MATHS

BOOKS - NAGEEN PRAKASHAN ENGLISH

THREE-DIMENSIONAL GEOMETRY
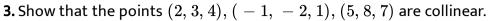
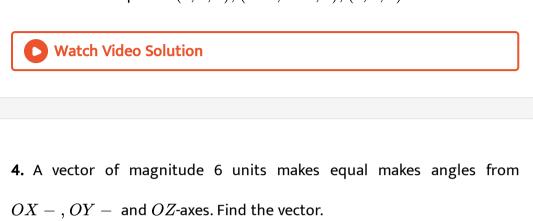**Solved Example**

1. A line makes $30°$, $120°$ and $90°$ angles from the positive direction of x-axis, y-axis and π-axis respectively. Find its direction cosines.

▶ **Watch Video Solution**

2. Find the direction ratios and direction cosines of a line joining the points $(3, \ -4, 6)$ and $(5, 2, 5)$.
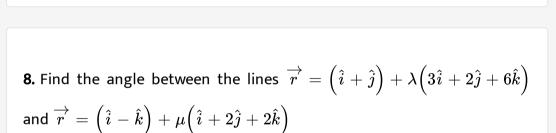
▶ **Watch Video Solution**

3. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

4. A vector of magnitude 6 units makes equal makes angles from $OX-$, $OY-$ and OZ-axes. Find the vector.

5. Find the equation of a line passing through the point $(2, -3, 5)$ and parallel to vector $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$.

6. Find the vector and cartesian equation of a line passes through the points $(1, 3, 2)$ and origin.

7. Find the value of λ. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, \lambda, 10)$ are collinear.

8. Find the angle between the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(\hat{i} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

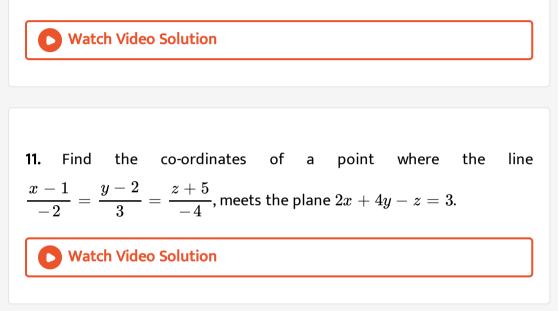**9.** Show that the lines
$$\frac{x-1}{1} = \frac{y}{-5} = \frac{z}{3} \text{ and } \frac{x+1}{7} = \frac{y}{2} - \frac{z-3}{1} \text{ are perpendicular.}$$

10. Show that the lines
$$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7} \text{ and } \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \text{ intersect.}$$
Find the coordinates of their point of intersection.

11. Find the co-ordinates of a point where the line
$$\frac{x-1}{-2} = \frac{y-2}{3} = \frac{z+5}{-4}, \text{ meets the plane } 2x + 4y - z = 3.$$

12. Find the co-ordinates of the foot of perpendicular drawn from point $A(1, 6, 3)$ to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

13. Find the shortest distance between the lines

$$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$$ and

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$$

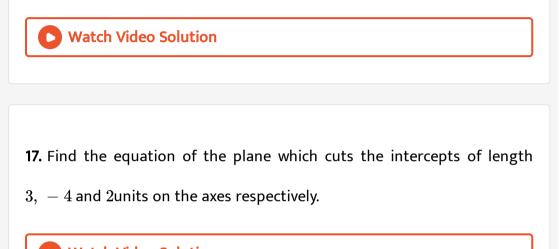**14.** Find the shortest distance between the lines $\dfrac{x+2}{-4} = \dfrac{y}{1} = \dfrac{z-7}{1}$ and $\dfrac{x+3}{-4} = \dfrac{y-6}{3} = \dfrac{9}{2}$

15. Find the shrotest distance between the lines

$$\vec{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \text{ and } \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(2\hat{i} - \hat{j} + \hat{k}\right).$$

16. Show that the points $A(0, 4, 3)$, $B(-1, -5, -3)$, $C(-2, -2, 1)$ and $D(1, 1, 1)$ are coplanar. Also find the equation of the plane in which these points lie.

▶ Watch Video Solution

17. Find the equation of the plane which cuts the intercepts of length 3, -4 and 2units on the axes respectively.

▶ Watch Video Solution

18. Convert the equation of plane $2x - 4y + 3z = 24$ into intercept from and find the intercepts cuts from the axes.

▶ Watch Video Solution

19. A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the three coordinate axes is constant. Show that the plane passes through a fixed point.

20. A plane cuts the co-ordinate axes at A, B and C respectively. If the centroid of $\triangle ABC$ is $(2, -3, 4)$, find the equation of the plane.

21. Find the vector and equation of a plane which is at a distance of 10 units from origin and normal vector from origin to this plane is $2\hat{i} - \hat{j} + 2\hat{k}$.

22. The vector equation of a plane is $\vec{r} \cdot \left(3\hat{i} + 2\hat{j} - 6\hat{k}\right) = 56$. Convert it into normal form. Also find the length of perpendicular from origin and direction cosines of normal to the plane.

▶ Watch Video Solution

23. The co-ordiantes of the foot of perpendicular from origin to a plane are $(1, 2, -3)$. Find the eqution of the plane.

▶ Watch Video Solution

24. Find the normal form of the plane $2x + 3y - z = 5$. Also find the length of perpendicular from origin and d.c's of the normal to the plane.

▶ Watch Video Solution

25. Find the vector and cartesian equation of plane which passes through the point $(1, 3, -2)$ and normal to the vector $\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$.

26. Find the vector equation of the following plane in product form $\vec{r} = \left(\hat{i} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j}\right) + \mu\left(\hat{i} + 3\hat{j} - \hat{k}\right).$

27. Find the cartesian form of the equation of the plane. $\vec{r} = (\lambda - \mu)\hat{i} + (1 - \mu)\hat{j} + (2\lambda + 3\mu)\hat{k}$

28. Find the angle between the planes $3x + y + 2z = 1$ and $2x - y + z + 3 = 0$.

29. Find the value of $'\lambda'$ if the planes $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + 3 = 0$ and $\vec{r}.\left(\lambda\hat{i} + 2\hat{j} + 7\hat{k}\right) = 10$ are perpendicular.

30. Find the equation of a plane passes through the point $(1, -2, 4)$ and parallel to the plane $3x - y + 3 + 2z + 5 = 0$.

31. Find the equation of the plane which passes through the point $(2, 2, 2)$ and through the intersection of the planes $3x - y + 2z = 4$ and $x + y + z = 2$.

