



# MATHS

# **BOOKS - NAGEEN PRAKASHAN ENGLISH**

# LINEAR PROGRAMMING

Solved Example Type

**1.** Maximise Z=3x+4y

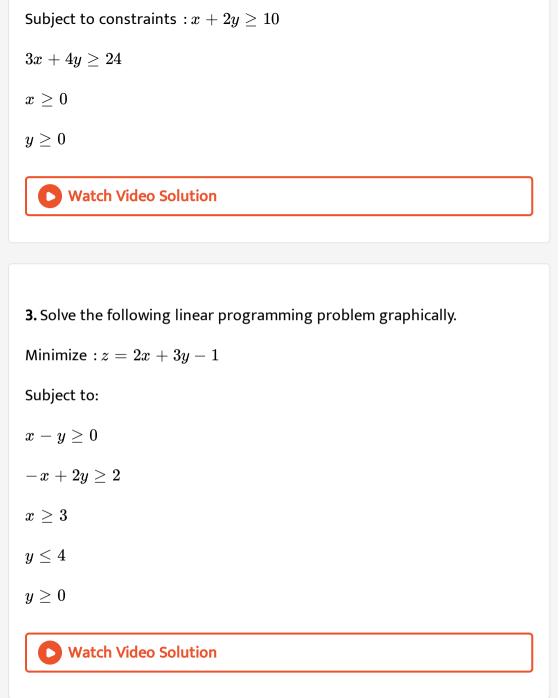
Subject to the constraints

 $x+y\leq 4, x\geq 0, y\geq 0$ 



2. Solve the following linear programming problem graphically:

 $\mathsf{Minimize}: z = 100x + 50y$ 



Minimize: z = x + 3y

Subject to:

 $x+y\leq 8$ 

 $3x+5y\geq 15$ 

 $x \ge 0$ 

 $y \geq 0$ 

Watch Video Solution

5. Solve the following linear programming problem graphically:

Maximize : z = 5x + 3y

Subject to:

 $x + 3y \leq 5$ 

 $x+y\leq 3$ 

 $x \ge 0$ 

 $y \geq 0$ 

**6.** A man wishes to mix two types of food X and Y in such a way that the vitamins contents of the mixture contain at least B units of vitamin A and 11 units of vitamin B. Food X costs Rs. 50 per kg and food Y costs Rs. 60 per kg. Fod X contains 2 units per kg of vitamin A and 5 kg per unit of vitamin B while food Y contains 5 units per kg of vitamin A and 2 units per kg of vitamin B. Determine the minimum cost of the mixture.

Watch Video Solution

7. (Allocation problem) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be

**8.** A factory makes tennis rackets and cricket bats. A tennis racket takes 1. 5 hours of machine time and 3 hours of craftmans time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftmans time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsmans time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically.

### Watch Video Solution

**9.** A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machine for at most 12 hours a day?

**10.** There are factories located one at place P and other at place Q from which a certin commodity is to be delivered to each of three depot situated at A,B and C. The weekly requirement of the depots are respectively 5,5 and 4 units of the commodity while the production capacity of the factories P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below.

From/To		Cost (in	
	Λ	В	С
Р	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. Also find the minimum transportation cost.

View Text Solution



Maximize : z = 60x + 15ySubject to :  $x + y \le 50$  $3x + y \le 90$  $x \ge 0$  $y \ge 0$ 

Watch Video Solution

2. Solve the following linear programming problem graphically:

 $\mathsf{Minimize}: z = 3x + 5y$ 

Subject to:  $x + y \geq 2$ 

 $x+3y\geq 3$ 

 $x \ge 0$ 

```
y \geq 0
```

Maximize : z=30x+25ySubject to:  $x+y\leq 6$  $3x+2y\leq 15$  $x\geq 0$  $y\geq 0$ 

Watch Video Solution

4. Solve the following linear programming problem graphically: Minimize : z=200x+500ySubject to:  $x+2y\geq 10$  $3x+4y\leq 24$  $x\geq 0$  $y\geq 0$ 

Maximize : z = x + 9ySubject to:  $x + 3y \le 60$  $x + y \ge 10$  $x \le y$  $x \ge 0$  $y \ge 0$ 

Watch Video Solution

6. Solve the following linear programming problem graphically:

minimize : z = x + 2y

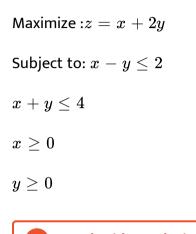
Subject to:  $2x + y \geq 3$ 

 $x + 2y \ge 6$ 

 $x \ge 0$ 

 $y \ge 0$ 

Show that z is minimum at two points.



