MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Maxima and Minima

Example

1. Determine the points of maxima and minima of the function, $f(x) = \frac{1}{8}\log x - bx + x^2, x > 0$ when $b \geq 0$ is a constant.

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2. Q. Let $f(x) = \sin^3 x + \lambda \sin^2 x$ where $\frac{\pi}{2} < x < \frac{\pi}{2}$. The interval in which λshould lie in order that f (x) has exactly one minimum and exactly one maximum is

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3. Find the polynomial f(x) of degree 6, which satisfies $\underset{x \to 0}{\text{Limit}} \left(1 + \frac{f(x)}{3}\right)^{1/x} = e^2$ and has local maximum at x = 1 and local minimum at x = 0 & 2.

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4. Find the point on the curve $4x^2 + a^2y^2 = 4a^2, 4 < a^2 < 8$ that is farthest from the point (0, –2).

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5. Let $A(p^2, -p), B(q^2, q), C(r^2, -r)$ be the vertices of triangle ABC. A parallelogram AFDE is drawn with D,E, and F on the line segments BC, CA and AB, respectively. Using calculus, show that the maximum area of such a parallelogram is $\frac{1}{2}(p+q)(q+r)(p-r)$.

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6. What normal to the curve $y = x^2$ forms the shortest chord?

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7. A conical vessel is to be prepared out of a circular sheet of gold of unit radius. How much sectorial area is to be removed from the sheet so that the vessel has maximum volume?

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8. Two towns A and B are situated on the same side of a straight road at distances a and brespectively perpendiculars drawn from A and B meet the road at point C and D respectively.The distance between C and D is c. A hospital is to be built at a point P on the road such thatthe distance APB is minimum. Find position of P.

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9. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the

colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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10. A lane runs at right angles out of a road a metres wide. Find how many metres wide the lane should beif it is just possible to carry a pole b metres long $(b > a)$ from the road into the lane keeping it horizontal.

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11. A tyre company is able to manufacture x (hundred) grade A tyres and y (hundred) grade B tyres per day, where $y(5-x)=10(4-X), 0 \leq x \leq 4.$ If the profit oneach grade A tyre is twice the profit on grade B tyre,what is the most profitable number of grade A tyresper day to manufacture?

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Exercise

1. Find the maxima and minimum value of the function $y = \frac{40}{3x^4 + 8x^3 - 18x^2 + 60}$.

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2. $f(x) = 2x - \tan^{-1} x - \ln\left(x + \sqrt{1 + x^2}\right)$

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3. Show that $f(x) = \sqrt{e^{x^2} - 1}$ has its extremum at $x = 0$. Also mention whether it is a maximum or minimum.

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4. Using examples, prove that the condition $\frac{dy}{dx} = 0$ is neither a necessary nor a sufficient condition for a function $y = f(x)$ to have a maximum or a minimumat a point.

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5. Let $f(x) = x^3 - 12x$. Prove that the equation $x^3 - 12x - k = 0$ has a root -2 if $k = \max f(x)$ while it will have a root 2 if $k = \min f(x)$.

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6. If $y = a\log x + bx^2 + x$ has its exteme values at x=-1 and x=2, then find a and b.

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7. If $y = \frac{ax - b}{(x-1)(x-4)}$ has a turning point $P(2, -1)$, find the value of $aandb$ and show that y is maximum at P.

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8. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \leq c \leq 5$.

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9. Find the coordinates of a point on the parabola $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

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10. Given an interior point A on the axis of the parabola $y^2 = 2px, (p > 0)$ at a distance a from its vertex. Find the abscissa of the point on the curve nearest to the given point.

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11. Find the coordinates of the point on the curve $y = \dfrac{x}{1+x^2}$ where the tangent to the curve has the greatest slope.

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12. The parametric equations of the curve are $x = 2t\left(t^2+3\right) - 3t^2, y = 2t\left(t^2+3\right) + 3t^2$ Find the maximum slope of the curve and the corresponding point on it.

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13. A function is defined parametrically as follows : $x = t^5 - 5t^3 - 20t + 7y = 4t^3 - 3t^2 - 18t + 3\}, |t| < 2$ .Finf the the maximum and the mimimum of the funtion and mention the values of t where they are attained.

