

# MATHS

# **BOOKS - PRADEEP PUBLICATION**

# **APPLICATIONS OF DERIVATIVES**

#### Example

**1.** A balloon, which always remains spherical, has a variable diameter  $\frac{3}{2}(2x+1)$ . Find the rate of change of its volume with respect to x.

**2.** The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long?

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3. The radius of a circle is increasing at the rate of 0.7 cm/s. What

is the rate of increase of its circumference ?



**4.** A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing?

5. A circular disc of radius 3cm is being heated. Due to expansion, its radius increases at the rate of  $0.05c\frac{m}{s}$ . Find the rate at which its area is increasing when radius is 3.2cm.

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6. Find the rate of change of the area of a circle per second with

respect to its radius r when r = 5 cm.

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7. The volume of a balloon is changing at the rate of 25  ${
m cm}^3/{
m sec.}$ 

Find the rate of change of its surface area when its radius is 5 cm.

**8.** The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

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**9.** If x and y are the sides of two squares such that  $y = x - x^2$ ,

find the rate of the change of the area of the second square with

respect to the first square.

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**10.** Find the point on the curve  $y^2 = 8x$  for which the abscissa and ordiante change at the same rate.

**11.** A man 2m high walks at a uniform speed of 6k/h away from a lamp ost 6 metres high. Find the rate at which the length of his shadow increases.

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**12.** Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?



**13.** An inverted conical vessel whose height is 10 cm and the radisu of whse base is 5 cm is being filled with water at the uniform rate of 1.5 cu cm/m. Find the rate at which the level of water in the vessel is rising when the depth is 4 cm.

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**14.** The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.



**15.** The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per

minute, when x = 8 cm and y = 6 cm, find the rate of change of the

perimeter of the rectangle.

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<b>16.</b> Without using the derivative, prove that $f(X) = 2x + 5$ is increasing for all $x \in R$
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<b>17.</b> Prove that the function $f(X) = ax + b$ is increasing iff $a > 0$ .
<b>Vatch Video Solution</b>

**18.** Prove that 
$$f(X) = \frac{1}{x}$$
 is decreasing for all x >0, without using the

derviative.



**22.** Show that the function  $f(x) = x^2 - 6x + 3$  is increasing in

the interval [4, 6]





function of x. throughout its domain.

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**28.** Prove that the function  $f(x) = \tan x - 4x$  is strictly decreasing on

$$\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$$

**29.** Prove that  $y = 4 \frac{\sin \theta}{2 + \cos \theta} - \theta$ , is an increasing function of  $\theta$  in  $\left[0, \frac{\pi}{2}\right]$ .

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**30.** Prove that the function f given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$ .

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**31.** Prove that the function  $x^2 + x + 1$  is neither increasing nor

decreasing on (-1,0)



**32.** Find the values of x for which the function  $f(x) = x^2 - 6x + 9$  is increasing or decreasing.

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**33.** Show that the following functions are increasing in the indicated intervals

$$f(x)=x^2-8x, x\geq 4$$

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**34.** Show that the following functions are increasing in the indicated intervals

$$f(x) = x^3 - 27x, x < -3 \, \, {
m or} \, \, x > 3$$

35. Show that the following functions are increasing in the

indicated intervals

f(x) = 
$$rac{1}{1+x^2}$$
 ,  $x\leq 0$ 

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**36.** Show that the following functions are increasing in the indicated intervals

$$f(x) = \ - rac{x}{2} + \sin x, \ - rac{\pi}{3} < x < rac{\pi}{3}$$

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**37.** Show that the following functions are increasing in the indicated intervals

$$f(x) = rac{x}{\sin x}, 0 < x < rac{\pi}{2}$$

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**38.** Show that the following functions are increasing in the indicated intervals

$$f(x)=1-rac{1}{x}, x<0$$

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**39.** Show that the following functions are increasing in the indicated intervals

$$f(x) = x^5 + 20x^3 + 25x, x \in R$$



**40.** Show that the following functions are increasing in the indicated intervals





**41.** Determine the intervals in which the following functions  $f(x) = x^2 - 4x + 6$  are strictly increasing or strictly decreasing.

