MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

RELATIONS AND FUNCTIONS

Solved Examples

1. If A = {1, 2, 3, 4}, define relations on A which have properties of being

(i) reflexive, transitive but not symmetric.

(ii) symmetric but neither reflexive nor transitive.

(iii) reflexive, symmetric and transitive.

2. A relation R is defined on the set of integers as follows :

${}_aR_b \Leftrightarrow (a - b)$, is divisible by 6 where a, b, $\in$ I. prove that R is an equivalence relation.

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3. What is an equivalence relation?

Show that the relation of'similarity' on the set S of all triangle in a plane is an equivalence relation.

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4. Prove that the relation R on the set $N \times N$ defined by $(a, b)R(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation. Also, find the equivalence classes [(2, 3)] and [(1, 3)].

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5. In the set of straight lines in a plane, for the relation 'perpendicular' check whether it is reflexive, symmetric and transitive.

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6. A relation R on set of complex numbers defined by $Z_1 R Z_2 \Leftrightarrow \frac{Z_1 - Z_2}{Z_1 + Z_2}$ is real then which of the following is not true ?

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7. Let a relation R be defined by relation The $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$ $R^{-1} o R$ is given by

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8. Which types of the following functions are ?

(i) {(a,1),(b,1),(c,1),(d,1),(e,1)}

(ii) {(3,2),(6,4)(9,2),(12,4)}

(iii) {(a,1),(b,2),(c,3),(d,4)}.

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9. prove that a function f = $\{(x, 2x+1) : x \in N\}$ defined on the set of natural numbers NxxN is one - one function.

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10. Prove that the function $f : N \rightarrow N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

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11. Show that the function $f: R \rightarrow R: f(x) = \sin x$ is neither one-one nor onto

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12. Check the injectivity and surjectivity of the following functions:(i) $f: N \rightarrow N$given by $f(x) = x^2$(ii) $f: Z \rightarrow Z$given by $f(x) = x^2$(iii) $f: R \rightarrow R$given by $f(x) = x^2$(iv) $f: N \rightarrow N$given by $f(x) = x^3$(v) `f : Z ->

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13. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are two mappings such that $f(x) = 2x$ and $g(x) = x^2 + 2$ then find fog and gog.

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14. If $f: R \rightarrow R$ and $g: R \rightarrow R$ be two mapping such that f(x) = sin x and g(x) = x^2, then

prove that fog $\neq$ gof.

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15. If f and g two functions are defined as :

f= {(1,2),(3,6),(4,5)} and g = {(2,3),(6,7),(5,8)}, then find gof.

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16. A function $f: R \rightarrow R$ is defined as $f(x) = x^2 + 2$, then evaluate each of the following :

(i) $f^{-1}(-6)$ (ii) $f^{-1}(18)$

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17. Prove that the function f : $R \rightarrow R$ where R is the set of all real numbers, defined as f (x) = $3x + 4$ is one-one and onto . Also find the inverse function of f.

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18. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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19. Show that subtraction and division are not binary operations on N.

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20. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \rightarrow P$given by $(A, B) \rightarrow A \cup B$and $\cap : P \times P \rightarrow P$ given by $(A, B) \rightarrow A \cap B$are binary operations on the set P.

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21. Show that the operation $\vee$ and $\wedge$ on R defined as $a \vee b =$ Maximum of a and b ; $a \wedge b =$ Minimum of a and b are binary operations of R.

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22. Show that $+ : R \times R \rightarrow R$and $\times : R \times R \rightarrow R$are commutative binary operations, but $- : R \times R \rightarrow R$and $\div : R_{\cdot} \times R_{\cdot} \rightarrow R_{\cdot}$are not commutative.

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23. Prove that $*: R \times R \to R$ defined as $a * b = a + 2ab$ is not commutative .

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24. Prove that in the set of real numbers '+' and $' \times '$ are associative but '-' and $' \div '$ are not associative.

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25. Prove that $*: R \times R \to R$ defined as $a * b = a + 2ab$ is not associative

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26. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $\div R \times R \to R$ and $\div : R_{.} \times R_{.} \to R_{.}$.

