MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Parabola

Example

1. Find the equation of the parabola whose focus is $(1, 1)$ and tangent at the vertex is

$x + y = 1.$

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2. Find the equation of the parabola whose axis is parallel to X-axis and which passes through the point (0,4),(1,9) and (-2,6) . Also, find its latusrectum.

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3. Find vertex, focus, directrix and latus rectum of the parabola $y^2 + 4x + 4y - 3 = 0$.

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4. Prove that on the axis of any parabola there is a certain point 'k' which has the property that, if a chord PQ of parabola be drawn through it then $\frac{1}{(PK)^2} + \frac{1}{(QK)^2}$ is the same for all positions of the chord.

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5. The number of integral values of a for which the point (-2a,a+1) will be interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$ is:

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6. Show that the tangents at the extremities of any focal chord of a parabola intersect at right angles at the directrix.

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7. The Circumcircle of the triangle formed by any three tangents to a parabola passes through

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8. Three normals are drawn from the point (c,0) to the curve $y^2 = x$. Show that c must be greater than $\frac{1}{2}$. One normal is always the X-

axis. Find c for which the other two normals are perpendicular to each other.

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9. Let $(x_r, y_r), r$=1,2,3,4 ne the points of intersection the parabola "y^2=4ax" and the circle"x^2+y^2+2gx+2fy+c=0"prove that $y_1 + y_2 + y_3 + y_4 = 0$

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10. find the common tangents of the circle $x^2 + y^2 = 2a^2$ and the parabola$y^2 = 8ax$

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11. A parabola is drawn to pass through A and B, the ends of a diameter of a given circle of radius a, and to have as directrix a tangent to a concentric circle of radius the axes of reference being AB and a perpendicular

diameter, prove that the locus of the focus of parabola $\frac{x^2}{a^2} + \frac{y^2}{b^2 - a^2} = 1$

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12. The locus of the middle points of normal chords of the parabola $y^2 = 4ax$ is-

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13. Show that the locus of a point that divides a chord of slope 2 of the parabola $y^2 = 4x$

internally in the ratio $1:2$ is parabola. Find the vertex of this parabola.

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14. A variable chord PQ of the parabola $y = 4x^2$ subtends a right angle at the vertex. Then the locus of points of intersection of the tangents at P and Q is

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15. Find the locus of the foot of the perpendicular drawn from a fixed point to any tangent to a parabola.

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16. The locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4ax$ is

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17. The general eqaution to a system of parallel chords of the parabola $y^2 = \frac{25}{7} x is 4x - y + k = 0$.What is the equation to the corresponding diameter?

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18. A ray of light is coming along the line $y = b$ from the positive direction of x-axis and striks a concave mirror whose intersection with xy-plane is a parabola $y^2 = 4ax$. Find the equation of the reflected ray and show that it

passes through the focus of the parabola. Both a and b are positive.

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19. A parabola is drawn touching the axis of x at the origin and having its vertex at a given distance k form this axis Prove that the axis of the parabola is a tangent to the parabola $x^2 = -8k(y-2k)$.

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Exercise

1. Find the equation of the parabola, if the focus is at $(-6, -6)$ and the vertex is at $(-2, 2)$

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2. Prove that the equation $y^2 + 2ax + 2by + c = 0$ represents a parabola whose axis is parallel to x-axis. Find

its vertex and the equation of the double ordinate through the focus.

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3. Prove that the equation $y^2 - 2y + 8x - 23 = 0$reprsents a parabola and find its focus and directrix.

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4. If $(a^2, a-2)$ be a point interior to the region of the parabola $y^2 = 2x$ bounded by the chord joining the points $(2, 2)$ and $(8, -4)$, then the set of all possible real values of a is

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5. Show that the tangents at the extremities of any focal chord of a parabola intersect at right angles at the directrix.

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6. The orthocenter of a triangle formed by 3 tangents to a parabola $y^2 = 4ax$ lies on

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7. Tangents are drawn from any point on the line $x + 4a = 0$ to the parabola $y^2 = 4ax$. Then find the angle subtended by the chord of contact at the vertex.

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8. Prove that the area of triangle formed by the tangents to the parabola $y^2 = 4ax$ from the point (x_1, y_1) and the chord of contact is $\frac{1}{2a}\left(y_1^2 - 4ax_1\right)^{3/2}$ sq. units.

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9. Points A, B, C lie on the parabola $y^2 = 4ax$ The tangents to the parabola at A, B and C, taken in pair, intersect at points P, Q and R.