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42. Find the interval in which the following function is decreasing

$$f(x) = 2x^3 - 15x^2 + 36x + 1.$$

:



$$rac{x}{2}+rac{2}{x}$$

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**44.** Find the intervals in which the following functions are increasing or decreasing :

$$rac{x}{x^2+1}$$

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**45.** Find the intervals in which the following functions are increasing or decreasing :

 $\sin 3x, 0 \leq x \leq \frac{\pi}{2}$ 

 $5x^{3\,/\,2}-3x^{5\,/\,2}, x>0$ 

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**47.** Find the intervals in which the following functions are increasing or decreasing :

$$x^4-rac{1}{3}x^3$$



**48.** Find the intervals in which the following functions are strictly

increasing or decreasing:  $-2x^3 - 9x^2 - 12x + 1$ 



 $2x^3 - 9x^2 + 12x - 5$ 



**50.** Find the intervals in which the following functions are increasing or decreasing :

 $5x^3 - 15x^2 - 120x + 3$ 

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51. Find the intervals in which the following functions are strictly

increasing or decreasing:  $-2x^3 - 9x^2 - 12x + 1$ 



 $x^3 - 4x$ 



53. Find the intervals in which the following functions are strictly

increasing or strictly decreasing

$$x^3 - 12x^2 + 36x + 17$$



**54.** Find the intervals in which the following functions are increasing or decreasing :

$$x^3 - 9x^2 + 15x + 11$$

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**55.** Find the intervals in which the following functions are increasing or decreasing :

 $x^3 - 6x^2 + 9x + 15$ 

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**56.** Find the intervals in which the following functions are increasing or decreasing :

 $x^4 - 2x^2$ 

$$\cos^{-1}igg(rac{1-x^2}{1+x^2}igg)$$

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**58.** Find the intervals in which the following functions are increasing or decreasing :

 ${(x+1)}^{3}{(x-3)}^{3}$ 

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**59.** Find the intervals in which the following functions are increasing or decreasing :

$$rac{3}{10}x^4 - rac{4}{5}x^3 - 3x^2 + rac{36}{5}x + 11$$



 $\sin x + \cos x, 0 \le x \le 2\pi$ 

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**61.** Find the slope of the normal to the curve  $y = x^3 - xatx = 2$ 



62. Find the equation of the tangent and normal to the curve

$$rac{X^2}{a^2}-rac{y^2}{b^2}=1$$
 at the point  $ig(\sqrt{2}a,big)$ 



**64.** Find the points on the curve  $4x^2 + 9y^2 = 1$ ,where the tangents are perpendicular to the line 2y+x=0

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65. Find the equation of the tangent line to the curve

$$x= heta+\sin heta,y=1+\cos heta$$
 at  $heta=rac{\pi}{4}$ 

**66.** Find the point on the curve  $y = x^2$  where the slope of the tangent is equal to the x-coordinate of the point.



**69.** At what points on the curve  $y = x^2 - 4x + 5 = 0$  is the tangent perpendicular to line 2y+x=7?



72. Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line x + 14y + 4 = 0

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73. Find the equation of the tangent line to the curve  $y = \sqrt{5x - 3} - 2$  which is paralel to the line 4x - 2y + 3 = 0

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74. Find the point at which the tangent to the curve  $y = \sqrt{4x - 3} - 1$  has its slope =  $\frac{2}{3}$ 

75. Find the equation of tangents to the curve $y=\cos(x+y),\ -2\pi\leq x\leq 2\pi$  that are parallel to the linex+2y=0.

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76. Find the co-ordinates of the points on the curvey  $y = 2x^2 + 3x + 18$ , the tangents at which pass through origin.

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77. For the curve  $y = 4x^3 - 2x^5$ , find all the points at which the

tangent passes through the origin.



**78.** Find the condition that the line x  $\cos lpha + y \sin lpha = p$  may be

a tangent to the curve 
$$rac{x^2}{a^2}+rac{y^2}{b^2}=1$$



**80.** Show that the normal at any point  $\theta$  to the curve

 $x=a\cos heta+a heta\sin heta, y=a\sin heta-a heta\cos heta$  is at a constant

distance from the origin.

