrix (if it exists)given in $[123024005]$

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8. Find the inverse the matrix (if it exists)given in $[10033052 - 1]$

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9. Find the inverse the matrix (if it exists) given in $[2134-10-721]$

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10. Find the inverse the matrix (if it exists)given in $[1-1202-33-24]$

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11. Find the inverse the matrix (if it exists)given in $[0000\cos\alpha\sin\alpha 0\sin\alpha-\cos\alpha]$

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12. If $A=\begin{vmatrix}3 & 2\\ 7 & 5\end{vmatrix}$ and $B=\begin{vmatrix}6 & 7\\ 8 & 9\end{vmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$.

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13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}.

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14. Solve system of linear equations, using matrix method, $x - y + 2z = 7$, $3x + 4y - 5z =$ - 5 $2x - y + 3\ z = 12$

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15. For the matrix $A = [11112 - 32 - 13]$. Show that $A^3 - 6A^2 + 5A + 11\ I_3 = O$. Hence, find A^{-1} .

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16. If $A = [2 - 11 - 12 - 11 - 12]$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

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17. Let A be a non-singular square matrix of order 3×3. Then |adj A| is equal to (a) $|A|$ (B) $|A|^2$ (C) $|A|^3$ (D) $3|A|$

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18. If A is an invertible matrix, then $\det(A^{-1})$ is equal to $\det(A)$ (b) $\frac{1}{\det(A)}$ (c) 1 (d) none of these

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Exercise 4 6

1. Examine the consistency of the system of equations $x + 2y = 2$

$2x + 3y = 3$

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2. Examine the consistency of the system of equations $2x - y = 5$

$x + y = 4$

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3. Examine the consistency of the system of equations $x + 3y = 5$

$2x + 6y = 8$

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4. Examine the consistency of the system of equations

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

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5. Examine the consistency of the system of equations $3xy2z = 22yz = 1$

$3x5y = 3$

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6. Examine the consistency of the system of equations $5xy + 4z = 5$

$2x + 3y + 5z = 25x2y + 6z = 1$

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7. Solve system of linear equations, using matrix method, $5x + 2y = 4$

$7x + 3y = 5$

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8. Solve system of linear equations, using matrix method, $2.x \quad y = 2\ 3x + 4y = 3$

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9. Solve system of linear equations, using matrix method,$4x3y = 3$

$3x5y = 7$

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10. Solve system of linear equations, using matrix method,$5x + 2y = 3$

$3x + 2y = 5$

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11. Solve system of linear equations, using matrix method,$2x + y + z = 1$

$x - 2y - z = \frac{3}{2} 3y - 5z = 9$

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12. Solve system of linear equations, using matrix method,

$x - y + z = 4, \quad 2x + y - 3z = 0$

$x + y + z = 2$

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13. Solve system of linear equations, using matrix method,

$2x + 3y + 3z = 5x - 2y + z = -4, 3x - y - 2z = 3$

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14. Solve system of linear equations, using matrix method.

$x - y + 2z = 7, 3x + 4y - 5z = -5, 2x - y + 3z = 12$

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15. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1}. Use it to solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$ and $x + y - 2z = -3$

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16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

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Miscellaneous Exercise

1. Prove that the determinant $[x \sin\theta \cos\theta - \sin\theta - x 1 \cos\theta 1 x]$ is independent of 0.

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2. Without expanding the determinant, prove that $|aa^2bc \hat{} 2ca \hat{} 2ab| = |1a^2a^31b^2b^31c^2c^3|$

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3. Ecaluate

$$\begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{bmatrix}$$

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4. if a,b, and c are real number and

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$a+b+c=0$ or $a=b=c$

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5. Solve the following determinant equation: $\begin{vmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{vmatrix} = 0$

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6. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

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7. If $A^{-1}=[3-11-156-55-22]$ and $B=[12-2-1300-21]$, find $(AB)^{-1}$.

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8. Let $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$. Verify that ltbtgt (i) $[adjA]^{-1}=adj(A^{-1})$

(ii) $(A^{-1}\hat{}(-1)=A$

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9. Evaluate $|xyx+yyx+y\times\ +yxy|$.

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10. Evaluate the following: $\begin{vmatrix}1 & x & y\\ 1 & x+y & y\\ 1 & x & x+y\end{vmatrix}$

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11. Using properties of determinants. Prove that

$$\left|\alpha\alpha^2\beta+\gamma\rho\rho^2\gamma+\alpha\gamma\gamma^2\alpha+\beta\right| = (\rho-\gamma)(\gamma-\alpha)(\alpha-\rho)(\alpha+\rho+\gamma)$$

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12. For any scalar p prove that

$$= \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)\,.$$

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13. show that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$$= 3(a+b+c)(ab+bc+ca)$$

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14. Show that $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1.$

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15. Show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$

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16. Solve the system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,$
$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

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17. Choose the correct answer in questions 17 to 19:

If a, b, c are in A.P., then the determinant $\begin{bmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{bmatrix}$ is :

(a) 0

(b) 1

(c) x

(d) 2x

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18. If x, y, z are non-zero real numbers, then the inverse of matrix $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ is

(A) $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(B) $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

(C) $\frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

(D) $\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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19. Let $A = |1 \sin\theta 1 - \sin\theta 1 \sin\theta - 1 - \sin\theta 1|$, where $0 \leq \theta \leq 2\pi$.

Then $Det(A) = 0$ (b) $Det(A) \in (2, \infty)$ $Det(A) \in (2, 4)$ (d) $Det(A) \in [2, 4]$

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