32. Find the vector equation of a plane passing through the intersection of the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k} \right) = 6$ and $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} + 4\hat{k} \right) - 5 = 0$ and through the point $(2, 2, 1)$.

33. Find the angle between the line $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k} \right) + \lambda \left(2\hat{i} + 2\hat{j} + \hat{k} \right)$ and the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} + 2\hat{k} \right) + 1 = 0$.

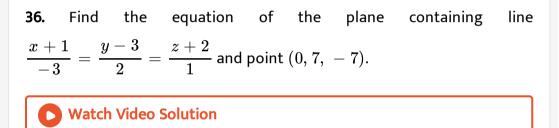**34.** If the line $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k} \right) + \lambda \left(\hat{i} + m\hat{j} - 2\hat{k} \right)$ is parallel to the plane $\vec{r} \cdot \left(2\hat{i} + \hat{j} + m\hat{k} \right) = 1$ then find the value of m.

35. Find the equation of a plane passes through the points $(0, 0, 0)$ and $(1, 3, 5)$ and parallel to the line $\dfrac{x}{-2} = \dfrac{y}{1} = \dfrac{z+3}{4}$.

36. Find the equation of the plane containing line $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$ and point $(0, 7, -7)$.

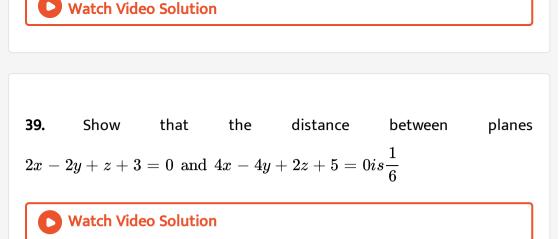**37.** Find the perpendicular distance from the point $(1, -3, 4)$ to the plane $3x - 4y + 12z - 1 = 0$.

38. Show that the point $(1, 2, 1)$ is equidistant from the planes $\vec{r} \cdot \left(\hat{i} + 2\hat{j} - 2\hat{k} \right) = 5$ and $\vec{r} \cdot \left(2\hat{i} - 2\hat{j} + \hat{k} \right) + 3 = 0$.

39. Show that the distance between planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0 is \dfrac{1}{6}$

40. Find the equations of het planes parallel to the plane $x + 2y - 2z + 8 = 0$ which are at distance of 2 units from the point $(2, 1, 1)$.

41. Find the co-ordinates of the foot of perpendicular and its perpendicular distance drawn from the point $(1, 3, 4)$ to the plane $2x - y - 2z + 3 = 0$. Also find the image of the point $(1, 3, 4)$ in the plane.

42. Find the image of the point $(1, \backslash 2, \backslash 3)$ in the plane

$x + 2y + 4z = 38$

▶ Watch Video Solution

43. A variable plane which remains at q constant distance $3p$ from the origin cut the coordinate axes at A, B, C. Show that the locus o the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.
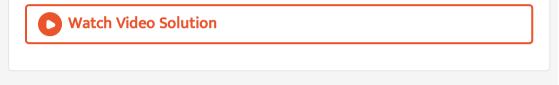
▶ Watch Video Solution

44. Find the distance of the point $(2, 3, 4)$ from the plane $3x + 2y + 2z + 5 = 0$ measured parallel to the line $\dfrac{x+3}{3} = \dfrac{y-2}{6} = \dfrac{z}{2}$.

▶ Watch Video Solution

45. Show that the line

$$\overrightarrow{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\overrightarrow{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$ are coplanar. Also find the equation of plane in which these lines lie.

46. Prove that the lines

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \ and \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$$ are coplanar .

Aslo, find the plane containing these two lines.

47. Find the equation of a plane passing through the parallel lines

$$\frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

and $$\frac{z-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5}$$ are coplanar. Also find the equation of plane in which these lines lie.

1. If a line makes angles $90o$, $135o$, $45o$ with the x, y and z-axes respectively, find its direction cosines.

▶ **Watch Video Solution**

2. Can a line make angle $45°, 60°, 120°$ with $x-$, $y-$ and z-axes respectively.

▶ **Watch Video Solution**

3. Find the direction cosines of that line whose direction ratios are as follows :

(i) $1, -2, 2$, (ii) $2, 6, 3$

(iii) $3, 1, -2$

4. Find the direction cosines of the line joining the following points :

(i) $A(2, \ -1, 3), B(3, 1.1)$

(ii) $A(2, \ -1, 2), B(\ -4, 2, 0)$

(iii) $A(4, 3, \ -5), B(\ -2, 1, \ -8)$

5. Show that the point $A(2, \ -3, \ -4), B(1, 2, 3), C(3, \ -8, \ -11)$ are

collinear.

6. Find the angle between those lines whose direction ratios are as

follows :

(i) $(2, 3, 6)$ and $(1, 2, 2)$

(ii) $(4, -3, 5)$ and $(3, 4, 5)$

(iii) $(1, 2, 1)$ and $(4, -3, 2)$

7. Find the angle between the following vectors :

(i) $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

(ii) $\vec{a} = 6\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = 4\hat{i} - 2\hat{j} + 9\hat{k}$

8. Show that the joint of the points (1,2,3), (4,5,7) is parallel to the join of the points (-4,3,-6),(2,9,2).

9. If A (6,-6,0), B(-1,-7,6), C (3,-4, 4) and D (2,-9,2) be four points in space. Show that AB is perpendicular to CD.

10. If $\vec{r}$ is a vector of magnitude 21 and has direction ratios 2, -3 and 6, then find $\vec{r}$.

11. Find the angles which the following vectors, makes form the co-ordinates axes :

(i) $2\hat{i} + \hat{j} + 3\hat{k}$, (ii) $3\hat{i} - 4\hat{j} + 5\hat{k}$

12. The pair of lines whose direction cosines are given by the equations

$3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$ are

13. The direction cosines of two lines satisfying the conditions $l + m + n = 0$ and $3lm - 5mn + 2nl = 0$ where l, m, n are the direction cosines.

The value of $(l - m)^2 + (m - n)^2 + (n - l)^2$ is

14. If the direction cosines of two lines are l_1, m_1, n_1 and l_2, m_2, n_2, then find the direction cosine of a line perpendicular to these lines.

15. Find the angle between any two diagonals of a cube.

16. Find the angle between two lines whose direction ratios are proportional to $1, 1, 2 and (\sqrt{3} - 1), (-\sqrt{3} - 1), 4$.

