



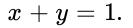
# MATHS

# BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

# Parabola



**1.** Find the equation of the parabola whose focus is(1, 1) and tangent at the vertex is



## Watch Video Solution

**2.** Find the equation of the parabola whose axis is parallel to X-axis and which passes through the point (0,4),(1,9) and (-2,6) . Also, find its latusrectum.



3. Find vertex, focus, directrix and latus rectum

of the parabola  $y^2 + 4x + 4y - 3 = 0$ .

# Watch Video Solution

**4.** Prove that on the axis of any parabola there is a certain point 'k' which has the property that, if a chord PQ of parabola be drawn through it then  $\frac{1}{(PK)^2} + \frac{1}{(QK)^2}$  is the

same for all positions of the chord.

5. The number of integral values of a for which the point (-2a,a+1) will be interior point of the smaller region bounded by the circle  $x^2 + y^2 = 4$  and the parabola  $y^2 = 4x$  is:

Watch Video Solution

6. Show that the tangents at the extremities of

any focal chord of a parabola intersect at right

angles at the directrix.

**7.** The Circumcircle of the triangle formed by any three tangents to a parabola passes through

Watch Video Solution

8. Three normals are drawn from the point (c,0) to the curve  $y^2 = x$ . Show that c must be greater than  $\frac{1}{2}$ . One normal is always the X-

axis. Find c for which the other two normals

are perpendicular to each other.



**9.** Let  $(x_r, y_r)$ , r=1,2,3,4 ne the points of intersection the parabola "y^2=4ax" and the circle"x^2+y^2+2gx+2fy+c=0"prove that

 $y_1 + y_2 + y_3 + y_4 = 0$ 

10. find the common tangents of the circle  $x^2 + y^2 = 2a^2$  and the parabola $y^2 = 8ax$ 

# Watch Video Solution

**11.** A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius the axes of reference being AB and a perpendicular

diameter, prove that the locus of the focus of

parabola 
$$\displaystyle rac{x^2}{a^2} + \displaystyle rac{y^2}{b^2-a^2} = 1$$

Watch Video Solution

12. The locus of the middle points of normal

chords of the parabola  $y^2=4ax$  is-

Watch Video Solution

13. Show that the locus of a point that divides

a chord of slope 2 of the parabola  $y^2=4x$ 

internally in the ratio 1:2 is parabola. Find the

vertex of this parabola.



14. A variable chord PQ of the parabola  $y = 4x^2$  subtends a right angle at the vertex. Then the locus of points of intersection of the tangents at P and Q is

**15.** Find the locus of the foot of the perpendicular drawn from a fixed point to any tangent to a parabola.



16. The locus of the poles of tangents to the parabola  $y^2 = 4ax$  with respect to the parabola  $y^2 = 4ax$  is

17. The general eqaution to a system of parallel chords of the parabola $y^2=rac{25}{7}xis4x-y+k=0.$ What is the

equation to the corresponding diameter?



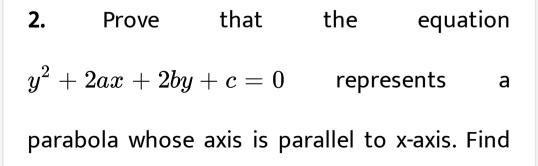
18. A ray of light is coming along the line y = bfrom the positive direction of x-axis and striks a concave mirror whose intersection with xyplane is a parabola  $y^2 = 4ax$ . Find the equation of the reflected ray and show that it passes through the focus of the parabola.

Both a and b are positive.



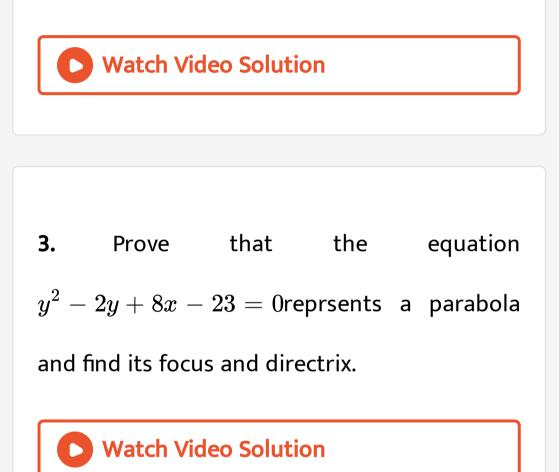
19. A parabola is drawn touching the axis of x at the origin and having its vertex at a given distance k form this axis Prove that the axis of the parabola is a tangent to the parabola  $x^2 = -8k(y - 2k).$ 

1. Find the equation of the parabola, if the focus is at (-6, -6) and the vertex is at (-2, 2)



its vertex and the equation of the double

ordinate through the focus.