**81.** Show that the curves  $y = a^x$  and  $y = b^x$ , a > b > 0 intersect

at an angle of 
$$\tan^{-1}\left(\left|\left(\frac{\log\left(\frac{a}{b}\right)}{1+\log a \log b}\right)\right|\right)$$

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82. Find the angle of intersection of curves  $y^2=4ax$  and  $x^2=4by, ab
eq 0$ 

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**83.** Show that the curves  $x^2 + y^2 - 2x = 0$  and

 $x^2 + y^2 - 2y = 0$  cut orthogonally at the point (0,0).

**84.** Find the condition for the curves  $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$  and  $xy = c^2$  to intersect orthogonally. Watch Video Solution

85. Find the value of p for which the curves  $x^2 = 9p(9-y)$  and  $x^2 = p(y+1)$  cut each other at right

angles.

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86. Evaluate:

 $\sqrt{25.3}$ 

#### 87. Evaluate:

 $\sqrt{.037}$ 



**90.** Use differentials to approximate the cube root of 66.

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<b>91.</b> Find the approximate value of $(1.999)^5$
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<b>92.</b> Find an approximate value of $\left(25 ight)^{1/3}$ using differentials.
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<b>93.</b> Find the value of $\log_{10}$ (10.1), given that $\log_{10}$ e = 0.4343.
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**94.** Given sin  $30^{\circ} = 0.5$  and  $\cos 30^{\circ} = 0.866$ , find an approximate value of sin  $31^{\circ}$ .

**95.** If  $y = x^3 - 4x$  and x chagnes form 2 to 1.99, find the approximate change in the value of y.

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**96.** If  $f(x) = 3x^2 + 5x + 3$ , find an approximate value of (3.02)



97. If the radius of a circle increases from 5 cm to 5.1 cm, find the

increase in area.



**98.** If the error committed in measuring the radius of a circle is 0.01 %, find the corresponding error in calculating the area.

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**99.** Show that the relative error in the volue of a sphere due to an error in measurement of radius is thhree times the relative error in radius.



**100.** If in a  $\triangle ABC$ , the side c and the angle C remain constant, while the remaining elements are changed slightly. Using differential, show that  $\frac{da}{\cos A} + \frac{db}{\cos B} = 0$ .



101. Find the maximum and minimum values of the function

 $f(x) = 3 - 2\sin x$ 

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102. Find the maximum and minimum values of the function

 $f(x) = a \sin x + b \cos x$ 



103. Find the maximum and minimum values of the function

$$f(x) = 2 + x - x^2$$

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104. Find the maximum and minimum values of the function

 $f(x) = \sin(\sin x)$ 

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105. Find the maximum and minimum values of the function

$$f(x) = |\sin 4x + 3|$$
106. Find the local maximum and local minimum values of the

function

$$f(x)=\cos^4x+\sin^4x$$
, (0,pi/2)

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107. Find the maximum and minimum values of the function

$$f(x)=x+rac{4}{x},x<0$$

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108. Show that the function  $f(x) = x^3 + x^2 + x + 1$  has neither

a maximum value nor a minimum value.



**109.** Find the absolute maximum and minimum values of the function.

$$f(x)=rac{x}{x^2+1},\;-3\leq x\leq 3$$

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**110.** Find the absolute maximum and minimum values of the function.

$$f(x) = 12 x^{4/3} - 6 x^{1/3} \in (\,-1,1)$$

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**111.** Find the absolute maximum and minimum values of the function.

$$f(x)=3x^4-8x^3+12x^2-48x+25\in(0,3)$$

**112.** Find the absolute maximum and minimum values of the function.

$$2x^3 - 15x^2 + 36x + 1 \in (1,5)$$

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113. Find the (absolute) maximum and minimum values of

$$f(x)= -4\sin x+2x\mathrm{in}\Big(0,rac{\pi}{2}\Big)$$

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114. Find the absolute maximum and minimum values of

 $f(x)=x+\sin 2xon(0,2\pi)$  and also find the points of (absolute

maximum and minimum.