17. Find the angles of a triangle whose verties are $A(3, 2, 1), B(35, 2)$ and $C(5, \ -2, 3)$.

18. If a line makes angles $90o, \quad 135o, \quad 45o$ with the x, y and z-axes respectively, find its direction cosines.

19. Find the direction cosines of a line which makes equal angles with the coordinate axes.

20. If a line has the direction ratios $18, 12, 4$, then what are its direction cosines?

> ▶ **Watch Video Solution**

21. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

> ▶ **Watch Video Solution**

22. Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, 4), (1, 1, 2)$ and $(5, 5, 2)$.

> ▶ **Watch Video Solution**

Exercise 11 B

1. Find the vector equation of a lin e passes through the point whose position vector is $\left(2\hat{i} - \hat{j} - \hat{k}\right)$ and parallel to vector $\hat{i} + 5\hat{k}$.

2. Find the vector equation of a line passes through the point $\hat{i} + 3\hat{j} + \hat{k}$ and parallel to vector $3\hat{i} - 2\hat{j} + k$.

3. Find the equation of a line passes through the point $(2, 3, 4)$ and whose direction ratios are $3, \ -1, \ -2$.

4. Find the equation of a line parallel to the line $\dfrac{x-5}{3} = \dfrac{y+1}{-2} = \dfrac{z}{1}$ and passes through the point $(0, \ -1, 2)$.

5. The cartesian equation of a line is $\dfrac{x+2}{1} = \dfrac{y+3}{-2} = \dfrac{z}{3}$, find its vector equation.

6. Find the vector equation of the line through $A(3, 4, -7) and B(1, -1, 6).$ Find also, its Cartesian equations.

7. Find the equation of a line passes through the points whose position vectors are $\left(\hat{i} + 4\hat{j} + \hat{k}\right)$ and $\left(2\hat{i} - \hat{j} + 5\hat{k}\right)$.

8. Prove that the points $A(2, 0, -3)$, $B(1, -2, -5)$ and $C(3, 2, -1)$ are collinear.

9. Prove that the points $A(9, -1, 4)$, $B(-1, -3, 2)$ and $C(4, -2, 3)$ are collinear.

10. Show that the points $A(-2, 3, 5)$, $B(1, 2, 3)$ and $C(7, 0, -1)$ are collinear.

11. Find the values of λ ad μ if the points $A(-1, 4, -2)$, $B(\lambda, \mu, 1)$ and $C(0, 2, -1)$ are collinear.

12. Find the equation of a line passes through the point $\hat{i} + \hat{j} + 5\hat{k}$ and parallel to line joining the points $(2, -4, 1)$ and $(0, 1, 3)$.

13. The cartesian equation of a line is $6x + 1 = 3y - 2 = 3 - 2x$. Find its direction ratios.

14. Show that the line $\dfrac{x + 3}{2} = \dfrac{y + 1}{-1} = \dfrac{z + 3}{3}$ and $\dfrac{x}{5} = \dfrac{y - 5}{1} = \dfrac{z - 3}{-3}$ are perpendicular.

15. Find the values of λ if the following of lines perpendicular :

$$\frac{1-x}{3} = \frac{7y-14}{3\lambda} = \frac{z+1}{2} \text{ and } \frac{7-7x}{3\lambda} = \frac{y}{1} = \frac{1-z}{5}$$

> ▶ Watch Video Solution

16. Show that the following pairs of lines intersect. Also find their point of

intersection :

(i) $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = z$

(ii) $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$ and $\dfrac{x-1}{2} = \dfrac{y+1}{-3} = \dfrac{z+10}{8}$

> ▶ Watch Video Solution

17. Show that the lines

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1} \text{ and } \frac{x-1}{1-} = \frac{y-1}{-1} = \frac{z-1}{2} \text{ do not}$$

intersect.

> ▶ Watch Video Solution

18. Find the co-ordinates of that point at which the lines joining the points $(1, 1, 2)$ and $(3, 5, -1)$ meets the xy-plane.

19. Find the co-ordinates of that point at which the line joining the points $(-2, 1, 4)$ and $(2, 0, 3)$ meets the yz – plane.

20. Find the co-ordinates of a point at which the line $\dfrac{x+1}{2} = \dfrac{y-3}{-2} = \dfrac{z+5}{6}$, meets the plane $3x - y + z = 3$.

21. Find the co-ordinates of a point at which the line $\dfrac{x+1}{2} = \dfrac{y-1}{-2} = \dfrac{z+5}{6}$, meets the plane $x - 2y + 3z = 8$.

22. Find the co-ordiantes of the foot of perpendicular drawn from the point $(1, 2, 3)$ to the line $\dfrac{x - 6}{3} = \dfrac{y - 7}{2} = \dfrac{z - 7}{-2}$.

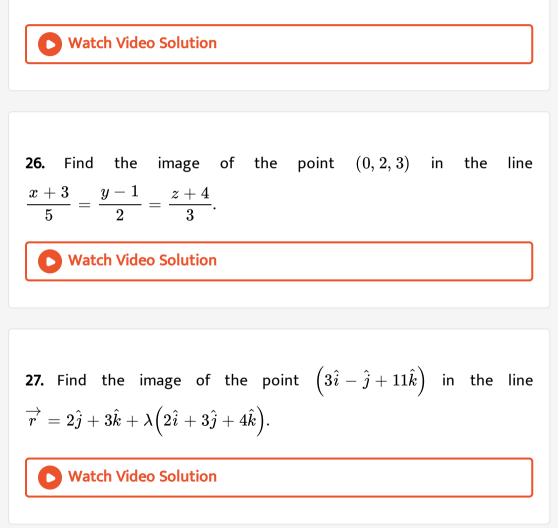**23.** Find the length and the foot of the perpendicular drawn from the point $(2, -1, 5)$ to the line $\dfrac{x - 11}{10} = \dfrac{y + 2}{-4} = \dfrac{z + 8}{-11}$

24. Find the co-ordinates of the foot of perpendicular and length of perpendicular drawn from point $\left(\hat{i} + 6\hat{j} + 3\hat{k} \right)$ to the line $\vec{r} = \hat{j} + 2\hat{k} + \lambda \left(\hat{i} + 2\hat{j} + 3\hat{k} \right)$.

25. Find the image of the point $(1, 6, 3)$ in the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$

. Also, write the equation of the line joining the given point and its image

and find length of the segment joining the given point and its image.

26. Find the image of the point $(0, 2, 3)$ in the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$.

27. Find the image of the point $\left(3\hat{i} - \hat{j} + 11\hat{k}\right)$ in the line $\overrightarrow{r} = 2\hat{j} + 3\hat{k} + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$.