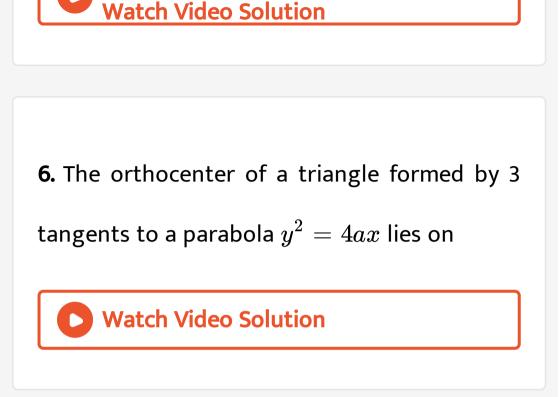
4. If  $(a^2, a - 2)$  be a point interior to the region of the parabola  $y^2 = 2x$  bounded by the chord joining the points (2, 2) and (8, -4), then the set of all possible real values of a is

Watch Video Solution

**5.** Show that the tangents at the extremities of any focal chord of a parabola intersect at right

angles at the directrix.





7. Tangents are drawn from any point on the line x + 4a = 0 to the parabola  $y^2 = 4ax$ . Then find the angle subtended by the chord of contact at the vertex. 8. Prove that the area of triangle formed by the tangents to the parabola  $y^2 = 4ax$  from the point  $(x_1, y_1)$  and the chord of contact is  $rac{1}{2a} ig(y_1^2 - 4ax_1ig)^{3/2}$  sq. units.

Watch Video Solution

**9.** Points A, B, C lie on the parabola  $y^2 = 4ax$ The tangents to the parabola at A, B and C, taken in pair, intersect at points P, Q and R. Determine the ratio of the areas of the

riangle ABC and riangle PQR



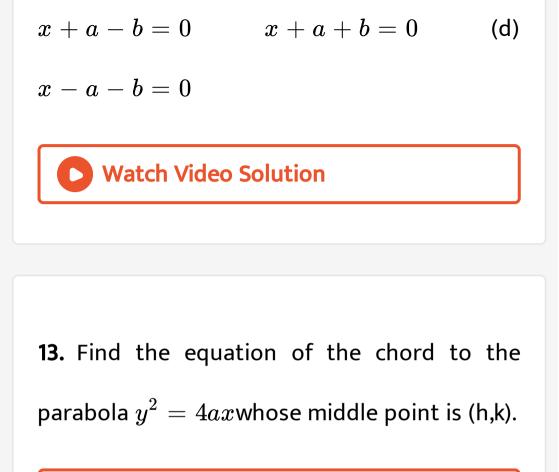
10. If P, Q, R are three points on a parabola  $y^2 = 4ax$  whose ordinates are in geometrical progression, then the tangents at P and R meet on :

**11.** Prove that any three tangents to a parabola whose slopes are in harmonic progression

enclose a triangle of constant area.



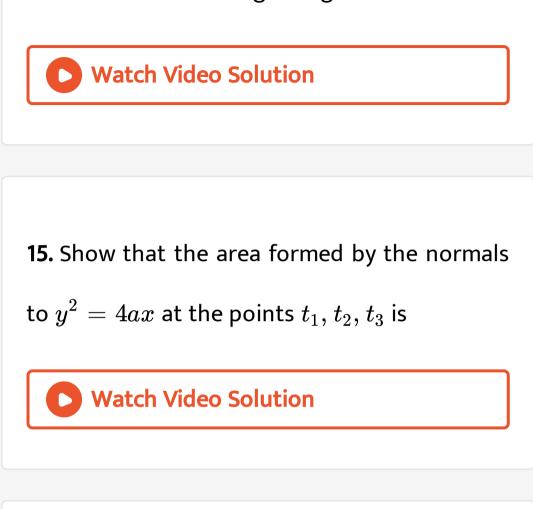
12. Two straight lines are perpendicular to each other. One of them touches the parabola  $y^2 = 4a(x + a)$  and the other touches  $y^2 = 4b(x + b)$ . Their point of intersection lies on the line. x - a + b = 0 (b)



# Watch Video Solution

**14.** Prove that the normal chord to a parabola at the point whose ordinate is equal to the

abscissa subtends a right angle at the focus.



**16.** P & Q are the points of contact of the tangents drawn from the point T to the parabola  $y^2 = 4ax$ . If PQ be the normal to the

parabola at P, prove that TP is bisected by the

directrix.