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27. Show that a is the inverse of a for the addition operation + on R and $\frac{1}{a}$ is the inverse of $a \neq 0$for the multiplication operation $\times$ on R.

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28. Show that a is not the inverse of $a \in N$ for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation $\times$ on N, for $a \neq 1$.

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Exercies 1 A

1. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm

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2. If A= {x,y,z}, B=(1,2,3} and R= {(x,2),(y,3),(z,1),(z,2), then find R^{-1}.

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3. Prove that the relation R on Z defined by $(a,\ b) \in R \Leftrightarrow\ a-b$ is divisible by 5 is an equivalence relation on Z .

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4. Prove that the relation $R = \{(x, y) : x, y \in N \text{ and } x - y \text{ is divisible by } 7\}$ defined on positive integers N is an equivlance relations.

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5. (i) Prove that the relation $_xR_y \Leftrightarrow 3$ is a factor of (x-y) , defined on the set of integers is an equivalence relation.

(ii) If x is the set of real numbers, then prove that the relation $R = \{(a, b) : a \in x, b \in x \text{ and } a = b\}$ is an equivalence relation.

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6. If I is the set of real numbers, then prove that the relation $R = \{(x, y) : x, y \in I \text{ and } x - y \text{ is an integer }\}$, then prove that R is an equivalence relations.

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7. If A={a,b,c,d}, then on A .

(i) write the identity relation I_A .

(ii) write a reflexive relation which is not the identity relation.

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8. Show that the relation $\geq$ on the set R of all real numbers is reflexive and transitive but not symmetric.

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9. Let O be the origin. We define a relation between two points P and Q in a plane if $OP = OQ$. Show that the relation, so defined is an equivalence relation.

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10. Let N be the set of all natural numbers and let R be a relation on $N \times N$, defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on $N \times N$

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11. (i) Show that in the set of positive integer, the relation ' greater than ' is transitive but it is not reflexive or symmetric.

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12. Show that the relation is congruent to on the set of all triangles in a plane is an equivalence relation

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13. Show that the relation $\geq$ on the set R of all real numbers is reflexive and transitive but not symmetric.

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14. Let R be relation difined on the set of natural number N as follows, $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

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15. Let a relation R be defined by relation The $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$ $R^{-1}oR$ is given by

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16. If $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 8\}$ and $C = \{4, 16, 36, 39\}$ are three sets and R is a relation from A to B and S from B to C defined as

${}_aR_b \Leftrightarrow b = 2a$ where $a \in A, b \in B$

${}_bS_c \Leftrightarrow c = b^2$ where $b \in B, c \in C$

$\therefore$ Find SoR.

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17. Show that the relation R in the set R of real numbers, defined at $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

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18. If $R_1$and $R_2$are equivalence relations in a set A, show that $R_1 \cap R_2$is also an equivalence relation.

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Exercies 1 B

1. $f: R \rightarrow R$ is a function where f(x)= 2x-3 . Check whether f is noe - one ?

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2. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

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3. Show that the function $f: N \to N$, given by $f(x) = 2x$, is one-one but not onto.

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4. The function $f: R \to R$ defined as $f(x) = x^2$. The function f is

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5. If Q is the set of rational numbers, then prove that a function $f: Q \to Q$ defined as $f(x) = 5x - 3, x \in Q$ is one -one and onto function.

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6. If R is the set of real numbers then prove that a function $f: R \to R$ defined as $f(x) = \frac{1}{x}, x \ne 0, x \in R,$ is one-one onto.

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7. Prove that the function $f: R^{+} \to R$ which is defined as $f(x) = \log_e x$ is one - one .

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8. If R is the set of real numbers prove that a function $f: R \to R, f(x) = e^{x}, x \in R$ is one to one mapping.

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9. A function $f: R \rightarrow R$ is defined as $f(x) = 4x - 1, x \in R,$ then prove that f is one - one.

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10. Let $A = R - \{3\} and B = R - \{1\}.$ Consider the function $f: A^{\rightarrow}$ defined by $f(x) = \frac{x-2}{x-3}.$ Show that is one-one and onto and hence find f^{-1}

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11. Let the function $f: R \rightarrow R$ be defined by $f(x) = \cos x, \forall x \in R.$ Show that f is neither one-one nor onto.