28. Find the shortest distance between the following lines :

(i) $\vec{r} = 4\hat{i} - \hat{j} + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and

$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

(ii) $\vec{r} = -\hat{i} + \hat{j} - \hat{k} + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$ and

$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

(iii) $\dfrac{x-1}{-1} = \dfrac{y+2}{1} = \dfrac{z-3}{-2}$ and

$\dfrac{x-1}{1} = \dfrac{y+1}{2} = \dfrac{z+1}{-2}$

(iv) $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-5}{5}$

(v) $\vec{r} = \vec{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(-\hat{i} + \hat{j} - \hat{k}\right)$

▶ Watch Video Solution

---

29. Find the co-ordinates of the point at a distance of $\sqrt{5}$ units from the point $(1, 2, 3)$ on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$.

▶ Watch Video Solution

30. Find the co-ordinates of the point at a distance of $\sqrt{14}$ from the mid-point of AB on the line joining the point $A(1, 2, 3)$ and $B(3, 6, 9)$.

> ▶ Watch Video Solution

Exercise 11 C

1. Find the equations of the plane passing through the following points :

(i) $A(2, 1, 0), B(3, -2, -2), C(3, 1, 7)$

(ii) $A(1, 1, 1), B(1, -1, 2), C(-2, -2, 2)$

(iii) $A(0, -1, 0), B(2, 1, -1), C(1, 1, 1)$

(iv) $A(1, -2, 5), B(0, -5, -1), C(-3, 5, 0)$

(v) $(4, -1, -1), B(2, 0, 2), C(3, -1, 2)$

> ▶ Watch Video Solution

2. Show that the points

$A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, 2, 1)$

are coplanar.

3. Show that the point $A(4, -1, 2), B(-3, 5, 1), C(2, 3, 4)$ and $D(1, 6, 6)$ are coplanar. Also find the equation of the plane passing through these points.

4. Find the vector equation of the plane with intercepts $3, -4$ and 2 on x, y and z-axis respectively.

5. Find the equation of a plane passes through the point $(1, 2, 3)$ and cuts equal intercepts on the co-ordinate axes.

6. Find the equation of a plane passes through the point $(4, 4, 1)$ and the ratio of intercepts cuts on axes from this plane is $2 : 1 : 1$.

▶ **Watch Video Solution**

7. A plane meets the coordinate axes at A, $BandC$ respectively such that the centroid of triangle ABC is $(1, -2, 3)$. Find the equation of the plane.

▶ **Watch Video Solution**

Exercise 11 D

1. Find th equation of the plane which is at distance of 8 units from origin and the perpendicular vector from origin to this plane is $\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$.

▶ **Watch Video Solution**

2. Find the equation of the plane which is at a distance of $\sqrt{29}$ units from origin and the perpendicular vector from origin to this plane is $\left(4\hat{i} - 2\hat{j} + 3\hat{k}\right)$.

> ▶ Watch Video Solution

3. The vector equation of a plane is $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 2\hat{k}\right) = 12$. Convert it into normal form. Find the d.c.s' of the perpendicular vector drawn from origin to this plane and length of perpendicular.

> ▶ Watch Video Solution

4. The vector equation of a plane is $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 2 = 0$. Convert it into normal form. Also find the length of perpendicular from origin and the d.c.'s of this perpendicular vector.

> ▶ Watch Video Solution

5. Find the equation of a plane which is a distance of 2 units from origin and the d.r's of perpendicular vectors are 2, $-1, 2$.

6. Find the angle between the planes $\overrightarrow{r} \cdot \left(\hat{i} + \hat{j} - 2\hat{k} \right) = 3$ and $\overrightarrow{r} \cdot \left(2\hat{i} - 2\hat{j} + \hat{k} \right) = 22$

7. Find the vector equation of the following planes whose Cartesian equations are $x + 2y + 3z + 5 = 0$

8. The co-ordiantes of the foot of perpendicular from origin to a plane are $(3, -2, 1)$. Find the equation of the plane.

9. Find the normal form of the plane $x + 2y - 2z + 6 = 0$. Also find the length of perpendicular from origin to this plane and the d.c.'s of the normal.

10. Find the d.c.'s of the normal and length of perpendicular from origin to the plane $x = 2$.

11. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.(a) $z = 2$ (b) $x + y + z = 1$ (c) $2x + 3yz = 5$(d) $5y + 8 = 0$

12. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

▶ Watch Video Solution

13. Find the coordinates of the foot of perpendicular drawn from origin to the planes: $x + y + z = 1$

▶ Watch Video Solution

14. Find the vector and Cartesian equation of the plane that passes through the point (1,4,6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

▶ Watch Video Solution

15. Find the vector and cartesian equation of a plane which passes through the point $(2, -1, 3)$ and perpendicular to a line whose d.r.'s are $1, -3, 5$.

16. Find the vector equation of the following plane in non-parametric form: $\vec{r} = (\lambda - 2\mu)\hat{i} + (3 - \mu)\hat{j} + (2\lambda + \mu)\hat{k}$.

17. Convert the equation of the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(-\hat{i} + \hat{j} + 2\hat{k}) + \mu(\hat{i} + 2\hat{j} + \hat{k})$ into scalar product form.

18. Find the vector equation of the plane passing through the points $P(2\,5,\ -3),\ Q(-2,\ -3,\ 5)\ and\ R(5,\ 3,\ -3).$

19. Find the equation of the plane passing through $A(2, 2,\ -1), B(3, 4, 2)$ and $C(7, 0, 6).$ Also find a unit vector perpendicular to this plane.

20. Find the cartesian equation of plane passing through the points $(1, 1, 1), (1,\ -1, 1)$ and $(-7,\ -3,\ -5).$

21. Find the angle between the folowing planes :-

(i) $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} + 4\hat{k} \right) = 5$ and $\vec{r} \cdot \left(- \hat{i} + \hat{j} \right) = 1$

(ii) $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 6\hat{k} \right) = 1$ and

$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 2\hat{k} \right) + 3 = 0$

(iii) $x + y - 2z = 3$ and $2x - 2y + z + 1 = 0$

▶ **Watch Video Solution**

---

22. Find the value of $'\lambda'$ if the following planes are perpendicular.

(i) $2x - 4y + 3z + 1 = 0$ and $x + 2y + \lambda z = 3$

(ii) $\vec{r} \cdot \left(3\hat{i} - 6\hat{j} - 2\hat{k} \right) = 1$ and

$\vec{r} \cdot \left(2\hat{i} + \hat{j} - \lambda\hat{i} \right) = 2$

▶ **Watch Video Solution**

---

23. Find the equation of the plane passes through the point $(2, 3, 5)$ and

parallel to the plane $x - 3y + z = 8$.

