MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Product of three or more Vectors

Example

1. The position vectors of three A,B, and C in space are respectively $2\vec{i}+3\vec{j}-\vec{k}, \vec{i}-2\vec{j}+3\vec{k}$ and $4\vec{i}+\vec{j}+\vec{k}$. Find the volume of the parallelepiped whose three concurrent edges are OA, OB and OC where O is the origin.

2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero vectors,prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right]=2\left[\vec{a}, \vec{b}, \vec{c}\right]$

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3. If the four points $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar, then show that $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\left[\vec{b}\,\vec{c}\,\vec{d}\right]+\left[\vec{c}\,\vec{a}\,\vec{d}\right]+\left[\vec{a}\,\vec{b}\,\vec{d}\right]$

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4. If $\vec{b}$ and $\vec{c}$ be any two non-collinear vectors, and $\vec{a}$ be any vector then $\left(\vec{a}.\vec{b}\right)\vec{b}+\left(\vec{a}.\vec{c}\right)\vec{c}+\frac{\vec{a}.\left(\vec{b}\times\vec{c}\right)}{\left|\vec{b}\times\vec{c}\right|^2}\left(\vec{b}\times\vec{c}\right)$ is equal to

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5. Let $\hat{x}, \hat{y}$ and $\hat{z}$ be unit vectors such that $\hat{x} + \hat{y} + \hat{z} = a. \hat{x} \times (\hat{y} \times \hat{z}) = b, (\hat{x} \times \hat{y}) \times \hat{z} = c, a \cdot \hat{x} = \frac{3}{2}, a \cdot \hat{y} = \frac{7}{4}$ a

. Find x, y and z in terms of a, b and c.

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6. $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors ,prove that $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\vec{a} & \vec{a}\vec{b} & \vec{a}\vec{c} \\ \vec{b}\vec{a} & \vec{b}\vec{b} & \vec{b}\vec{a} \end{vmatrix} = 0$

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7. If $\vec{e}_1, \vec{e}_2, \vec{e}_3 and \vec{E}_1, \vec{E}_2, \vec{E}_3$ are two sets of vectors such that $\vec{e}_i \dot{\vec{E}}_j = 1$, if $i = j and \vec{e}_i \dot{\vec{E}}_j = 0 and$ if $i \neq j$, then prove that $\left[\vec{e}_1 \vec{e}_2 \vec{e}_3\right]\left[\vec{E}_1 \vec{E}_2 \vec{E}_3\right] = 1.$

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Exercise

1. If three concurrent edges of a parallelopiped of volume V represent vectors $\vec{a}, \vec{b}, \vec{c}$ then the volume of the parallelopiped whose three concurrent edges are the three concurrent diagonals of the three faces of the given parallelopiped is

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2. The position vectors of the points A,B,C , D are respectively $2\vec{i}+\vec{j}-\vec{k}, \vec{i}+\vec{j}+\vec{k}, \vec{i}-2\vec{j}+3\vec{k}$ and $3\vec{i}-\vec{j}+2\vec{k}$. Evaluate $\left[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}\right]$.

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3. If the three vectors $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.

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4. Prove that $\left(\vec{a}-\vec{b}\right)\left(\vec{b}-\vec{c}\right)\dot{\times}\left(\vec{c}-\vec{a}\right)=0$

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5. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k};\ \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k};\ \vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}\ \&\ \vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2=$

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6. If $\vec{a},\vec{b},\vec{c},\vec{a}',\vec{b}',\vec{c}'$, are two sets of non-coplanar vectors such that $\vec{a}.\vec{a}'=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'=1$, then the two systems are called Reciprocal System of vectors and $\bar{a}'=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]},\ \vec{b}'=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$ and $\vec{c}'=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$ Find the value of $\vec{a}\times\vec{a}'+\vec{b}\times\vec{b}'+\vec{c}\times\vec{c}'$.