Determine the ratio of the areas of the $\triangle ABC$ and $\triangle PQR$

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10. If P, Q, R are three points on a parabola $y^2 = 4ax$ whose ordinates are in geometrical progression, then the tangents at P and R meet on :

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11. Prove that any three tangents to a parabola whose slopes are in harmonic progression enclose a triangle of constant area.

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12. Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x + a)$ and the other touches $y^2 = 4b(x + b)$. Their point of intersection lies on the line. $x - a + b = 0$ (b)

$x + a - b = 0$ $x + a + b = 0$ (d)

$x - a - b = 0$

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13. Find the equation of the chord to the parabola $y^2 = 4ax$whose middle point is (h,k).

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14. Prove that the normal chord to a parabola at the point whose ordinate is equal to the

abscissa subtends a right angle at the focus.

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15. Show that the area formed by the normals to $y^2 = 4ax$ at the points t_1, t_2, t_3 is

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16. P & Q are the points of contact of the tangents drawn from the point T to the parabola $y^2 = 4ax$. If PQ be the normal to the

parabola at P, prove that TP is bisected by the directrix.

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17. For what values of 'a' will the tangents drawn to the parabola $y^2 = 4ax$ from a point, not on the y-axis, will be normal to the parabola $x^2 = 4y$.

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18. Find the centre and radius of the smaller of the two circles that touch the parabola $75y^2 = 65(5x - 3)$ at $\left(\frac{6}{5}, \frac{8}{5}\right)$ and the X axis.

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19. Prove that the length of the intercept on the normal at the point $P\left(at^2, 2at\right)$ of the parabola $y^2 = 4ax$ made by the circle

described on the line joining the focus and P as diameter is $a\sqrt{1+t^2}$.

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20. Prove that the locus of the middle pointsof chords of the parabola$y^2 = 4ax$through the vertex is also a parabola.Find focus and latus rectum of the locus.

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21. Prove that the locus of a point, which moves so that its distance from a fixed line is equal to the length of the tangent drawn from it to a given circle, is a parabola.

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22. 5. The locus of point of intersection of two tangents to the parabola $y^2 = 4x$ such that their chord of contact subtends a right angle at the vertex is

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23. Find the locus of the intersection of normals to the parabola $y^2 = 4ax$ at the extremities of a focal chord.


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24. Find the locus of the point of intersection of those normals to the parabola $x^2 = 8y$ which are at right angles to each other.


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25. Find the locus of point of intersection of tangent to the parabola $y^2 = 4ax$ (i)which are inclined at an angle θ to each other. (ii) which intercept constant length c on the tangent at the vertex (iii) such that the area of ΔABC is constant c; where A and B are the points of intersection of tangents with the y-axis and R is a point of intersections of tangents.

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26. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which subtend a right angle at the vertex of the parabola.

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27. Show that the locus of points such that two of the three normals drawn from them to the parabola $y^2 = 4ax$ coincide is $27ay^2 = 4(x - 2a)^3$.

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28. Prove that the locus of the point of intersection of the normals at the ends of a system of parallel chords of a parabola is a straight line which is a normal to the curve.

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29. Find the locus of the middle points of the chords of the parabola $y^2 = 4ax$ which

subtend a right angle at the vertex of the parabola.

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30. If two tangents to the parabola $y^2 = 4ax$ from a point P make angles θ_1 and θ_2 with the axis of the parabola, then find the locus of P in each of the following cases. $\tan^2\theta_1 + \tan^2\theta_2 = \lambda$ (a constant)

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31. Locus of the feet of the perpendiculars drawn from vertex of the parabola $y^2 = 4ax$ upon all such chords of the parabola which subtend a right angle at the vertex is

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32. If the focus =(2,3)and directrix is x+y=1 then the equation of the parabola is ____.

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33. The line x+y+1=0touches the parabola $y^2 = kx$ if k=_____.

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34. If the normals to the parabola $y^2 = 4ax$ at the ends of the latus rectum meet the parabola at $QandQ'$, then $\mathbb{Q}'$ is $10a$ (b) $4a$ (c) $20c$ (d) $12a$

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35. Write the length of het chord of the parabola $y^2 = 4ax$ which passes through the vertex and in inclined to the axis at $\frac{\pi}{4}$.

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36. Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$.