▶ Watch Video Solution

24. Find the equation of the plane passes through the point $(1, \ -3, 1)$ and parallel to the plane $2x + 3y + z = 1.$

25. Find the equation of the plane passes through the point $(2, 1, \ -2)$ and parallel to the plane $\vec{r} . \left(3\hat{i} + \hat{j} - \hat{k} \right) = 0.$

26. Equation of plane passing through the points $(2, 2, 1)(9, 3, 6)$ and perpendicular to the plane $2x + 6y + 6z - 1 = 0$ is

27. Find the equation of a plane passes through the point $(0, 0, 0)$ and perpendicular to each to the planes $x + 2y - z = 1$ and $3x - 4y + z = 5$.

▶ Watch Video Solution

28. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

▶ Watch Video Solution

29. Find the equation of the plane through the intersection of the planes. $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k} \right) = 9$ and $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k} \right) = 3$ and passing through the origin.

▶ Watch Video Solution

30. Find the equation of a plane containing the line of intersection of the plane $x + y + z - 6 = 0$ and $2x + 3y + 4z + 5 = 0$ and passing through (1, 1, 1).

▶ Watch Video Solution

31. Find the equation of the plane passing through the intersection of the planes $2x - 3y + z - 4 = 0 and x - y + z + 1 = 0$ and perpendicular to the plane $x + 2y - 3z + 6 = 0$.

▶ Watch Video Solution

32. Find the equation of a plane passing through the intersection of the planes $\vec{r} \cdot \left(2\hat{i} - 7\hat{j} + 4\hat{k} \right) = 3$ and $\vec{r} \cdot \left(3\hat{i} - 5\hat{j} + 4\hat{k} \right) + 11 - 0$ and passes through the point $\left(-2\hat{i} + \hat{j} + 3\hat{k} \right)$.

▶ Watch Video Solution

33. Find the equation of a plane passing through the intersection of the planes $x - 3y + 2z - 5 - 0$ and $2x - y + 3z - 1 = 0$ and passes through the point $(1, -2, 3)$.

34. Prove that the equaton of a plane through point $(2, -4, 5)$ and the line o lintersection of the planes $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$ and $\vec{r} \cdot \left(3\hat{i} + \hat{j} - 2\hat{k}\right) = 2$ is $\vec{r} \cdot \left(2\hat{i} + 8\hat{j} + 7\hat{k}\right) = 7$.

35. The vector equation of the plane through the point $(2, 1, -1)$ and passing through the line of intersection of the plane $r \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$ and $r \cdot \left(\hat{j}2\hat{k}\right) = 0$, is

1. Find the angle between the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 4$.

> ▶ **Watch Video Solution**

2. Find the value of $'m'$ for which the line $\vec{r} = \left(\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + \hat{k}\right)$ is parallel to the plane $\vec{r} \cdot \left(3\hat{i} - 2\hat{j} + m\hat{k}\right) = 5$.

> ▶ **Watch Video Solution**

3. Find the value of 'm' for which the line $\vec{r} = \hat{i} + \lambda\left(2\hat{i} - m\hat{j} - 3\hat{k}\right)$ is parallel to the plane $\vec{r} \cdot \left(m\hat{i} + 3\hat{j} + \hat{k}\right) = 1$.

> ▶ **Watch Video Solution**

4. Find the equation of a plane passing through the points (0,0,0) and (3, -1, 2) and parallel to the line $\dfrac{x-4}{1} = \dfrac{y+3}{-4} = \dfrac{z+1}{7}$.

5. Find the equation of the plane through (2, 3, -4) and (1, -1, 3) and parallel o x-axis.

6. Find the equation of a line passing through the point (1,2,3) and perpendicular to the plane $\overrightarrow{r} \cdot \left(2\hat{i} - 3\hat{j} + 4\hat{k} \right) = 1$.

7. The equation of the line passing through $(1, 2, 3)$ and parallel to the planes $x - y + 2z = 5$ and $3x + y + z = 6$ is.

8. Find the perpendicular distance from the point $\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ to the plane $\vec{r} \cdot \left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) = 1$.

9. Find the perpendicular distance from the point $\left(2\hat{i} + \hat{j} - \hat{k}\right)$ to the plane $\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 4\hat{k}\right) = 3$.

10. Find the distance of the point $(21, 0)$ from the plane $2x + y + 2z + 5 = 0$.

11. Find the distance of each of the following points from the corresponding given plane: $(2, 3, -5)$, $x + 2y - 2z = 9$

12. If the points $(1, 1, \lambda)$ and $(-3, 0, 1)$ are equidistant from the plane, $3x + 4y - 12z + 13 = 0$, then λ satisfies the equation :

13. Find the distance between the parallel planes $2x - y + 3 - 4 = 0 \; and \; 6x - 3y + 13 = 0$.

14. Find the distance between the parallel planes, $\overrightarrow{r} = 2\hat{i} - 3\hat{j} + 6\hat{k} = 5 and \, \overrightarrow{r} = 6\hat{i} - 9\hat{j} + 18\hat{k} + 20 = 0$.

15. Find the equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which is at a unit distance from the point $(1, 2, 3)$.

16. Find the length of the foot of the perpendicular from the point (1,1,2) to the plane $\vec{r} \cdot \left(2\hat{i} - 2\hat{j} + 4\hat{k} \right) + 5 = 0$

17. Find the coordinates of the foot of the perpendicular from the point (2, 3, 7) to the plane $3x - y - z = 7$. Also, find the length of the perpendicular.

18. Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.

> ▶ Watch Video Solution

19. Find the image of the point O(0,0,0) in the plane 3x+4y-6z+1=0

> ▶ Watch Video Solution

20. A variable plane which remains at q constant distance $3p$ from the origin cut the coordinate axes at A, B, C. Show that the locus o the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

> ▶ Watch Video Solution

21. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}$.

