MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Miscellaneous Exercises

Exercise

1.

2. Column I	Column II
(i) $x + \sin x$	(a) increasing
(ii) $\sec x$	(b) decreasing
	(c) neither increasing nor decreasing

2. If $u \sin t + v \cos t = 5$ and $u \cos t - v \sin t = 7$ then $(\ddot{u})(\dot{v}) - (\dot{u})(\ddot{v})$ = ____ where dots denote differentiation w.r.t. t.

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3. If g is the inverse function of $f and f'(x) = \sin x, then g'(x)$ is $\cos ec\{g(x)\}$ (b) $\sin\{g(x)\}$ $-\frac{1}{\sin\{g(x)\}}$ (d) none of these

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4. Let $f(x) = x + 1$ and $\phi(x) = x - 2$. Then the value of x satisfying $|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ are :

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5. One diagonal of a square is the portion of the line $7x+5y=35$ intercepted by the axes. Obtain the extremities of the other diagonal.

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6. Let $f(x)=2\sin^2\beta+4\cos(x+\beta)\cdot\sin x\cdot\sin\beta+\cos 2(x+\beta)$. Then the value of $|f(\alpha)|^2+\left\{f\left(\frac{\pi}{4}-\alpha\right)\right\}^2$=______.

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7. A positive integral power of the imaginary quantity i is taken at random. The probability of the value being real is __________.

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8. If z is a complex number such that $z\bar{z} = 1$ and amp $\frac{z}{\bar{z}} = \frac{\pi}{2}$ then z= _______.

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9. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then: x=

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10. The solution set of $f'(x) > g'(x)$ where $f(x) = \left(\frac{1}{2}\right)5^{2x+1}$ and $g(x) = 5^x + 4x \log 5$ is

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11. $\lim_{n \to \infty} \frac{a^n}{n!}$ is equal to ______.

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12. The differential coefficient of 2^x w.r.t. x^2 is_____.

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13. A natural number less than or equal to 200 is written down at random. The probability of the number being a perfect square is ______.

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14. A box contains 100 tickets numbered 1, 2, 3, ... ,100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. the minimum number on them is 2 with probability________.

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15. Prove that two parabolas having the same focus and their axes in opposite directions, cut at right angles.

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16. If a, b, c and d are positive real numbers such that a+b+c+d= 1 then prove that $ab + bc + cd + da \leq \frac{1}{4}$.

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17. A vertical lamp-post of height h stands at a point on the boundary of a circular field. A man of height a is running round the boundary. Prove that the end of the shadow of the man will also travel on a circle. Find the ratio of the radii of the two circles.

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18. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0, 7x + y = 50$ &

divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2:1.

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19. The circle $x^2 + y^2 = 1$ cuts the x-axis at $PandQ$. Another circle with center at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S. Find the maximum area of triangle QSR.

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20. Solve - $\dfrac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \dfrac{\cos x}{3}$

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21. let f(x) be the polynomial function. It satisfies the equation $2 + f(x) \cdot f(y) = f(x) + f(y) + f(xy)$ for all x and y. If $f(2) = 5$ find $f|f(2)|$.

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22. Evaluate $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx.$

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23. Find the co-ordinates of all the points P on the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,for which the area of the triangle PON is maximum, where O denotes the origin and N, the foot of the perpendicular from O to tangent at P.

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24. Find the equation of a curve passing through the point (1.1) if the perpendicular distance of the origin from the normal at any point P(x, y) of the curve is equal to the distance of P from the x-axis.

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25. Solve: $\left(x^2 + 4y^2 + 4xy\right)dy = (x + 2y + 1)dx.$

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26. If the normal to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ which is farthest from its centre be $\frac{x_1}{a} + \frac{y}{b} = 1$ then value of

$\left[a^2 + b^2\right]$ is equal to (where [.] represents the GIF)

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27. $\sum_{n=0}^{\infty} \frac{1}{n!}\left[\sum_{k=0}^{n} (k+1) \int_0^1 2^{-(k+1)x} dx\right]$

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28. Consider the family of circles $x^2 + y^2 = r^2 \; 2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ meets the axes at A and B then find the equation of the locus of middle point of AB.

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29. Let T_1, T_2 and be two tangents drawn from (-2, 0) onto the circle $C: x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles when taken two at a time

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30. The equation of two equal sides AB and AC of an isosceies triangle ABC are $x + y = 5$ and $7x - y = 3$respectively Find the equations of the side BC if the area of the triangle ofABC is 5 units

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31. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$

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32. Prove that the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$ touch. Find the common tangent at the point of contact.

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33. Find the equation of the ellipse whose foci are the points (1, 2) and (-3,2), and the length of the minor axis is $4\sqrt{3}$.