**115.** Find the absolute maximum and minimum values of the function.

 $f(x)=2\cos 2x-\cos 4x\in (0,\pi)$ 

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**116.** Find the absolute maximum and minimum values of the function.

$$f(x)=\sin^2x-\cos x, x\in (0,\pi)$$

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**117.** Find the maximum and minimum values of the function.

$$f(x)=\sec x+\log\cos^2 x,\,\,\in(0,\,2\pi)$$



**118.** Find the points of local maxima and minima, if any, of he

function :

$$f(x) = \ - \left(rac{3}{4}
ight) x^4 - 8 x^3 - \left(rac{45}{2}
ight) x^2 + 105$$

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119. Find local minimum value of the function f given by  $f(x)=3+|x|, x\in R$ 

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120. Find the local maximum and minima of the function  $f(x)=2x^3-21x^2+36x-20$ 

**121.** Find all points of local maxima and minima and the corresponding maximum and minimum values of the function given below:

$$f(x) = \ - \ rac{3}{4} x^4 + 2 x^3 + rac{9}{2} x^2 + 100$$

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122. Prove that the function  $f(x)=2x^3-6x^2+6x+5$  has no

maximum or minimum.



123. Show that a local minimum value of  $f(x)=x+rac{1}{x}, x
eq 0$  is

greater than a local maximum value.



124. Find the local maximum and local minimum of  $f(x) = \sin 2x - x, \ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ 

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**125.** Find the local maximum and local minimum of  $f(x) = 2\sin x - x \ln \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$ Watch Video Solution

126. Find the points of local maximum and local minimum, if any, of the function:  $f(x) = \sin x - \cos x$ ,  $0 < x < 2\pi$ . Also find the local maximum and local minimum values. 127. Find the points of local maxima and minima of the function

$$f(x) = 3x^4 - 4x^3 + 5 {\rm in}(\,-1,2)$$



128. Discuss local maxima and minima of the function  $f(x) = \left(4-x^2
ight)^{2/3}$ 

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129. Discuss local maxima and minima of the function  $f(x)=x^x, x>0$ 

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130. If  $y = a \log x + bx^2 + x$  has its extreme values at x = -1 and x

= 2, then find a and b.



where a,b>0.



132.  $y = rac{ax-b}{(x-1)(x-4)}$  has a turning point P (2,-1). Find the

values of 'a' and 'b' and show that y is maximum at P.

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133. Prove that for x > -c, the minimum value of f(x) =  $(a+x)\frac{b+x}{c+x}$  is  $2\sqrt{(c-a)(c-b)}$  + a + b - 2c , given(c-a)(c-b) > 0.

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134. Find the points of local maxima and minima of the function

 $f(x) = 4 \sin x + \cos 2 x x \in (0, 2 \pi)$  Also find absolute maxima

and minimum values.

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135. Show that f(x)  $= x^5 - 5x^4 + 5x^3 - 1$  has a local maximum

at x = 1, a local minimum at x = 3 and neither at x = 0

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**136.** How should we choose two numbers, each greater than or equal to -2 whose sum is  $\frac{1}{2}$  so that the sum of square of the first and cube of the second is minimum?



**137.** Find two positive numbers whose sum is 24 and whose product is maximum.



138. Find two positive numbers whose sum is 15 and the sum of

whose squares is minimum.



139. Find two numbers whose sum is 15 and the square of one

multiplied by the cube of the other is maximum.

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<b>140.</b> Show that of all rectangles with given perimeter square has maximum area
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<b>141.</b> Show that, of all the rectangles with a given area, the square

has the smallest perimeter.



**142.** Use the functio  $f(x) = x^{1/x}$ , x > 0 to determine the greater

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of e^i and \pi^e
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**143.** Find the area of the largest isosceles triangle whose perimeter is 36 untis.

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144. Two sides of a triangle are a and b. Find the angle between

them such that area shall be maximum

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**147.** Prove that the area of right-angled triangle of given

hypotenuse is maximum when the triangle is isosceles.

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**148.** A wire of length 28 cm is to be cut off into two pieces. One piece is to be made into a circle and other into a square. What should be the lengths of two pieces so that combined area of circle and square is minimum.

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**149.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.



**150.** Show that the right circular cone of least curved surface and given volume has an altitude equal to  $\sqrt{2}$  time the radius of the base.



**151.** A figure consists of a semi-circle with a rectangle on its diameter. Given perimeter of the figure, find the dimensions in order that the area may be maximum.



**152.** Show that the height of the circular cylinder of maximum volume that can be inscribed in a given right-circular cone of height h is  $\frac{1}{3}$ h

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**153.** Show that the curved surface of a right circualr cylinder inscribed in a right circular cone is maximum when the radius of

its base is half that of the cone.



**154.** A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each comer and folding up the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum also find the volume ?