> ▶ Watch Video Solution

22. Find the distance of the point (0.-3. -2) from the plane $x + 2y - z = 1$ measured parallel to $\dfrac{x+1}{2} = \dfrac{y+1}{2} = \dfrac{z}{3}$

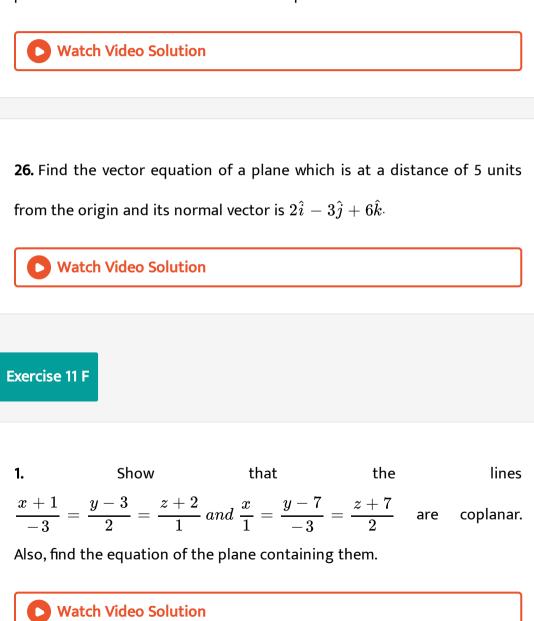**23.** Find the equation of the plane passing through the intersection of the planes $2x - 3y + z - 4 = 0 and x - y + z + 1 = 0$ and perpendicular to the plane $x + 2y - 3z + 6 = 0$.

24. Find the equation of the plane through the intersection of the planes $3x - 4y + 5z = 10 and 2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z.$

25. Find the equation of a line passing through the point $\left(2\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and perpendicular to the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0$. Also find the point of intersection of this line and the plane.

▶ Watch Video Solution

26. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$.

▶ Watch Video Solution

Exercise 11 F

1. Show that the lines
$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} \; and \; \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$$ are coplanar.
Also, find the equation of the plane containing them.

▶ Watch Video Solution

2. Show that the lines $\dfrac{x+3}{-3} = \dfrac{y-1}{1} = \dfrac{z-5}{5}$ and $\dfrac{x+1}{-1} = \dfrac{y-2}{2} = \dfrac{z-5}{5}$ are coplanar.
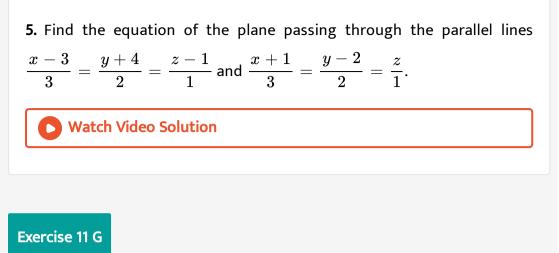
3. The length of the perpendicular drawn from the point (2,1,4) to the plane containing the lines $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(\hat{i} + \hat{j}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$ is $\sqrt{l}$. The value of l is_____.

4. Show that the lines

$\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and

$\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$

are coplanar. Also the find the equation of the plane passing through these lines.

5. Find the equation of the plane passing through the parallel lines $\dfrac{x-3}{3} = \dfrac{y+4}{2} = \dfrac{z-1}{1}$ and $\dfrac{x+1}{3} = \dfrac{y-2}{2} = \dfrac{z}{1}$.

▶ Watch Video Solution

Exercise 11 G

1. Show that the line through the points $(1, 1, 2), (3, 4, 2)$ is perpendicular to the line through the points $(0, 3, 2)$ and $(3, 5, 6)$.

A. $30°$

B. $60°$

C. $90°$

D. $180°$

Answer: C

▶ Watch Video Solution

2. Find the shortest distance

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right) \text{ and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j}\right)$$

.

A. $\dfrac{4}{\sqrt{19}}$

B. $\dfrac{3}{\sqrt{19}}$

C. $\dfrac{2}{\sqrt{19}}$

D. $\dfrac{1}{\sqrt{19}}$

Answer: B

▶ **Watch Video Solution**

---

3. Find the distance of the plane $2x - 2y + 4z = 6$ from the origin.

A. $\dfrac{8}{\sqrt{29}}$

B. $\dfrac{6}{\sqrt{29}}$

C. $\dfrac{4}{\sqrt{29}}$

D. $\dfrac{2}{\sqrt{29}}$

Answer: A

---

4. A plane meets the coordinate axes at A, $B and C$ respectively such that the centroid of triangle ABC is $(1, \, -2, 3).$ Find the equation of the plane.

A. $\alpha x + \beta y + \gamma z = 1$

B. $\alpha^2 x + \beta^2 y + \gamma^2 z = 3$

C. $\dfrac{x}{\alpha} + \dfrac{y}{\beta} + \dfrac{z}{\gamma} = 1$

D. $\dfrac{x}{\alpha} + \dfrac{y}{\beta} + \dfrac{z}{\gamma} = 3.$

Answer: D

5. Find the equation of the plane which cuts intercepts 2,3,-4 on the axes.

A. $6x + 4y - 3z = 12$

B. $6x - 4y + 3z = 12$

C. $2x - 3y + 4z = 12$

D. None of these

Answer: B

▶ Watch Video Solution

6. Find the perpendicular distance of the point (1, 0, 0) from the line $\dfrac{x - 1}{2} = \dfrac{y + 1}{-3} = \dfrac{z + 10}{8}$.

A. $\sqrt{6}$ unit

B. $2\sqrt{6}$ unit

C. $3\sqrt{6}$ unit

D. None of these

Answer: B

---

7. Find the image of the point $(1, 6, 3)$ in the line $\dfrac{x}{1} = \dfrac{y - 1}{2} = \dfrac{z - 2}{3}$.

Also, write the equation of the line joining the given point and its image

and find length of the segment joining the given point and its image.

A. $(1, 0, 7)$

B. $(0, 1, 7)$

C. $(7, \; -1, 0)$

D. $(-7, \; -1, 0)$

Answer: A

---

8. Find the image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$.

A. $(3, 5, -2)$

B. $(2, 3, -5)$

C. $(-3, 5, 2)$

D. $(-2, 3, 5)$

Answer: C

▶ **Watch Video Solution**

9. The distance between the planes $3x + 5y + z = 8$ and $3x + 5y + z + 27 = 0$ is :

A. $\dfrac{8}{\sqrt{35}}$

B. $\dfrac{27}{\sqrt{35}}$

C. $\sqrt{35}$

D. $2\sqrt{35}$

Answer: C

10. Find the equation of the plane passing through the points $(1, -1, 2) and (2, -2, 2)$ and which is perpendicular to the plane $6 x - 2y + 2z = 9$