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14. Let (h, k) be a fixed point, where $h > 0, k > 0$. A straight line passing through this point cuts the positive direction of the coordinate axes at the point $PandQ$. Find the minimum area of triangle OPQ, O being the origin.

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15. A 12-cm-long wire is bent to form a triangle with one of the angles as 60°. Find the sides of the triangle if its area is the largest.

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16. The largest area of a rectangle which has one side on the x-axis and the two vertices on the curve $y = e^{-x^2}$ is

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17. Three sides of a trapezium are each equal to kcm. Find the greatest possible area of the trapezium.

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18. A statue 4 meters high sits on a column 5.6 meters high . How far from the column must a man, whose eye level is 1.6 meters from the ground, stand in order to see the statue at the greatest angle ?

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19. A wire of length l is cut into two parts. One part is bent into a circle and the other into a square. Prove that the sum of the areas of the circle and the square is the least, if the radius of the circle is half of the side of the square.

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20. A point P is given on the circumference of a circle of radius r. Chords QR are parallel to the tangent at P. Determine the maximum possible area of triangle PQR.

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21. The circle $x^2 + y^2 = 1$ cuts the x-axis at $PandQ$. Another circle with center at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S. Find the maximum area of triangle QSR.

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22. Find the point $(\alpha,)\beta$ on the ellipse $4x^2 + 3y^2 = 12,$ in the first quadrant, so that the area enclosed by the lines $y = x, y = \beta, x = \alpha$, and the x-axis is maximum.

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23. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$with its vertex at one end of the major axis.

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24. Prove that the minimum length of the intercept made by the axes on the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to $(a+b)$

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25. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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26. Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

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27. It is desired to construct a cylindrical vessel of capacity $500cum$, open at the top. What should be the

dimensions of the vessel so that the material used is minimum, given that the thickness of the material used is $2cm$?

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28. A manufacturer plans to construct a cylindrical can to hold one cu m of liquid. If the cost of constructing the top and bottom of the can is twice the cost of constructing the sides, what are the dimensions of the most economical can?

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29. A square-based tank of capacity 250 cu m has to bedug out. The cost of land is Rs 50 per sq m. The cost of digging increases with the depth and for the whole tank the cost is Rs $400 \times (depth)^2$. Find the dimensions of the tank for the least total cost.

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30. A box without lid having maximum volume is made out of square metal sheet of edge$60cms$ by cutting equal square pieces from the four corners and turning up the projecting pieces to make the sides of the box. The height of the box is

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31. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2\frac{R}{\sqrt{3}}$. Also find maximum volume.

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32. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to 2/3 of the diameter of the sphere.

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33. Prove that the cone, circumscribing a sphere of radius r, has the minimum volume if its altitude is 4r and its semivertical angle is $\sin^{-1}\left(\frac{1}{3}\right)$.

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34. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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35. A tree trunk I metres long, is in the shape of a frustum of a cone, the radii of its ends being a and meters $(a > b)$. It is required to cut from it a beam of uniforms quare section. Prove that the beam of the greatest volume that can be cut is $\frac{1}{3}\frac{al}{a-b}$ metres long.

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36. A swimmer S is in the sea at a distance dkm from the closest point A on a straight shore. The house of the swimmer is on the shore at distance Lkm /h per hour and walk at a speed of vkm/h , `u

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37. A private telephone company serving a small community makes a profit of Rs. 12.00 per subscriber, if it has 725 subscribers. It decides to reduce the rate by a fixed sum for each subscriber over 725, thereby reducing the profit by 1 paise per subscriber. Thus, there will be profit of Rs. 11.99 on each of the 726 subscribers, Rs. 11.98 on each of the 727 subscribers, etc. What is the number of subscribers which will give the company the maximum profit?

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38. A factory D is to be connected by a road with a straight railway line on which a town A is situated. The distance DB of the factory to the railway line is $5\sqrt{3}km$. Length AB of the railway line is 20km. Freight charges on the road are twice the charges on the railway. The point `P(A P

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39. Fill in the blanks. $f(x) = x^2 + ax^{-1}$ will have a minimum at $x = 2$ if $a =$_

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40. What is the area of the largest triangle that can be inscribed in a semicircle of radius r unit.