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37. The point of intersection of the tangents at the ends of the latus rectum of the parabola $y^2 = 4x$ is_____

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38. Find the angle between the tangents drawn from (1, 3) to the parabola $y^2 = 4x$.

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39. Find the equation of the circle described on the line segment joining the foci of the parabolas $x^2 - 4ay$ and $y^2 = 4a(x - a)$ as diameter.

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40. A double ordinate of the parabola $y^2 = 8px$ is of length $16p$. The angle subtended by it at the vertex of the parabola is

A. a) $\frac{\pi}{4}$

B. b) $\frac{\pi}{2}$

C. c) $\frac{\pi}{3}$

D. d)none of these

Answer:

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41. The equation of parabola whose vertex and focus lie on the axis of x at distances a and a_1 from the origin respectively, is

A. a) $y^2 = 4(a' - a)(x - a)$

B. b) $y^2 = 4(a' - a)(x - a')$

C. c) $x^2 = 4(a' - a)(y - a)$

D. d) $y^2 = 4(a - a')(x - a)$

Answer:

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42. f the normal at the point $P(at_1, 2at_1)$ meets the parabola $y^2 = 4ax$ aguin at $(at_2, 2at_2)$, then

A. a) $t_1 = -t_2 - \frac{2}{t_2}$

B. b) $\frac{t_1 + t_2}{2} = \frac{1}{t_1}$

C. c) $t_2 = -t_1 - \frac{2}{t_1}$

D. d) $\frac{t_1 + t_2}{2} = \frac{1}{t_2}$

Answer:

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43. If the tangents to the parabola $y^2 = 4ax$ at the points (x_1, y_1) and $((x_2, y_2)$ meet at the point (x_3, y_3) then

A. $y_3 = \sqrt{y_1 y_2}$

B. $2y_3 = y_1 + y_2$

C. $\frac{2}{y_3} = = \frac{1}{y_1} + \frac{1}{y_2}$

D. none of these

Answer: B

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44. Find the condition that the line $x\cos\alpha + y\sin\alpha = p$ touches the parabola $y^2 = 4ax$.

A. a)$p\cos\alpha + a\sin^2\alpha = 0$

B. b)$p\sin\alpha + a\cos^2\alpha = 0$

C. c)$a\cos\alpha + p\sin^2\alpha = 0$

D. d)$a\sin\alpha + p\cos^2\alpha = 0$

Answer:

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45. The angle between the tangents drawn from the origin to the parabola $y^2 = 4a(x - a)$ is

A. a)90°

B. b)30°

C. c)$\tan^{-1}(1/2)$

D. d)45°

Answer:

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46. The equation of a tangent to the parabola $y^2 = 8xisy = x + 2$. The point on this line from which the other tangent to the parabola

is perpendicular to the given tangent is (1) $(-1,1)$ (2) $(0,2)$ (3) $(2,4)$ (4) $(-2,0)$

A. a)(2,4)

B. b)(-2,0)

C. c)(-1,1)

D. d)none of these

Answer:

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47. If $'t_1'$ and $'t_2'$ be the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1 t_2$ is equal to

A. 1

B. -1

C. 2

D. none of these

Answer: B

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48. The general equation to a system of parallel chords of the parabola $y^2 = 4x$is y=2x+k.The equation of the corresponding diameter is ____.

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49. P is a point on the parabola $y^2 = 4ax$ and PQ is its focal chord. If PT is tangent at P and QN is normal at Q, the angle α, between PT and QN , distance between PT and QN is 'd' then

A. a)$0^\circ < \alpha < 90^\circ$

B. b)$\alpha = 0^\circ$

C. c)d=0

D. d)$\frac{a\left(1+t^2\right)^{3/2}}{t^2}$

Answer:

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50. The radius of the circle whose centre is (-4,0) and which cuts the parabola $y^2 = 8x$ at

A and B such that the common chord AB subtends a right angle at the vertex of the parabola is equal to

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51. Let C_1 and C_2 be parabolas $x^2 = y - 1$ and $y^2 = x - 1$ respectively. Let P be any point on C_1 and Q be any point C_2. Let P_1 and Q_1 be the reflection of P and Q, respectively w.r.t the line y = x then prove that P_1 lies on C_2 and Q_1 lies on C_1 and $PQ \geq [PP_1, QQ_1]$.

Hence or otherwise , determine points P_0 and Q_0 on the parabolas C_1 and C_2 respectively such that $P_0Q_0 \leq PQ$ for all pairs of points (P,Q) with P on C_1 and Q on C_2

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