A. $x + y - 2z = -4$

B. $x + y - 2z + 1 = 0$

C. $x + y - 2z = 4$

D. None of these

Answer: A

Exercise 11 H

1. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

    A. $x - 2y + 2z - 3 = 0$

    B. $x - 2y + 2z + 1 = 0$

    C. $zx - 2y + 2z - 1 = 0$

    D. $x - 2y + 2z + 5 = 0$

Answer: A

▶ **Watch Video Solution**

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying

$\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 9$ then find the value of $\left|2\vec{a} + 5\right|$

    A. 1

    B. 2

C. 3

D. 4

Answer: C

3. If the straight lines $\dfrac{x-1}{k} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{k} = \dfrac{z-1}{2}$ intersect at a point, then the integer k is equal to (1) -5 (2) 5 (3) 2 (4) -2

A. -5

B. 5

C. 2

D. -2

Answer: A

4. Find the line of intersection of the planes $\vec{r} \cdot \left(3\hat{i} - \hat{j} + \hat{k}\right) = 1$ and $\vec{r} \cdot \left(\hat{i} + 4\hat{j} - 2\hat{k}\right) = 2$

A. $2\hat{i} + 7\hat{j} + 13\hat{k}$

B. $-2\hat{j} + 7\hat{k} + 13\hat{k}$

C. $2\hat{i} - 7\hat{j} + 13\hat{k}$

D. $-2\hat{i} - 7\hat{j} + 13\hat{k}$

Answer: B

5. The vector equation of the plane through the point $\hat{i} + 2\hat{j} - \hat{k}$ and perpendicular to the line of intersection of the plane $\vec{r} \cdot \left(3\hat{i} - \hat{j} + \hat{k}\right) = 1$ and $\vec{r} \cdot \left(\hat{i} + 4\hat{j} - 2\hat{k}\right) = 2$, is

A. $\vec{r} \cdot \left(2\hat{i} - 7\hat{j} - 13\hat{k}\right) = 1$

B. $\vec{r} \cdot \left(2\hat{i} + 7\hat{j} + 13\hat{k} \right) = 1$

C. $\vec{r} \cdot \left(2\hat{i} - 7\hat{j} - 13\hat{k} \right) = 4$

D. None of these

Answer: A

▶ Watch Video Solution

---

6. The plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ meets the coordinate axes at A,B and C respectively. Find the equation of the sphere OABC.

A. $x^2 + y^2 + z^2 + az + by + cz = 0$

B. $x^2 + y^2 + z^2 + 2az + 2by + 2cz = 0$

C. $x^2 + y^2 + z^2 - ax - by - cz = 0$

D. None of these

Answer: C

▶ Watch Video Solution

7. Equations of the line passing through $(1, 1, 1)$ and perpendicular to the plane $2x + 3y + z + 5 = 0$ are

A. $\dfrac{x-1}{-1} = \dfrac{y+2}{1} = \dfrac{z-3}{-1}$

B. $\dfrac{x-1}{3} = \dfrac{y+2}{2} = \dfrac{z-3}{-1}$

C. $\dfrac{x-1}{1} = \dfrac{y+2}{-2} = \dfrac{z-3}{3}$

D. None of above

Answer: A

▶ **Watch Video Solution**

---

8. The perpendicular distance between the line $\overrightarrow{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $\overrightarrow{r} \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5$ is :

A. $\dfrac{10}{3}$

B. $\dfrac{5}{3\sqrt{3}}$

C. $\dfrac{10}{3\sqrt{3}}$

D. $\dfrac{5}{3}$

Answer: C

▶ **Watch Video Solution**

---

9. The vector equation of the plane through the point $(2, 1, -1)$ and passing through the line of intersection of the plane $r \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$ and $r \cdot \left(\hat{j}2\hat{k}\right) = 0$, is

A. $\overrightarrow{r} \cdot \left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$

B. $\overrightarrow{r} \cdot \left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 6$

C. $\overrightarrow{r} \cdot \left(\hat{i} - 9\hat{j} - 11\hat{k}\right)0$

D. None of the above

Answer: A

10. A line makes angles $\angle, \beta, \gamma$ and δ with the diagonals of a cube. Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$.

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. $\dfrac{4}{3}$

D. $\dfrac{5}{3}$

Answer: C

Exercise 11 2

1. Show that the three lines with direction cosines $\dfrac{12}{13}, \dfrac{-3}{13}, \dfrac{-4}{13}; \dfrac{4}{13}, \dfrac{12}{13}, \dfrac{3}{13}; \dfrac{3}{13}, \dfrac{-4}{13}, \dfrac{12}{13}$ are mutually perpendicular.

2. Show that the line through the points (1, -1, 2) and (3, 4,-2) is perpendicular to the through the points (0, 3,2) and (3, 5, 6).

3. Show that the line through the points $(4, 7, 8), (2, 3, 4)$ is parallel to the line through the points $(1, 2, 1),(1, 2, 5)$.

4. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

5. Find the equation of the line in vector and in cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

6. Find the Cartesian equation of the line which passes through the point (-2, 4, -5) and is parallel to the line $\dfrac{x+3}{3} = \dfrac{4-y}{5} = \dfrac{z+8}{6}$.

7. The cartesian equation of a line is $\dfrac{x-5}{3} = \dfrac{y+4}{7} = \dfrac{z-6}{2}$. Write its vector form.

8. Find the vector and the cartesian equations of the lines that passes through the origin and $(5, 2, 3)$.

9. Find the vector and the cartesian equations of the line that passes through the points $(3, 2, 5)$, $(3, 2, 6)$.

10. Find the angle between the following pairs of lines.

(i) $\hat{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\overrightarrow{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

(ii) $\overrightarrow{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and

$\overrightarrow{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

11. Find the angle between the following pair of lines:

$$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

12. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and

$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5} \text{ are at right angles.}$$

13. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are

perpendicular to each other.

14. Find the shortest distance between the lines

$$\vec{r} = \left(\hat{i}\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} = 2\hat{k}\right) \text{ and } \vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j}\right)$$

.

15. Find the shortest distance between the following lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}; \frac{3-x}{-1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

16. Find the shortest distance

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right) \text{ and } \vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j}\right)$$

.

17. Find the shortest distance between the lines whose vector equations

are $\rightarrow r = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and

$\rightarrow r = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$

1. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.(a) $z = 2$ (b) $x + y + z = 1$ (c) $2x + 3yz = 5$(d) $5y + 8 = 0$

▶ Watch Video Solution

2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$.