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41. x and y be two variables such that $x > 0$ and $xy = 1$. Then the minimum value of $x + y$ is

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42. The value of a so that the sum of the squares of the roots of the equations $x^2 - (a-2)x - a + 1 = 0$ assume the least value is

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43. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq\dot{m}$) of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10

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44. The total cost of making n pocket radios per day is $Rs\left\{\frac{n^2}{4}+35n+25\right.$and the rate at which they may be sold to a distributor is $Rs\left\{\frac{1}{2}(100-n)\right\}$. For

amaximum total profit, the daily output should be many of a small town has 500

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45. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300 per subscriber. The company proposes to increase the annual subscription and it is believed that every increase of Rs 1 one subscriber will discontinue the service. Find what increase will bring maximum revenue?

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46. If $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ is a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. Then, the function $P(x)$ has

A. exactly one maximum

B. exactly one minimum

C. one maximum and one minimum

D. none of these

Answer:

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47. Find the minimum length of radius vector of the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

A. abs(a - b)

B. ab

C. a + b

D. none of these

Answer:

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48. A right cylindrical vessel of a given capacity is formed using least possible material. Then the ratio of

the height to the radius of the base is

A. pi : 1

B. sqrt (pi) : 1

C. 0.0840277777777778

D. 0.0423611111111111

Answer:

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49. Let $f(x) = (x-1)^p.(x-2)^q wherep > 1, q > 1.$ Each critical point of $f(x)$ is a point of extremum when

A. $p = 3, q = 4$

B. $p = 4, q = 2$

C. $p = 2, q = 3$

D. $p = 2, q = 4$

Answer:

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50. The point $(0, 3)$ is closest to the curve $x^2 = 2y$ at

A. (2sqrt 2 , 0)

B. (0,0)

C. (2,2)

D. none of these

Answer:

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51. For the function $f(x) = xe^x$, the point

A. $x = -1$is a `minimum

B. $x = 0$ is a minimum

C. $x = -1$ is a maximum

D. $x = 0$ is a maximum

Answer:

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52. Let f(x) be a function such that $f'(a) \neq 0$. Then , at x=a, f(x)

A. f(x) cannot have maximum at $x = a$

B. f(x) cannot be minimum at $x = a$

C. f(x) must have neither a maximum nor a minimum at $x = a$.

D. none of these

Answer:

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53. N Characters of information are held on magnetic tape, in batches of x characters each, the batch processing time is $\alpha + \beta x^2$seconds, α and β are constants. The optical value of x for fast processing is

A. frac{alpha}{beta}

B. frac{beta}{alpha}

C. sqrt(frac{alpha}{beta})

D. sqrt[frac{beta}{alpha}]

Answer:

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54. The function $f(x) = x^2 + \frac{a}{x}$ cannot have a maximum at $x = -3$ for any real value of a. (True/false)

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55. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$.

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56. Two points A(1, 4) & B(3, 0) are given on the ellipse $2x^2 + y^2 = 18$. The co-ordinates of a point C on the ellipse such that the area of the triangle ABC is greatest is

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57. If $f(x) = a\log|x| + bx^2 + x$ has its extremum values at $x = -1 and x = 2$, then $a = 2, b = -1$ $a = 2, b = -1/2$ $a = -2, b = 1/2$ (d) none of these

A. $a = \frac{-1}{2}$,`b = frac{1}{4}

B. $a = -2, b = \frac{1}{4}$

C. $a = 2$, `b = frac{-1}{2}

D. $a = -2, b = \frac{-1}{2}$

Answer:

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58. P is a variable point on the curve y= f(x) and A is a fixed point in the plane not lying on the curve. If PA^2 is minimum, then the angle between PA and the tangent at P is

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59. The fuel charges for running a train are proportional to the square of the speed generated in km/h, and the cost is Rs. 48 at 16 km/h. If the fixed charges amount to Rs. 300/h, the most economical speed is 60 km/h (b) 40 km/h 48 km/h (d) 36 km/h

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