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34. Let ABC and PQR be any two triangles in the same plane. Assume that the perpendiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise,prove that the perpendiculars from $P, Q, R \rightarrow BC, CA, AB$ respectively are also concurrent.

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35. An unbiased dice, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers shown up is noted. Then find the probability that among the numbers 1, 2, 3, 4, 5, 6 only three numbers appear in this list and each number appears at least once.

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36. Find the smallest positive values of $x and y$ satisfying $x - y = \frac{\pi}{4} and \cot x + \cot y = 2$

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37. $\log_{\frac{3}{4}} \log_8 (x^2 + 7) + \log_{\frac{1}{2}} \log_{\frac{1}{4}} (x^2 + 7)^{-1} = -2.$

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38. Total number of solutions of the equation $x^2 - 4 - [x] = 0$ are (where (.) denotes the greatest integer function)

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39. Let a, b be the roots of the equation $x^2 - kx + k = 0$, $k \in R$. If $a^2 + b^2$ is the minimum then find the roots of the equation.

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40. Two parabola have the focus (3, 2). Their directrices are the x-axis and the y-axis respectively. Then the slope of their common chord is

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41. Let $\alpha \in R$. prove that a function $f: R - R$ is differentiable at α if and only if there is a function $g: R - R$

which is continuous at α and satisfies

$f(x) - f(\alpha) = g(x)(x - \alpha), \forall x \in R.$

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42. Find the $\lim_{x \to 0} \frac{\sqrt{x+2} - \sqrt{2}}{\sqrt[3]{1-x} - \sqrt{1+x}}$.

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43. Two parabolas $y_2 = 4a(x - 1l_1)$ and $x_2 = 4a(y - l_2)$ always touch one another, the quantities l_1 and l_2 are both variable. Locus of their point of contact has the equation

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44. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, then the length of OA is

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45. Let $f(x), x \geq 0,$ be a non-negative continuous function, and let $f(x) = \int_0^x f(t)dt, x \geq 0,$ if for some $c > 0, f(x) \leq cF(x)$ for all $x \geq 0,$ then show that $f(x) = 0$ for all $x \geq 0.$

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46. Let $b \neq 0$ and for $j = 0, 1, 2, \ldots, n$. Let S_j be the area of the region bounded by Y_axis and the curve $x \cdot e^{ay} = \sin by, \frac{j\pi}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that $S_0, S_1, S_2, \ldots S_n$ are in geometric progression. Also, find their sum for a=-1 and $b = \pi$.

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47. Let P be a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 < b < a$ and let the line parallel to y-axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x-axis. For two positive real numbers r and s, find the locus of the point R on PQ such that $PR : RQ = r : s$ and P varies over the ellipse.

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48. A hemi-spherical tank of radius 2 m is initially full of water and has an outlet of $12cm^2$ cross-sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $v(t) = \sqrt{0.62gh(t)}$, where $v(t)$ and $h(t)$ are, respectively, the velocity of the flow through the outlet and the height of water level above the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank.

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49. I_n is the area of n sided refular polygon inscribed in a circle unit radius and O_n be the area of the polygon circumscribing the given circle, prove that

$$I_n = \frac{O_n}{2}\left(1 + \sqrt{1 - \left(\frac{2I_n}{n}\right)^2}\right)$$

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50. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P(6,8) to the circle and the chord of contact and the chord of contact is maximum.

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51. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant k is positive). Suppose that r(t) is the radius of the liquid cone at time t. The time after which the cone is empty is

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52. If $\overrightarrow{u}, vercv, \overrightarrow{w}$ be three noncoplanar unit vectors and α, β, γ the angles between $\overrightarrow{u}$ and $\overrightarrow{v}$, $\overrightarrow{v}$ and $\overrightarrow{w}$, $\overrightarrow{w}$ and $\overrightarrow{u}$ respectively, $\overrightarrow{x}, \overrightarrow{y}, \overrightarrow{z}$ unit vector along the bisectors of the angles α, β, γ respectively. Prove that:

$$\left[\overrightarrow{x} \times \overrightarrow{y}\overrightarrow{y} \times \overrightarrow{z}\overrightarrow{z} \times \overrightarrow{x}\right] = \frac{1}{16}\left[\overrightarrow{u}\overrightarrow{v}\overrightarrow{w}\right]^2 \frac{\sec^2\alpha}{2}\frac{\sec^2\beta}{2}\frac{\sec^2\gamma}{2}$$

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53. If $P(1) = 0 and \frac{dP(x)}{dx}, \sin x + 2x \geq \frac{3x(x+1)}{\pi}$.

Explain the identity, if any, used in the proof.