17. For what values of 'a' will the tangents drawn to the parabola  $y^2 = 4ax$  from a point, not on the y-axis, will be normal to the parabola  $x^2 = 4y$ .

**18.** Find the centre and radius of the smaller of the two circles that touch the parabola  $75y^2 = 65(5x-3)$  at  $\left(\frac{6}{5}, \frac{8}{5}\right)$  and the X axis.

Watch Video Solution

19. Prove that the length of the intercept on the normal at the point  $Pig(at^2,2atig)$  of the parabola  $y^2=4ax$  made by the circle

described on the line joining the focus and P

as diameter is  $a\sqrt{1+t^2}$  .

## Watch Video Solution

**20.** Prove that the locus of the middle pointsof chords of the parabola $y^2 = 4ax$ through the vertex is also a parabola.Find focus and latus rectum of the locus.

**21.** Prove that the locus of a point, which moves so that its distance from a fixed line is equal to the length of the tangent drawn from it to a given circle, is a parabola.

**Watch Video Solution** 

**22.** 5. The locus of point of intersection of two tangents to the parabola  $y^2 = 4x$  such that their chord of contact subtends a right angle at the vertex is



**23.** Find the locus of the intersection of normals to the parabola $y^2 = 4ax$ at the extremities of a focal chord.

Watch Video Solution

**24.** Find the locus of the point of intersection of those normals to the parabola  $x^2 = 8y$  which are at right angles to each other.

25. Find the locus of point of intersection of tangent to the parabola  $y^2 = 4ax$  (i)which are inclined at an angle  $\theta$  to each other. (ii) which intercept constant length c on the tangent at the vertex (iii) such that the area of  $\Delta ABC$  is constant c; where A and B are the points of intersection of tangents with the y-axis and R is a point of intersections of tangents.



**26.** Find the locus of the middle points of the chords of the parabola  $y^2 = 4ax$  which subtend a right angle at the vertex of the parabola.

27. Show that the locus of points such that  
two of the three normals drawn from them to  
the parabola 
$$y^2 = 4ax$$
 coincide is  
 $27ay^2 = 4(x - 2a)^3$ .

**28.** Prove that the locus of the point of intersection of the normals at the ends of a system of parallel chords of a parabola is a straight line which is a normal to the curve.

Watch Video Solution

**29.** Find the locus of the middle points of the chords of the parabola  $y^2 = 4ax$  which

subtend a right angle at the vertex of the parabola.

# Watch Video Solution

**30.** If two tangents to the parabola  $y^2 = 4ax$ from a point P make angles  $\theta_1$  and  $\theta_2$  with the axis of the parabola, then find the locus of P in each of the following cases.  $\tan^2 \theta_1 + \tan^2 \theta_2 = \lambda$  (a constant)

**31.** Locus of the feet of the perpendiculars drawn from vertex of the parabola  $y^2 = 4ax$  upon all such chords of the parabola which subtend a right angle at the vertex is

Watch Video Solution

**32.** If the focus =(2,3)and directrix is x+y=1 then

the equation of the parabola is \_\_\_\_.

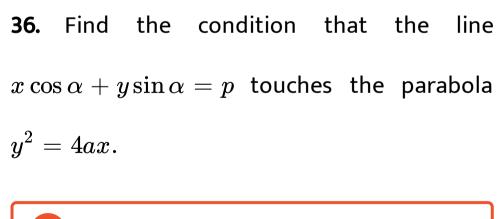
**33.** The line x+y+1=Otouches the parabola

$$y^2 = kx$$
 if k=\_\_\_\_

# Watch Video Solution

**34.** If the normals to the parabola  $y^2 = 4ax$  at the ends of the latus rectum meet the parabola at QandQ', then  $\mathbb{Q}'$  is 10a (b) 4a(c) 20c (d) 12a

**35.** Write the length of het chord of the parabola  $y^2 = 4ax$  which passes through the vertex and in inclined to the axis at  $\frac{\pi}{4}$ .



Watch Video Solution

37. The point of intersection of the tangents at

the ends of the latus rectum of the parabola

$$y^2=4x$$
 is\_\_\_\_\_

Watch Video Solution

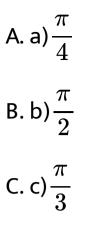
**38.** Find the angle between the tangents drawn from (1, 3) to the parabola  $y^2 = 4x$ .

**39.** Find the equation of the circle described on the line segment joining the foci of the parabolas  $x^2 - 4ay$  and  $y^2 = 4a(x - a)$  as diameter.