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12. If a function $f: R \to R$ is defined as $f(x) = x^3 + 1$, then prove that f is one-one onto.

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13. If $A = \left\{x : x \in R, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}\right\}, B = \{y : y \in R, -1 \leq y \leq 1\}$, then show that the function $f: A \to B$ defined as $f(x) = \sin x, x \in A$ is one-one onto function.

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14. If $f: R \to R$ and $g: R \to R$ be two functions defined as respectively $f(x) = 2x$ and $g(x) = x^2 + 2$, then prove that

(i) f is one-one onto

(ii) g is many-one into.

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15. Function $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined as $f(x) = \sin x$ and $g(x) = e^x$.

Find (gof)(x) and (fog)(x).

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16. If $f : R \rightarrow R$ and $g : R \rightarrow R$ be two functions defined as f(x)=2x+1 and $g(x) = x^2 - 2$ respectively , then find (gof) (x) and (fog) (x) and show that (fog) (x) $\neq$ (gof) (x).

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17. If f and g are two functions from R to R which are defined as $f(x) = x^2 + x + 1$ and $g(x) = 2x - 1$ for each $x \in R$, then show that (fog) (x) $\neq$ (gof) (x).

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18. If $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions defined as $f(x) = x^2$ and $g(x) = 5x$ where $x \in R$, then prove that (fog)(2) $\neq$ (gof) (2).

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19. If $f: R \rightarrow R$ defined as $f(x) = 3x + 7$, then find $f^{-1}(-2)$

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20. If Q is the set of rational numbers and a function $f: Q \rightarrow Q$ is defined as $f(x) = 5x - 4, x \in Q$, then show that f is one-one and onto.

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Exercies 1 C

1. Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this. (i) $On Z^{+}, def \in e \cdot bya \cdot b = a - b$(ii) `O n Z^+,

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2. Check the commutativity and associativity of * on Z defined by $a \cdot b = a - b$ for all $a, \ b \in Z$.

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3. Construction a composition table for binary operation $\wedge$ defined as $a \wedge b$= minimum of {a,b} in the set {1,2,3,4,5} and

(i) evaluate $(2 \wedge 3) \wedge 4 \text{ and } 2 \wedge (3 \wedge 4)$

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4. Consider the infimum binary operation $\wedge$ on the set $S = \{1, 2, 3, 4, 5\}$ defined by $a \wedge b =$ Minimum of $a \text{ and } b$. Write the composition table of the operation $\wedge$.

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5. Let $' \cdot '$ be a binary operation on N given by $a \cdot b = L\dot{C}\dot{M}a\dot{,} b$ for all $a, b \in N$. Find $5 \cdot 7, 20 \cdot 16$ (ii) Is * commutative? Is * associative? Find the identity element in N Which element of N are invertible? Find them.

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6. Find which of the operations given above has identity.

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7. Let A = R R and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A.

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8. State whether the following statements are true or false. Justify. (i) For an arbitrary binary operation $\cdot$ on a set $N, \quad a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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9. If * is a binary operation in N defined as a*b $= a^3 + b^3$, then which of the following is true :

(i) * is associative as well as commutative.

(ii) * is commutative but not associative

(iii) * is associative but not commutative

(iv) * is neither associative not commutative.

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Exercies 1 D

1. Let R be a relation on the set of all line in a plane defined by $(l_1, l_2) \in R \Leftrightarrow l_1$ is parallel to line l_2. Show that R is an equivalence relation.

A. only reflexive

B. only symmetric

C. only transitive

D. equivalence relation

Answer: D

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2. In the set of straight lines in a plane, for the relation 'perpendicular' check whether it is reflexive, symmetric and transitive.

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: B

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3. Show that in the set of triangles in a plane , the relation 'similarity' is an equivalence relation.