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54. Normals are drawn from a point P with slopes m_1, m_2 and m_3 are drawn from the point p not from the parabola $y^2 = 4x$. For $m_1 m_2 = \alpha$, if the locus of the point P is a part of the parabola itself, then the value of α is (a) 1 (b)-2 (c) 2 (d) -1

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55. Find a point on the curve $x^2 + 2y^2 = 6$, whose distance from the line $x + y = 7$, is minimum.

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56. If a function $f:[-2a, 2a] \to R$ is an odd function such that, $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left-hand derivative at $x = a$ is 0, then find the left-hand derivative at $x = -a$.

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57. A is targeting to B, B and C are targeting to A. probability of hitting the target by A, B and C are 2/3, 1.2 and 1/3, respectively. If A is hit, then find the Probability that B hits the target and C does not.

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58. $f:[0.4] \to R$ is a differentiable function. Then prove that for some $a, b \in (0, 4)$, $f^2(4) - f^2(0) = 8f'(a) \cdot f(b)$.

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59. Given a function $f:[0, 4] \to R$ is differentiable ,then prove that for some $\alpha, \beta \varepsilon (0, 2)$, $\int_0^4 f(t)dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2)$.

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60. If $f: R \to [0, \infty)$ be a function such that $f(x-1) + f(x+1) = \sqrt{3}(f(x))$ then prove that $f(x+12) = f(x)$.

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61. Using Rolles theorem, prove that there is at least one root in $\left(45^{\frac{1}{100}}, 46\right)$ of the equation. $P(x) = 51x^{101} - 2323(x)^{100} - 45x + 1035 = 0.$

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62.

$$A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$$

If there is a vector matrix X, such that $AX = U$ has infinitely many solutions, then prove that $BX = V$ cannot have a unique solution. If $afd \neq 0$. Then,prove that $BX = V$ has no solution.

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63. Find the equation of circle touching the line $2x+3y+1=0$ at the point (1,-1) and is orthogonal to the circle which has the line segment having end poitns (0,-1) and (-2,3) as the diameter.

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64. $$\frac{\int\left(\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x\right)dx}{\sqrt{\sin^3 x\cos^3 x\sin(x-\alpha)}}$$

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65. T is a parallelepiped in which A,B,C and D are vertices of one face and the face just above it has corresponding

vertices A',B',C',D',T is now compressed to S with face ABCD remaining same and shifted to A",B",C",D", in S. The volume of parallelepiped S is reduced to 90% of T. Prove that locus of is a plane.

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66. $f'(0) = \lim_{n \to \infty} nf\left(\frac{1}{n}\right)$ and $f(0) = 0$ Using this, find `lim_(n->oo)((n+1)(2/pi)cos^(- 1)(1/n)-n)),|cos^(-1)1/n|

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67. If $y(x) = \int_{\frac{\pi^2}{16}}^{x^2} \frac{\cos x \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$ then find $y'(\pi)$

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68. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$ a tangent is drawn which meets the directrix at Q. Find the locus of point R which divides QP externally in the ratio $\frac{1}{2} : 1$

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69. If a ΔABC remains always similar to a given triangle and the point A is fixed and the point B always moves on a given straight line, then locus of C is (A) a circle (B) a straight line (C) a parabola (D) none of these

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70. Find the range of values of t for which

$$2 \sin t = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$$

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71. if $|f(x_1) - f(x_2)| \leq (x_1 - x_2)^2$Find the equation of gent to the curve $y = f(x)$ at the point $(1, 2)$.

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72. Find the equation of the common tangent to the circle $x^2 + y^2 = 16$ and the ellipse $4x^2 + 25y^2 = 100$ cutting off positive intercepts on the axes of reference. Also, find the intercept on the common tangent between the coordinate axes.

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73. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix}\begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$,$f(x)$ is a quadratic function and its maximum valueoccurs at a point V.A is a point of intersection of $y = f(x)$ with X-axis and point B is such that chord AB subtendsa right angle at V. Find the area enclosed by f(x) andchord AB.

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74. The total number of runs scored If n matches is $\frac{n+1}{4}\left(2^{n+1} - n - 2\right)$,where $n > 1$ and the runs scored in k^{th} match are given by $k.2^{n+1-k}$, where $1 \leq k \leq n$. Find n

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75. A square circumscribes the circle $|z-1|=\sqrt{2}$. If one of the vertices represents the complex number $2+i\sqrt{3}$ then find the complex numbers represented by the other vertices.

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76. Evaluate

$$\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x dx.$$

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77. If $f(x-y)=f(x).\,g(y)-f(y).\,g(x)$ and $g(x-y)=g(x).\,g(y)+f(x).\,f(y)$ for all $x\in R$. If right handed derivative at x=0 exists for f(x) find the derivative of g(x) at x =0

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