Watch Video Solution

40. A double ordinate of the parabola  $y^2 = 8px$  is of length 16p. The angle subtended by it at the vertex of the parabola .

is



D. d)none of these

#### Answer:



41. The equation of parabola whose vertex and

focus lie on the axis of x at distances a and  $a_1$ 

from the origin respectively, is

A. a)
$$y^2=4(a\,{}^\prime-a)(x-a)$$

B. b)
$$y^2=4(a\,{}^\prime-a)(x-a\,{}^\prime)$$

C. c)
$$x^2=4(a\,{}^\prime-a)(y-a)$$

D. d)
$$y^2 = 4(a-a')(x-a)$$

#### Answer:

# Watch Video Solution

42. f the normal at the point  $P(at_1, 2at_1)$  meets the parabola  $y^2 = 4ax$  again at  $(at_2, 2at_2)$ , then

A. a)
$$t_1 = -t_2 - rac{2}{t_2}$$
  
B. b) $rac{t_1 + t_2}{2} = rac{1}{t_1}$   
C. c) $t_2 = -t_1 - rac{2}{t_1}$   
D. d) $rac{t_1 + t_2}{2} = rac{1}{t_2}$ 

#### Answer:

# Watch Video Solution

**43.** If the tangents to the parabola  $y^2 = 4ax$ at the points  $(x_1, y_1)$ and  $((x_2, y_2)$ meet at the point  $(x_3, y_3)$ then

A. 
$$y_3=\sqrt{y_1y_2}$$

B. 
$$2y_3 = y_1 + y_2$$
  
C.  $rac{2}{y_3} = = rac{1}{y_1} + rac{1}{y_2}$ 

D. none of these

#### **Answer: B**



44. Find the condition that the line  $x\coslpha+y\sinlpha=p$  touches the parabola  $y^2 = 4ax.$ 

A. a)
$$p\coslpha+a\sin^2lpha=0$$

B. b)
$$p\sinlpha+a\cos^2lpha=0$$

C. c)
$$a\coslpha+p\sin^2lpha=0$$

D. d) $a\sinlpha+p\cos^2lpha=0$ 

#### **Answer:**

**45.** The angle between the tangents drawn from the origin to the parabola 
$$y^2 = 4a(x-a)$$
 is

A. a) $90^{\circ}$ 

### B. b) $30^{\circ}$

- C. c) $\tan^{-1}(1/2)$
- D. d) $45^{\circ}$

#### **Answer:**

# Watch Video Solution

**46.** The equation of a tangent to the parabola $y^2 = 8xisy = x+2$  . The point on this line

from which the other tangent to the parabola

is perpendicular to the given tangent is (1) (-1,1) (2) (0,2) (3) (2,4) (4) (-2,0)

A. a)(2,4)

B.b)(-2,0)

C. c)(-1,1)

D. d)none of these

#### Answer:



47. If ' $t_1$  'and ' $t_2$  'be the ends of a focal chord of the parabola  $y^2 = 4ax$  then $t_1t_2$ is equal to

A. 1

B. -1

C. 2

D. none of these

#### **Answer: B**

**48.** The general equation to a system of parallel chords of the parabola  $y^2 = 4x$  is y=2x+k.The equation of the corresponding diameter is .

Watch Video Solution

**49.** P is a point on the parabola  $y^2 = 4ax$  and PQ is its focal chord. If PT is tangent at P and QN is normal at Q, the angle  $\alpha$ , between PT and QN, distance between PT and QN is 'd' then

A. a)
$$0^\circ\,$$

B. b)
$$lpha=0^\circ$$

D. d)
$$rac{aig(1+t^2ig)^{3/2}}{t^2}$$

#### Answer:

# Watch Video Solution

50. The radius of the circle whose centre is (-4,0) and which cuts the parabola  $y^2=8x$  at

A and B such that the common chord AB subtends a right angle at the vertex of the parabola is equal to

Watch Video Solution

**51.** Let  $C_1$  and  $C_2$  be parabolas  $x^2 = y - 1$ and  $y^2 = x - 1$  respectively. Let P be any point on  $C_1$  and Q be any point  $C_2$ . Let  $P_1$  and  $Q_1$  be the reflection of P and Q, respectively w.r.t the line y = x then prove that  $P_1$  lies on  $C_2$ and  $Q_1$  lies on  $C_1$  and  $PQ \ge [PP_1, QQ_1]$ . Hence or otherwise , determine points  $P_0$  and  $Q_0$  on the parabolas  $C_1$  and  $C_2$  respectively such that  $P_0Q_0 \leq PQ$  for all pairs of points (P,Q) with P on  $C_1$  and Q on  $C_2$