A. reflexive, symmetric , transitive

B. reflexive, transitive but not symmetric

C. symmetric , transitive but not reflexive

D. none of the above

Answer: A

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4. If $f: N \rightarrow N$ defined as $f(x) = x^2 \forall x \in N,$ then f is :

A. many-one

B. one-one

C. onto

D. none of these

Answer: B

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5. The function f is defined as :

$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

The range of f is :

A. {1,0}

B. {0,-1}

C. {1,-1}

D. {1,0,-1}

Answer: D

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6. Let the function $f:R \to R$ be defined by $f(x)=\cos x, \forall x \in R$. Show that f is neither one-one nor onto

A. into

B. one-one

C. onto

D. none of these

Answer: C

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7. Let $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n is odd \frac{n}{2}, & \text{if } n is even \end{cases}$ for all $n \in N$.

State whether the function f is bijective. Justify your answer.

A. one-one into

B. one-one onto

C. many-one into

D. many-one onto

Answer: B

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8. If $f : R \to R$ is defined as f(x)=2x+5 and it is invertible , then $f^{-1}(x)$ is

A. $\frac{x-5}{2}$

B. $\frac{x-2}{5}$

C. $\frac{x+5}{2}$

D. none of these

Answer: A

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9. Is $\cdot$ defined on the $set\{1, 2,3, 4,5\}$ by $a^*b = L\dot{C}\dot{M}\cdot$ of a and b a binary operation? Justify your answer.

A. 6

B. 24

C. 36

D. none of these

Answer: C

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10. On the set Z of integers, if the binary operation * is defined by a*b=$a + b + 2$, then find the identity element.

A. commutative

B. associative

C. commutative and associative

D. none of above

Answer: D

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Exercies 1 E

1. In the set $A = \{1, 2, 3, 4, 5\}$ a relation R is defined by $R = \{(x, y) + x, y\varepsilon A$ and $x < \}$. Then R is

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: C

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2. Let R and S be two non-void relations on a set A. Which of the following statement is false?

A. R and S are reflexive , then $R \cap S$ is also reflexive .

B. R and S are symmetric , then $R \cup S$ is also symmetric

C. R and S are transitive , then $R \cap S$ is also transitive .

D. R and S are transitive , then $R \cup S$ is also transitive.

Answer: B

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3. If $f(x) = \frac{x-1}{x+1}, \quad \text{then f(2x) is:}$

A. $\frac{1+f(x)}{3+f(x)}$

B. $\frac{1+3f(x)}{3+f(x)}$

C. $\frac{3+f(x)}{1+f(x)}$

D. $\frac{1+3f(x)}{3-f(x)}$

Answer: B

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4. If n(A)=3 and n(B)=4 , then no. of of one-one function from A to B is

:

A. 12

B. 24

C. 36

D. none of these

Answer: B

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5. Let $f(X)=\frac{ax+b}{cx+d}$ then $f[f(x)]=x$ provided that

A. a=b=c=d=1

B. a=b=1

C. a=d

D. a=-d

Answer: D

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6. Let $f:[-1, \infty) \to [-1, \infty)$ is given by $f(x) = (x+1)^2 - 1$, $x \geq -1$. Show that f is invertible. Also, find the set $S = \{x, f(x) = f^{-1}(x)\}$.

A. $\left\{0, -1, \dfrac{-3 \pm \sqrt{3}}{2}\right\}$

B. {0,1,-1}

C. {0,-1}

D. {}

Answer: C

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7. If n(A)=10, then no of different functions from A to A is :

A. $\underline{|10}$

B. 10^{10}

C. 2^{10}

D. $2^{10} - 1$

Answer: B

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8. If $g(f(x)) = |\sin x| and f(g(x)) = (\sin \sqrt{x})^2$, then (a). $f(x) = \sin^2 x, g(x) = \sqrt{x}$ (b). $f(x) = \sin x, g(x) = |x|$ (c). $f\left(x = x^2, g(x) = \sin \sqrt{x}\right.$ (d).$f \ and g$ cannot be determined

A. $f(x) = \sin^2 x, g(x) = \sqrt{x}$

B. $f(x) = \sin x g(x) = |x|$

C. $f(x) = x^2, g(x) = \sin\sqrt{x}$

D. f(x) and g(x) cannot be determined

Answer: A

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9. If the function $f : A \to B$ is one-one onto and $g : B \to A$, is the inverse of f, then fog =?