**Vatch Video Solution** 

8. Solve the following linear programming problem graphically:

```
Maximize :z = 4x + 3y
Subject to: 2x + y \ge 40
x + 2y \ge 50
x + y \le 35
x \ge 0
y \ge 0
```

**1.** One kind of cake requires 200 g of flour and 25 g of fat and another kind of cake requires 100g of flour and 50 g of fat. Find the maximum number of cakes which can be in make from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of other in gradients used in making the cakes. Formulate the above as a linear programming problem and solve it graphically.

### Watch Video Solution

**2.** A woman wishes to mix two types of foods in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and at least 10 units of vitamin C. Food *I* contains 2 units/kg of vitamin A and 1 unit /kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 unit/kg of vitamin C. The cost of food I is Rs. 50/kg and of food II is Rs. 70/kg. Find the minimum cost of such a mixture.

**3.** Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60 / kg and Food Q costs Rs 80 / kg. Food P contains 3 units/kg

Watch Video Solution

**4.** Two tailors A and B earns 15 and 20 per dayrespectively. A can stitch 6 shirts and 4 paints while Bcan stitch 10 shirts and 4 paints per day. To minimise the cost to stitch 60 shirts and 32 paints, how manydays should they work?



**5.** (Manufacturing problem) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires

12 labour hours for fabricating a

Watch Video Solution

**6.** A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the nu

Watch Video Solution

**7.** A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture m order to maximise the profit?

#### Watch Video Solution

**8.** A dietician wishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units f Vitamin A and 10 units of vitamin C, Food I contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C while food II contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs Rs 50.00 per kg to purchase food I and Rs. 70.00 per kg to produce food II. Formulate the above linear programming problem to minimize the minimize the cost of such a mixture.

# Watch Video Solution

**9.** There are two types of fertilizers  $F_1$  and  $F_2$ .  $F_1$  consists of 10% nitrogen and 6% phosphoric acid and  $F_2$  contains 5% nitrogen and 10% phosphoric acid. After testing the soil condition a farmer finds that he

needs at least 14 kg of nitrogen and 14 kg of phosphoric acid forhis crop. If  $F_1$  costs Rs. 6 per kg and  $F_2$  costs Rs. 5 perkg, determine how muchofeach of fertilizers beused so that nutrient requirements are met at a minimum cost. What is the minimum cost?



**10.** A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the tune (in minutes) required for each toy on the machines is given below:

Types of toys	Machine		
	Ι	II	ш
A	12	18	6
В	6	0	9

Each machine is available for a maximum of Each machine

is available for a maximum of 6 hours per day. If the profit



**11.** Two godowns A and B have grain capacity of 100 quintal and 50 quintal respectively. They supply to 3 ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

How should the supplies be transpored in order that the transportation cost is minimum cost?

Watch Video Solution

**12.** An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps D,E,F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distance (in Km) between the depots and the petrol pumps are given in the following table :

From	Distance	🍒 (in Km)
То	- AR4	B
D 8	7	3
È E ⊰	6	4
F着	3	2

Assuming that the transportation cost of 10 litres of oil is  $\gtrless$  1 per km , how should the delivery be scheduled in order that the transportation cost is minimum ?