**155.** An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre. rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.



**156.** A cylinderical can is to be made to hold 1 litre of oil. Find the dimensions which will minimize the cost of the metal to make the can.

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**157.** Assuming that the petrol burnt per hour in driving a motor boat varies as the cube of its velocity, show that the most economical speed when going against a current of c km/hour is  $\frac{3c}{2}$  km/hour.

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**158.** Find the area of the greatest isocels triangle that can be inscried in a given ellipse having its vertex coincident with one extremly of major axis.

159. Find the area of the greatest rectangle that can be inscribed

in an ellipse 
$$rac{x^2}{a^2}+rac{y^2}{b^2}=1$$

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**160.** A telephone company in a town has 500 subsribers on its list and collects fixed charges of Rs. 300/- per year form each subsriber. The company proposes to increase the annual charges and it is believed that for every increases of Rs. 1/0, one subsriber will discontinue service. Find what increase will bring maximum

revenue?



**161.** Manufacturer can sell x items at a price of rupees  $Rs\left(5 - \left(\frac{x}{100}\right)\right)$  each. The cost price of x items is  $Rs\left(\left(\frac{x}{5}\right) + 500\right)$ . Find the number of items he should sell to earn maximum profit.



**162.** AB is a diameter of a circle and C is any point on the cirle. Show that the area of triangle ABC is maximum, when it is isosceles. **163.** The sum of the surface areas of a rectangualr parallelopiped with sides x, 2x and  $\frac{x}{3}$  and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.



164. An Apache helicopter of enemy is flying along the curve given by  $y = x^2 + 7$ . A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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2. The total cost C (x) and the total revenue R (X) associated with the production and sale of x units of an item are given by  $C(x) = 0.1x^2 + 30x + 1000$  and  $R(x) = 0.2x^2 + 36x - 100$ 

find the marginal cost and the marginal revenue when x = 20.



**3.** The total revenue in Rupees received from the sale of x units of a product is givne by  $R(x) = 3x^2 + 36x + 5$ . Find the marginal revenue, when x = 6, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.



**4.** A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing?



5. Find the rate of change of the volume V of a sphere of radius r

w.r.t change in the radius.



**6.** The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the surface area of the balloon increasing when its radius is 15 cm ?



**7.** The radius of a spherical soap bubble is increasing at the rate of 0.2 cm/sec. Find the rate of increase of its volume when the radius is 4 cm.



8. A ballon, which always remains spherical has a variable diameter  $\frac{3}{2}(2x+3)$ . Determine the rate of change of volume w.r.t.x.

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9. The volume of a spherical balloon is increasing at the rate of

 $20c \frac{m^3}{\text{sec}}$ . Find the rate at which its surface area is increasing at

the instant when its radius is 4 cm.

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**10.** An edge of a variable cube is increasing at the rate of 3 cm second. How fast is the volume of the cube increasing when the edge is 10 cm long?



11. A cone has a fixed radius r and a variable height h. Find the

rate of change of its curved surface S w.r.t change in the radius.

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12. A right-circular cone has a fixed radius r and a variable height

h. Find the rate of change of its curved surface S w.r.t. change in the height.

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**13.** A solid right circular cylinder has its radius equal to half of its height. Find the rate of change of its total surface area w.r.t. a change in the radius r.



**14.** From a cylindrical drum containing oil and kept vertical, the oil leaking at the rate of  $10cm^3/s$ . If the radius of the durm is 10 cm and height is 50 cm, then find the rate at which level of oil is changing when oil level is 20cm.



**15.** Two men A and B start with velocities v at the same time from the junction of two roads inclined at  $45^{\circ}$  to each other. If they travel by different roads, find the rate at which they are being separated.



**16.** A kite is moving horizontally at a height of 141.5 m. If the speed of kite is 10 m/s, how fast is the string being let out when the distance (direct) between the kite and the boy (who is flying it) is 250 m? The height of the boy is 1.5 m.

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17. For what x, sinx increases half as fast as x?



18. A particle moves along the curve  $6y = x^3 + 2$ . Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.



**19.** A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1m/sec. How fast is the length of his shadow increasing when he is 1 m away from the pole ?

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**20.** A man, 2 m tall, walks at the rate of  $1\frac{2}{3}$  m/sec towards a street light which is  $5\frac{1}{3}$  m above the ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is  $3\frac{1}{3}$  m from the light.