▶ Watch Video Solution

3. Find the Cartesian equations of the following planes whose vector equations are: $\overrightarrow{r} . \left[(s - 2t)\hat{i} + 93 - t)\hat{j} + (2s + t)\hat{k} \right] = 15$

▶ Watch Video Solution

4. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.(a) $2x + 3y + 4z - 12 = 0$ (b) $3y + 4z - 6 = 0$(c) $x + y + z = 1$ (d) $5y + 8 = 0$

▶ Watch Video Solution

5. Find the vector and Cartesian equation of the plane that passes through the point (1,4,6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

▶ Watch Video Solution

6. Find the equations of the planes that passes through three points. (a) $(b)(c)((d)(e)1, \quad 1, \quad 1(f)), \quad ((g)(h)6, \quad 4, \quad 5(i)), \quad ((j)(k) \quad 4$ (n) (o) `(p) (q)((r) (s)1," "1," "0(t))," "((

▶ Watch Video Solution

7. Find the intercepts cut off by the plane $2x + y - z = 5$.

> ▶ Watch Video Solution

8. Find the equation of the plane with intercept 3 on the y-axils and parallel to ZOX plane.

> ▶ Watch Video Solution

9. Find the equation of the plane through the intersection of the planes

$3x \quad y \quad + \quad 2z \quad 4 \quad = \quad 0$ and

$x \quad + \quad y \quad + \quad z \quad 2 \quad = \quad 0$ and the point $(2, 2, 1)$.

> ▶ Watch Video Solution

10. Find the equation of the plane passing through the intersection of the planes $\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k} = 7, \vec{r} = 2\hat{i} + 5\hat{j} + 3\hat{k} = 9$ the point $(2, 1, 3)$.

11. Find the vector equation of the plane through the line of intersection of the planes $x + y + z = 1 \text{ and } 2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

12. Find the direction cosines of the unit vector perpendcular to the plane $\overrightarrow{r} \cdot \left(2\hat{i} + 2\hat{j} - 3\hat{k} \right) = 5 \text{ and } \overrightarrow{r} \cdot \left(3\hat{i} - 3\hat{j} + 5\hat{k} \right) = 3$

13. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them. (a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$ (b) `2x" "+"

14. In the following cases, find the distance of each of the given points from the corresponding given plane.

	Point	Plane
(a)	(0, 0, 0)	$3x - 4y + 12z = 3$
(b)	(3,–2, 1)	$2x - y + 2z + 3 = 0$
(c)	(2, 3, –5)	$x + 2y - 2z = 9$
(d)	(–6, 0, 0)	$2x - 3y + 6z - 2 = 0$

▶ Watch Video Solution

Miscellaneous Exercise

1. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1)$ and $(4, 3, -1)$.

▶ Watch Video Solution

2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular   to   both   of   these   are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

▶ Watch Video Solution

3. Find the angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b$.

▶ Watch Video Solution

4. Find the equation of a line parallel to x-axis and passing through the origin.

▶ Watch Video Solution

5. If the coordinates of the points A, B, C, D are $(1, 2, 3), (4, 5, 7), (-4, 3, -6)$ and $2, 9, 2)$, then find the angle between AB and CD.

> ▶ Watch Video Solution

6. If the lines $\dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$ and $\dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-5}$ are perpendicular, find the value of k.
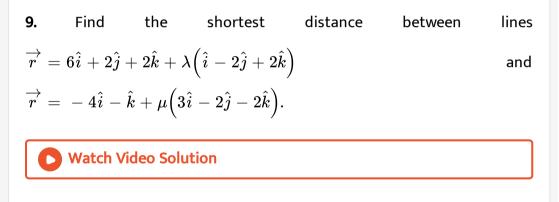
> ▶ Watch Video Solution

7. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\rightarrow r\hat{i} + 2\hat{j} - 5\hat{k} + 9 = 0$.

> ▶ Watch Video Solution

8. Write the vector equation of the plane, passing through the point $(a,\ b,\ c)$ and parallel to the plane $\to r\hat{i} + \hat{j} + \hat{k} = 2$.

9. Find the shortest distance between lines $\overrightarrow{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and $\overrightarrow{r} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$.

10. Find the coordinate of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the i. yz-plane ii. zx-plane.

11. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

12. Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

13. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$

14. If the points $(1, \quad 1, \quad p) \quad and \quad (\quad 3, \quad 0, \quad 1)$ be equidistant from the plane $\to r3\hat{i} + 4\hat{j} - 12\hat{k} + 13 = 0$, then find the value of p.

> ▶ Watch Video Solution

15. Find the equation of the plane passing through the line of intersection of the planes $\overrightarrow{r} . \left(\hat{i} + \hat{j} + \hat{k} \right) = 1$ and $\overrightarrow{r} . \left(2\hat{i} + 3\hat{j} - \hat{k} \right) + 4 = 0$ and parallel to x-axis.

> ▶ Watch Video Solution

16. If O be the origin and the coordinates of P be$(1, \quad 2, \quad 3)$, then find the equation of the plane passing through P and perpendicular to OP.

> ▶ Watch Video Solution

17. Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot \left(\hat{i} + 2\hat{j} + 3\hat{k} \right) - 4 = 0$ and $\vec{r} \cdot \left(2\hat{i} + \hat{j} - \hat{k} \right) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot \left(5\hat{i} + 3\hat{j} - 6\hat{k} \right) + 8 = 0$.

▶ Watch Video Solution

18. Find the distance of the point $(-1, -5, -10)$ from the point of the intersection of the line $\vec{r} = 2\hat{i} - 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k} \right)$ and the plane $\vec{r} \cdot \left(\hat{i} - \hat{j} + \hat{k} \right) = 5$.

▶ Watch Video Solution

19. Find the vector equation o the line passing through (1,2,3) and parallel to the planes $\vec{r} \cdot \left(\hat{i} + \hat{j} + 2\hat{k} \right) = 5$ and $\vec{r} \cdot \left(3\hat{i} + \hat{j} + \hat{k} \right) = 6$

▶ Watch Video Solution

20. Find the vector equation of line passing through the point $(1, 2, -4)$ and perpendicular to the two lines:

$$\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7} \quad \text{and} \quad \frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$$

21. Prove that if a plane has the intercepts $a,\ b,\ c\ and$ is at a distance of p units from theorigin, then $\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

22. Distance between the two planes $2x + 3y + 4z = 4 \text{ and } 4x + 6y + 8z = 12$ is

A. 2 units

B. 4 units

C. 8 units

D. $\dfrac{2}{\sqrt{29}}$ units

Answer: D

---

23. The planes $2x - y + 4z = 5$ and

$5x - 2.5y + 10z = 6$ are

    A. perpendicular

    B. parallel

    C. intersect y-axis

    D. passes through $\left(0, 0, \dfrac{5}{4}\right)$

Answer: B