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7. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors and let $\vec{p}, \vec{q}, \vec{r}$ be the vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$ Then the value of $\left(\vec{a}\right.$

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8. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $\left|\vec{a}\right| = 3, |b| = 4, \left|\vec{c}\right| = 5$ Find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

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9. Prove that $\hat{i} \times \left(\vec{a} \times \hat{i}\right)\hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a} \cdot$

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10. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a} \cdot \vec{b}\right)\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + \left(\beta^2 - 1\right)\vec{c}$ and being non-collinear then

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11. Let $\vec{a}$ be a unit vector and $\vec{b}$ a non-zero vector not parallel to $\vec{a}$ The angles of the triangle, two of whose sides are represented by $\sqrt{3}\left(\vec{a} \times \vec{b}\right)$ and $\left(\vec{b} - \left(\vec{a}\,.\,\vec{b}\right)\vec{a}\right.$ are

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12. For any vector $\vec{a}$, prove that $\left|\vec{a} \times \hat{i}\right|^2 + \left|\vec{a} \times \hat{j}\right|^2 + \left|\vec{a} \times \hat{k}\right|^2 = 2\left|\vec{a}\right|^2$

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13. If vectors $b, candd$ are not coplanar, then prove that vector $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{c} \times \vec{d}\right) + \left(\vec{a} \times \vec{c}\right) \times \left(\vec{d x x} \vec{b}\right) + \left(\vec{a} \times \vec{d}\right) \times \left(\vec{b}\right.$

is parallel to $\vec{a}$.

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14. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three conterminuous edges of a parallelopiped of the volume 6 then find the value of $\left[\vec{a} \times \vec{b} \quad \vec{a} \times \vec{c} \quad \vec{b} \times \vec{c}\right]$.

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15. Let $\vec{a}$, $\vec{b}$, $and \vec{c}$ be non-coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, find scalars $p, qandr$ in terms of θ.

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16. If $\overrightarrow{a} = \overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$, $\overrightarrow{b} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and $\overrightarrow{c} = \overrightarrow{i} + 2\overrightarrow{j} - \overrightarrow{k}$, then the value of $\begin{vmatrix} \overrightarrow{a}.\overrightarrow{a} & \overrightarrow{a}.\overrightarrow{b} & \overrightarrow{a}.\overrightarrow{c} \\ \overrightarrow{b}.\overrightarrow{a} & \overrightarrow{b}.\overrightarrow{b} & \overrightarrow{b}.\overrightarrow{c} \\ \overrightarrow{c}.\overrightarrow{a} & \overrightarrow{c}.\overrightarrow{b} & \overrightarrow{c}.\overrightarrow{c} \end{vmatrix}$ is equal to: (1) 2 (2) 4 (3) 16 (4) 64

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17. If the volume of a parallelopiped, whose three coterminous edges are

$-12\overrightarrow{i} + \lambda\overrightarrow{k}$;

$3\overrightarrow{j} - \overrightarrow{k}$ and $2\overrightarrow{i} + \overrightarrow{j} - 15\overrightarrow{k}$, is 546 then λ=______.

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18. Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\overrightarrow{c} = 3\hat{i} - \hat{j} - 2\hat{k}$

$\vec{a} = 11\hat{i}, \vec{b} = 2\hat{j} - \hat{k}, \vec{c} = 13\hat{k}$

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

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19. The points $(3, 6, 9), (1, 2, 3), (2, 3, 4)$ and $(4, 6, \lambda)$ are coplanar if λ =_______.

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20. If $\vec{x}\dot{\vec{a}} = 0, \vec{x}\dot{\vec{b}} = 0 and \vec{x}\dot{\vec{c}} = 0$ for some non-zero vector $\vec{x}$, then prove that $\left[\vec{a}\vec{b}\vec{c}\right] = 0.$

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21. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes $1, 1$ and 2 respectively. If $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = \vec{0}$, the acute angle between $\vec{a}$ and $\vec{c}$ is