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**21.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate

of 3 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 3 m away from the wall?

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**22.** Water is leaking from a conical funnel at a uniform rate of  $4cm^3/\sec$  thorugh at small hole at the vertex. When the slant height of the water in the funnel is 3 cm, find the rate of decrease of the slant height of water given that semi vertical angle of the cone (funnel) is  $30^{\circ}$ .

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**23.** Water is leaking from a conical funnel of semi-vertical angle  $\frac{\pi}{4}$  at a uniform rate of  $2cm^2/\sec$  in the surface area curved), through a tiny hole at the vertex of the bottom. When the slant

height of the cose is 4 cm, find the rate of decrease of the slant

height of water.



**24.** Find the marginal cost and the marginal revenue when x = 10, the total cost C(x) and the total revenue R (x) in Rupees, associated with production and sale of x units of an item are given by

$$C(x) = 0.003x^3 + 0.02x^2 + 20x + 1200R(x) = 0.1x^2 + 25x - 100x^2$$

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25. The total cost C(x) in Rupees, associated with the production of x units of an item is given by  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$  Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

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**26.** The radius of a cylinder is increasing at the rate of 2m/sec and its height is decreasintg at the rate of 3 m/sec. Find the rate at which the volume of the cylinder is changing when the radius is 3 m and height is 5 m.



**27.** An airforce plane is ascending vertically at the rate of  $100 \frac{km}{h}$ . If the radius of the earth is r km, how fast is the area of the earth, visible from the plane, increasing at 3 minutes after it started ascending? Given that the visible area A at height h is given by

$$A=2\pi r^2rac{h}{r+h}$$

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**28.** A spherical ball of salt is dissovlying water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radis is decreasing at a constant rate.



**29.** If the area of a circle increases at a uniform rate, then prove

that the perimeter varies inversely as the radius.



**30.** A swimming pool is to be drained for cleaning. If L represents the number of litres of water in the pool t seconds after the pool has been plugged off to drain and  $L = 200(10 - t)^2$ . How fast is the water running cost at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?



**31.** Prove that the function f(X) = ax + b is increasing iff a > 0.

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**32.** Show that the function f given by f(X) = 7x - 3 is strictly

increasing on R.

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**33.** Show that the function  $f(x) = rac{x}{1+|x|}$  is a strict increasing

function.

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**34.** Show that the function  $f(x) = x - \frac{1}{x}$  is a strict increasing

function fox x > 0

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35. Show that 
$$f(x)=2x+\cot^{-1}x+\log\Bigl(\sqrt{1+x^2}-x\Bigr)$$
 is

increasing in R.



**36.** Show that for a  $\geq$  1  $f(x) = \sqrt{3} \sin x - \cos x - 2ax + b$  is

decreasing in R.


**39.** Show that the function f(X)  $= (x^2 - 2x + 2)e^x + 3$  is an increasing function for all  $x \in R$ .

• Watch Video Solution 40. Show that the function  $x^2 - x + 1$  is neither increasing nor decreasing on (0,1). • Watch Video Solution

**41.** Indicate the interval in which the function f(X)

 $= \cos x, 0 \leq x \leq 2\pi$ , decreases.

**42.** Prove that the function f(X) =  $\cos x$  is strictly decreasing  $(0, \pi)$ 

and strictly increasing on  $(\pi, 2\pi)$ 

**43.** Determine whether the following function is increasing or  
decreasing in the given interval :  
$$f(X) = \cos\left(2x + \frac{\pi}{4}\right), \frac{3\pi}{8} \le x \le \frac{5\pi}{8}.$$
  
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**44.** Prove that the function  $f(x) = \tan x - 4x$  is strictly decreasing on

$$\left(-\frac{\pi}{3},\frac{\pi}{3}
ight)$$

**45.** Find for which values of 'x', the functions:  $y = x^4 - rac{4x^3}{3}$  is

increasing and for which values, it is decreasing.



**47.** Determine the values of x for which the function  $x + \frac{1}{x}$  is

strictly increasing or decreasing



**48.** Determine the values of x for which the function  $2x^3 - 24x + 5$  is increasing or decreasing.





51. Determine the intervals in which the function  $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$  is decreasing or increasing.