A. f

B. g

C. I_A

D. I_B

Answer: D

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10. If $f(x)=\left(ax^2+b\right)^3$, the function g such that $f(g(x))=g(f(x))$, is given by

A. $\left(\dfrac{x^{1/3}-b}{a}\right)^{1/2}$

B. $\dfrac{1}{\left(ax^2+b\right)^3}$

C. $\dfrac{1}{\left(ax^2+b\right)^{1/3}}$

D. none of these

Answer: A

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Exercise 1 1

1. Determine whether each of the following relations are reflexive, symmetric and transitive :

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

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2. Give an example of a relation. Which is (i) Symmetric but neither reflexive nor transitive. (ii) Transitive but neither reflexive nor symmetric. (iii) Reflexive and symmetric but not transitive. (iv) Reflexive and transitive but not symmetric. (v) Symm

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3. Show that the relation R on the set A of points in a plane, given by $R = \{(P,\ Q) :$ Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further show that the set of all points related to a point $P \neq (0,\ 0)$ is the circle passing through P with origin as centre.

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4. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 (is similar to T)_2\}$, is equivalence relation. Consider three right angle triangles $T_1$with sides 3, 4, 5, $T_2$with sides 5, 12, 13 and T_3w

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5. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3,4 and 5?

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6. Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1(\text{ i s p a r a l l e l t o L})_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

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7. Let R be the relation on the set $A = \{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Then, R is (a) reflexive and symmetric but not transitive (b) R is reflexive and transitive but not symmetric (c) R is symmetric and transitive but not reflexive (d) R is an equivalence relation

A. R is reflexive and symmetric but not transitive

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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8. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Choose the correct answer.(A) $(2, 4) \in R$ (B) $(3, 8) \in R$ (C) $(6, 8) \in R$(D) $(8, 7) \in R$

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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Exercise 1 2

1. Show that the function $f: R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set of all non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ?

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2. Check the injectivity and surjectivity of the following functions:(i) $f: N \to N$given by $f(x) = x^2$(ii) $f: Z \to Z$given by $f(x) = x^2$(iii) $f: R \to R$given by $f(x) = x^2$(iv) $f: N \to N$given by $f(x) = x^3$(v) `f : Z ->

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3. Prove that the Greatest Integer Function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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4. Show that the Modulus Function $f: R \rightarrow R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$is x, if x is positive or 0 and $|x|$ is x, if x is negative.

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5. Show that the Signum function $f: R \rightarrow R$, given by $f(x) = \{1, \quad \text{if} \quad x > 00, \quad \text{if} \quad x = 0 - 1, \quad \text{if} \quad x < 0$ is neither one-one nor onto.

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6. Let $A = \{1, 2, 3\},\ B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4),\ (2, 5),\ (3, 6)\}$ be a function from A to B. State whether f is one-one or not.

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7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.(i) $f: R \to R$, defined by $f(x) = 34x$(ii) $f: R \to R$, defined by $f(x) = 1 + x^2$

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8. Let A and B be two sets. Show that $f: A \times B \to B \times A$ defined by $f(a,\ b) = (b,\ a)$ is a bijection.

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9. Let $f : N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n is odd \frac{n}{2}, & \text{if } n is even \end{cases}$ for all $n \in N$.

State whether the function f is bijective. Justify your answer.

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10. Let $A = R - \{3\}$and $B = R - \{1\}$. Consider the function $f: A \to B$defined by $(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

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11. Let $f: R \Rightarrow R$ be defined as $f(x) = x^4$. Choose the correct option

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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12. Let $f: R \rightarrow R$be defined as $f(x) = 3x$. Choose the correct answer.(A) f is one-one onto (B) f is many-one onto(C) f is one-one but not onto (D) f is neither one-one nor onto.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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Exercise 1 3

1. Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$be given by $f = \{(1, 2), (3, 5), (4, 1)\}$and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof.