Watch Video Solution

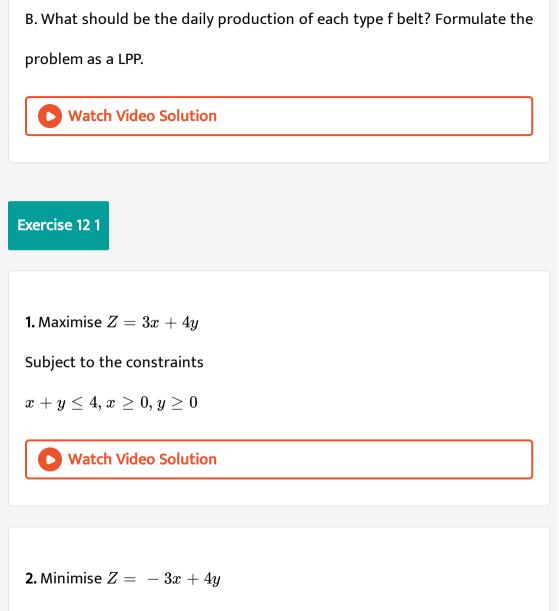
**13.** A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of sc



**14.** An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class then by first class. Determine how many tickets of each type must be sold to maximise profit for the airline. Form an LPP and solve it graphically.

Watch Video Solution

**15.** A company makes two kinds of leather belts, A and B. Belt A is high quality belt, and B is of lower quality. The respective profits are Rs. 40 and Rs. 30 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle, and only 400 buckles per day are available. There are only 700 buckles available for belt



Subject to  $x+2y\leq 8,$   $3x+2y\leq 12,$   $x\geq 0,$   $y\geq 0.$ 

**3.** Maximise Z = 5x + 3y

Subject to  $3x+5y\leq 15,$   $5x+2y\leq 10,$   $x\geq 0,$   $y\geq 0.$ 



4. Solve the following linear programming problem graphically:

 $\mathsf{Minimize}: z = 3x + 5y$ 

Subject to:  $x+y\geq 2$ 

 $x+3y\geq 3$ 

- $x \ge 0$
- $y \geq 0$

Watch Video Solution

5. Maximise Z = 3x + 2y

Subject to  $x+2y\leq 10,$   $3x+y\leq 15,$  x,  $y\geq 0$ 

minimize : z = x + 2ySubject to:  $2x + y \ge 3$  $x + 2y \ge 6$  $x \ge 0$  $y \ge 0$ 

Show that z is minimum at two points.



7. Minimise and Maximise Z=5x+10y

Subject to  $x+2y\leq 120, x+y\geq 60, x-2y\geq 0, y\geq 0.$ 

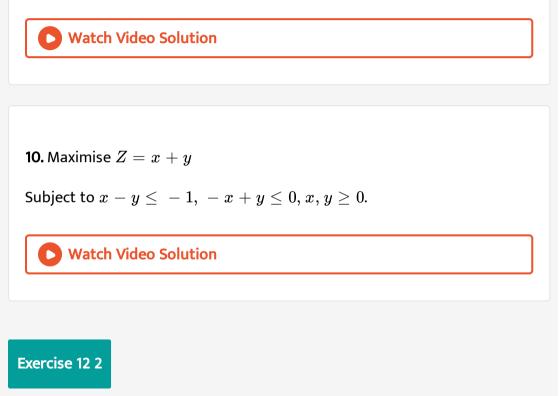
### Watch Video Solution

**8.** Minimise and Maximise Z=x+2y

Subject to  $x+2y\geq 100,$   $2x-y\leq 0,$   $2x+y\leq 200,$  x,  $y\geq 0.$ 

9. Maximise Z = -x + 2y

Subject to the constraints  $x \geq 3, x+y \geq 5, x+2y \geq 6, y \geq 0$ 



Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60 / kg and Food Q costs Rs 80 / kg.
 Food P contains 3 units/kg

**2.** One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Make an L.P.P. of the above and solve it graphically.

Watch Video Solution

**3.** A factory makes tennis rackets and cricket bats. A tennis racket takes 1. 5 hours of machine time and 3 hours of craftmans time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftmans time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsmans time. If the profit on a racket and on a bat is Rs. 20 and Rs. 10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P. and solve graphically.