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22. The scalar $\vec{A}\vec{B}\dot{+}\vec{C}\times\left(\vec{A}+\vec{B}+\vec{C}\right)$ equals 0 b. $\left[\vec{A}\vec{B}\vec{C}\right]+\left[\vec{B}\vec{C}\vec{A}\right]$ c. $\left[\vec{A}\vec{B}\vec{C}\right]$ d. none of these

A. 0

B. $\left[\vec{A}\quad\vec{B}\quad\vec{C}\right]+$ [[vec B, vec C, vec A]]

C. $\left[\vec{A}\vec{B}\vec{C}\right]$

D. none of these

Answer:

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23. Let a,b,c be distinct non zero numbers. If the vectors $a\vec{i}+a\vec{j}+c\vec{k}$, $\vec{i}+\vec{k}$ and $c\vec{i}+c\vec{j}+b\vec{k}$ lie in a plane then 'c' is

A. the AM of a and b

B. the GM of a and b

C. the HM of a and b

D. equal to zero

Answer: B

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24. The vectors $\vec{a} \times \left(\vec{b} \times \vec{c}\right), \vec{b} \times \left(\vec{c} \times \vec{a}\right)$ and $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are

A. unit vector

B. null vector

C. vector of magnitude $3\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right|$

D. none of these

Answer:

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25. $\overrightarrow{p}, \overrightarrow{q}, and \overrightarrow{r}$ are three mutually perpendicular vectors of the same magnitude. If vector $\overrightarrow{x}$ satisfies the equation $\overrightarrow{p} \times \left(\left(\overrightarrow{x} - \overrightarrow{q}\right) \times \overrightarrow{p}\right) + \overrightarrow{q} \times \left(\left(\overrightarrow{x} - \overrightarrow{r}\right) \times \overrightarrow{q}\right) + \overrightarrow{r} \times \left(\left(\overrightarrow{x} - \overrightarrow{p}\right) \times$ is given by $\frac{1}{2}\left(\overrightarrow{p} + \overrightarrow{q} - 2\overrightarrow{r}\right)$ b. $\frac{1}{2}\left(\overrightarrow{p} + \overrightarrow{q} + \overrightarrow{r}\right)$ c. $\frac{1}{3}\left(\overrightarrow{p} + \overrightarrow{q} + \overrightarrow{r}\right)$ d. $\frac{1}{3}\left(2\overrightarrow{p} + \overrightarrow{q} - \overrightarrow{r}\right)$

A. $\frac{1}{2}\left(\overrightarrow{p} + \overrightarrow{q} - 2\overrightarrow{r}\right)$

B. $\frac{1}{2}\left(\overrightarrow{p} + \overrightarrow{q} + \overrightarrow{r}\right)$

C. $\frac{1}{3}\left(\overrightarrow{p} + \overrightarrow{q} + \overrightarrow{r}\right)$

D. $\frac{1}{3}\left(2\overrightarrow{p} + \overrightarrow{q} - \overrightarrow{r}\right)$

Answer:

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26. In each of the following, one or more options ar correct. Choose the correct option(s). If $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ represent three concurrent edges of a rectangular parallelepiped whose lengths are 4,3,2 respectively then the value of $\left(\vec{a}+\vec{b}+\vec{c}\right)\cdot\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$ is

A. 0

B. 48

C. 72

D. none of these

Answer: C

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27. If $\vec{a}=\vec{p}+\vec{q}, \vec{p}\times\vec{b}=0 and \vec{q}\dot{\vec{b}}=0,$ then prove that $\frac{\vec{b}\times\left(\vec{a}\times\vec{b}\right)}{\vec{b}\dot{\vec{b}}}=\vec{q}.$

A. $\vec{q}$

B. $\vec{q}$

C. $\vec{p} \times \vec{q}$

D. none of these

Answer:

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28. Given $\left|\vec{a}\right| = \left|\vec{b}\right| = 1 and \left|\vec{a} + \vec{b}\right| = 3.$ If $\vec{c}$ is a vector such that $\vec{c} - \vec{a} - 2\vec{b} = 3\left(\vec{a} \times \vec{b}\right)$, then find the value of $\cdot \vec{b}$.

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