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2. Let f, g and h be functions from R to R. Show that 1. $(f + g)oh = foh + goh2. (f. g)oh = (foh). (goh)$

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3. Find fog and gof , if (i)$f(x) = |x|$ and $g(x) = |5x - 2|$(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

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4. If $f(x) = \left(\frac{4x+3}{(6x-4), x \neq \frac{2}{3}}\right.$, show that $fof(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f?

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5. State with reason whether following functions have inverse (i) $f: \{1, 2, 3, 4\} \rightarrow \{10\} with f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$(ii) $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\} with g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) `h :{2,3,4,5} ->{7,9

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6. Show that $f:[-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one.

Find the inverse of the function $f:[-1, 1] \to$ Range (f).

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7. Consider $f: R \to R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

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8. Consider $f: R^{\pm} > [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1}.

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9. Consider $f: R \overrightarrow{-5, \infty}$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.

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10. Let $f: X \rightarrow Y$ be an invertible function. Show that f has unique inverse. (Hint: suppose $g_1(\text{ and } g)_2$ are two inverses of f. Then for all $y \in Y, fog_1(y) = I_Y(y) = fog_2(y)$ Use one oneness of f).

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11. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a, f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $\left(f^{-1}\right)^{-1} = f$.

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12. Let $f: X \rightarrow Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1} = f$.

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13. If $f: R \rightarrow R$ be given by $f(x) = \left(3 - x^3\right)^{1/3}$, then $fof(x)$ is(a) $\frac{1}{x^3}$ (b) x^3 (c) x (d) $\left(3 - x^3\right)$

A. $x^{1/3}$

B. x^3

C. x

D. $\left(3 - x^3\right)$

Answer: C

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14. Let $f: R - \left\{-\frac{4}{3}\right\} \to R$be a function as $f(x) = \frac{4x}{3x+4}$. The inverse of f is map, $g: Ran \geq f \to R - \left\{-\frac{4}{3}\right\}$given by.(a) $g(y) = \frac{3y}{3-4y}$ (b) $g(y) = \frac{4y}{4-3y}$(c) $g(y) = \frac{4y}{3-4y}$ (d) $g(y) = \frac{3y}{4-3y}$

A. $g(y) = \frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4-3y}$

C. $g(y) = \frac{4y}{4-3y}$

D. $g(y) = \frac{4y}{3-4y}$

Answer: b

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Exercise 1 4

1. Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this. (i) $OnZ^+, def \in e \cdot bya \cdot b = a - b$(ii) `O n Z^+,

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2. For each binary operation * defined below, determine whether * is commutative or associative.(i) $OnZ, def \in ea \cdot b = a - b$(ii) $OnQ, def \in ea \cdot b = ab + 1$(iii) $OnQ, def \in ea \cdot b = \frac{ab}{2}$(iv) `O

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3. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$defined by $a \wedge b = m \in \{a, b\}$. Write the operation table of the operation $\wedge$.

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4. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table.

(i) Compute (2*3)*4 and 2*(3*4)

(ii) Is * commutative ?

(iii) Compute (2*3) *(4*5)

(Hint : uset the following table)

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5. Let $\cdot$ 'be the binary operation on the set $\{1, 2, 3, 4, 5\}$defined by $a \cdot 'b = H\dot{C}\dot{F}\cdot$of a and b. Is the operation $\cdot$ 'same as the operation $\cdot$ defined in Exercise 4 above? Justify your answer.

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6. Let $\cdot$ be the binary operation on N given by $a \cdot b = L\dot{C}\dot{M}.$of a and b. Find (i) $5 \cdot 7, 20 \cdot 16$(ii) Is $\cdot$ commutative? (iii) Is $\cdot$ associative? (iv) Find the identity of $\cdot$ in N (v) Which elements of N are invert

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7. Is $\cdot$ defined on the $set\{1, 2,3, 4,5\}$ by $a^*b=L\dot{C}\dot{M}.$ of a and b a binary operation? Justify your answer.

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8. Let * be the binary operation on N defined by a*b=HCF of a and b. Does there exist identity for this binary operation on N ?

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9. Find which of the operations given above has identity.