**4.** A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machine for at most 12 hours a day?



**5.** A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of sc

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs. 25 and that from a shade is Rs. 15. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit. Formulate an LPP and solve it graphically.

#### Watch Video Solution

**7.** A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20

minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture m order to maximise the profit?



**8.** A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the nu

## Watch Video Solution

**9.** A diet is to contain at leat 80 units of vitamin A and 100 units of minerals. Two foods  $F_1$  and  $F_2$  are available. Food  $F_1$  cost Rs. 4 per unit and  $F_2$  costs Rs. 6 per unit. One unit of food  $F_1$  contains 3 units of vitamin A and 4 units of minerals. One unit of food  $F_2$  contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming

problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

#### Watch Video Solution

10. There are two types of fertilizers  $F_1$  and  $F_2$ .  $F_1$  consists of 10% nitrogen and 6% phosphoric acid and  $F_2$  contains 5% nitrogen and 10% phosphoric acid. After testing the soil condition a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid forhis crop. If  $F_1$  costs Rs. 6 per kg and  $F_2$  costs Rs. 5 perkg, determine how muchofeach of fertilizers beused so that nutrient requirements are met at a minimum cost. What is the minimum cost?

# Watch Video Solution

**11.** The corner points of the feasible region determined by the following system of linear inequalities:  $2x + y \le 10, x + 3y \le 15, x, y \ge 0 are(0, 0), (5, 0), (3, 4)and(0, 5)Let$  . Condition on p and q so that the maximum of Z occurs at both (3,4) and

(0,5) is 
$$p=q$$
 b.  $p=2q$  c.  $p=3q$  d.  $q=3p$ 

Watch Video Solution

#### **Miscellaneous Exercise**

1. (Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30q) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calciums atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A? How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

**2.** A farmer mixes two brands *P* and *Q* of cattle feed. Brand *P* costing Rs. 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand *Q* costing Rs. 200 per bag contains 1.5 units of nutritonal element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A,B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Watch Video Solution

**3.** A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below:

Food	Vitamin	Vitamin	Vitamin
	А	В	С
X	1	2	3
Y	2	2	1

1 kg of food X costs Rs. 16 and 1 kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet?

Watch Video Solution

**4.** A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each purpose and the time (in minutes) required for each toy on the machines is given below:

		Machines	
Types of toys	Ĩ	Π	III
Α	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs. 7.50 and that on each toy of type B is Rs. 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

#### Watch Video Solution

**5.** An aeroplane can carry a maximum of two passengers. A profit of Rs. 1000 is made on each executive clas ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy clas than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

## Watch Video Solution

**6.** Two godowns A and B have grain capacity of 100 quintals and 50 qunitals respectively. They supply to 3 ration shops, D,E and F whose requirements are 60,50 and 40 quintals repectively. The cost of transportation per quintal from the godowns to the shops are given in

the following table:

Transporta	ation cost per q	uintal (in ₹)
From/To	Α	В
D	6	4
Ε	3	2
F	2.50	3

How should the supplies be transported in order that the transportation

cost is minimum? What is the minimum cost?



**7.** An oil company has two depots, A and B, with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three pumps D,E,F whose requirements are 4500 L, 3000 L and 3500 L respectively. The distance (in Km) between the depots and the petrol pumps are given in the following table :

From	Distance	👗 (in Km)
To	- AA	B
D K-	7	3
2 B-2	6	4
F着	3	2

Assuming that the transportation cost of 10 litres of oil is  $\gtrless$  1 per km , how should the delivery be scheduled in order that the transportation cost is minimum ?



**8.** A fruit grower canuse two types of fertilizer in his garden brand P and brand Q. The amounts (in kg) of nitrogen phosphoric acid, potash, and chlorine a bag of each brand are given in the stable. Tests indicate that the garden needs at least 240kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

	kg per bag Brand P	Brand Q
Nitro en	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

**9.** A toy company manufactures two types of dolls A and B. Market research and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 and Rs. 16 per doll respectively on dolls A and B how many of each should be produced weekly in order to maximise the profit?

View Text Solution