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10. Find the which of the operations given above has identity ?

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11. Let $A = N \times N$ and $\cdot$ be the binary operation on A defined by $(a, b) \cdot (c, d) = (a + c, b + d)$. Show that $\cdot$ is commutative and associative. Find the identity element for $\cdot$ on A, if any.

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12. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation $\cdot$ on a set N, $a \cdot a = a \forall a \in N$. (ii) If $\cdot$ is a commutative binary operation on N, then `a" "*" "(b" "*" "c)"

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13. If * is a binary operation in N defined as a*b $= a^3 + b^3$, then which of the following is true :

(i) * is associative as well as commutative.

(ii) * is commutative but not associative

(iii) * is associative but not commutative

(iv) * is neither associative not commutative.

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Miscellaneous Exercise

1. Let $f: R \rightarrow R$be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$such that$gof = fog = 1_R$

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2. Let $f: W \rightarrow W$be defined as $f(n) = n - 1$, if is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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3. If $f: R \rightarrow R$is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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4. Show that the function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1 + |x|}, x \in R$ is one-one and onto function.

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5. Show that the function $f: R \rightarrow R$given by $f(x) = x^3$is injective.

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6. Give examples of two functions $f: N \rightarrow Z$ and $g: Z \rightarrow Z$ such that gof is injective but g is not injective.

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7. Given examples of two functions $f: \quad N \rightarrow N \quad andg: \quad N \rightarrow N$ such that of is onto but f is not onto. (Hint: Consider `f(x)" "=" "x+1" "a n d""""""""""g(x)" "=" "{x-1 if x>1 1 if x=1}

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8. Given a non-empty set X, consider P(X) which is the set of all subsets of X . Define the relation R in P(X) as follows: For subsets A,B in P(X), ARB if and only if 1$A \subset B$. Is R an equivalence relation on P(X) ? Justify your answer.

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9. Given a non -empty set X, let $\cdot : P(X) \times P(X) \rightarrow P(X)$ be defined as A * B = (A B) ∪ (B A), ∀ A, B ∈ P(X) $A \cdot B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set φ is the identity for the

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10. Find the number of all onto functions from the set $A = \{1,\ 2,\ 3,\ ,\ n\}$ to itself.

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11. Let $S = \{a, b, c\} and T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.(i)$F = \{(a, 3), (b, 2), (c, 1)\}$(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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12. Consider the binary operations $\cdot : R \times R \rightarrow R$ and $o : R \times R \rightarrow R$ defined as $a \cdot b = |a - b|$ and $aob = a$ for all $a,\ b \in R$. Show that $\cdot$ is commutative but not associative, o is associative but not commutative.

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13. Given a non -empty set X, let

$\cdot : \quad P(X) \quad \times \quad P(X) \to P(X)$ be defined as A * B = (A B) ∪ (B

A), ∀ A, B ∈ P(X)

$A \quad \cdot \quad B \quad = \quad (A \quad - \quad B) \cup (B \quad - \quad A), \forall A, \quad B \in P(X)$

. Show that the empty set φ is the identity for the

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14. Define a binary operation * on the set {0, 1, 2, 3, 4, 5} as

$a \cdot b = \{a + b, \quad \text{if} \quad a + b < 6 a + b - 6, \quad \text{if} \quad a + b \geq 6$

Show that 0 is the identity for this operation and each element

$a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a.

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15. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g: A \rightarrow B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \left(\frac{1}{2}\right)\right| - 1, x \in A$. Are f and g equal? Justify your answer. (Hint: One may note that two functio

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16. Let $A = \{1, 2, 3\}$. Then, the number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is (a) 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer:

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17. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is (A) 1 (B) 2 (C) 3 (D) 4

A. 1

B. 2

C. 3

D. 4

Answer:

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18. Let $f: R \to R$be the Signum Function defined as $f(x) = \{1, x > 00, x = 0 - 1, x < 1$ and $g: R \to R$be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then does fo

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19. Number of binary operations on the set {a, b} are (A) 10 (B) 16 (C) 20 (D) 8

A. 10

B. 16

C. 20

D. 8

Answer:

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