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**52.** Find the intervals in which the function  $f(X) = (x-1)^3(x-2)^2$  is increasing or decreasing.

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53. Find the intervals in which the function 'f' given by:  $f(x) = \sin x - \cos x, 0 \le x \le 2\pi$  is strictly increasing or decreasing.

**54.** Show that the function  $f(x)=(\sin 3x)$  is strict decreasing on  $\left(0, \frac{\pi}{2}\right)$ 

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**55.** Show that the function f given by 
$$f(x) = \tan^{-1}(\sin x + \cos x), x > 0$$
 is always an increasing function in  $f, \left(0, \frac{\pi}{4}\right)$ 

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56. Find the intervals of increase and decrease of the function

$$f(x) = rac{\log x}{x}.$$

**57.** Find the intervals in which the function is increasing or decreasing:

$$f(x)=\log(1+x)-rac{2x}{2+x}$$



**58.** Prove that f(X) = x - [x] is increasing in (0,1)

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**59.** Find the interval of monotonocity of the function  $f(X) = 2x^2 - \log |x|, x \neq 0$ `

**60.** Integrate the function: 
$$\sin^{-1}\left(rac{2x}{1+x^2}
ight)$$



**63.** Find the equations of the tangent and the normal to each of

the following curves at the given point

$$y=x^3at(2,8)$$

**64.** Find the equations of the tangent and the normal to each of

the following curves at the given point

$$y = x^3 at(1,1)$$

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**65.** Find the equations of the tangent and the normal to each of the following curves at the given point

$$y=x^4-6x^3+13x^2-10x+5$$
 at  $(0,5)$ 

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66. Find the equations of the tangent and the normal to each of

the following curves at the given point



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67. Find the equations of the tangent and the normal to each of

the following curves at the given point

$$y=x^4-6x^3+13x^2-10x+5$$
 at  $(1,3)$ 

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68. Find the equations of the tangent and the normal to the

following curves at the given point:

 $20x^4+y^4=36$  at  $(\,-1,\,-2)$ 



69. Find the equations of the tangent and the normal to the

following curves at the given point:

$$9x^2+16y^2=144at(x_1.\ y_1)$$
 where  $y_1=2, x_1>0$ 



**70.** Find the equations of the tangent and the normal to the

following curves at the given point:

$$y=\sin^2 x$$
 at  $x=rac{\pi}{2}$ 

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**71.** Find the equations of the tangent and the normal to the

following curves at the given point:

$$y=rac{x^3}{4-x}at(2,4)$$

72. Find the equations of the tangent and the normal to the

following curves at the given point:

$$x=\cos t,y=\sin t$$
 at  $t=rac{\pi}{4}$ 

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**73.** Find the equations of the tangent and the normal to the following curves at the given point:

$$x=a\cos t,y=b\sin t$$
 at  $t=rac{\pi}{2}$ 

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**74.** Find the equations of the tangent and the normal to the following curves at the given point:

$$x=a(t+\sin t),y=a(1-\cos t)$$
 at  $t=rac{\pi}{2}$ 

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**75.** Find the equations of the tangent and the normal to the following curves at the given point:

 $x=1+\cos heta, y= heta+\sin heta$  at  $heta=\pi/4$ 

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76. Find the equations of the tangent and the normal to the

following curves at the given point:

$$x=1-\cos heta, y= heta-\sin heta at heta=rac{\pi}{2}$$

77. Find the equations of the tangent and the normal to the

following curves at the given point:

$$rac{x^2}{a^2} - rac{y^2}{b^2} = 1 a t(x_0,y_0)$$

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78. Find the equations of the tangent and the normal to the

following curves at the given point:

 $x=\sin 3t, y=\cos 2t$  at t =  $rac{\pi}{4}$ 

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**79.** Find the equations of the tangent and the normal to the

following curves at the given point:

$$x^{2\,/\,3} + y^{2\,/\,3} = 2at(1,1)$$

**80.** Find the equations of the tangent to the following curves at

the given point:

$$x=a\sin^3t, y=b\cos^3tatt=rac{\pi}{4}$$

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81. Find the equation of the tangent to the curve $rac{X^2}{a^2}+rac{y^2}{b^2}=1at(x_0,y_0)$ 

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82. Find the equation of the tangent to the curve  $\sqrt{x} + \sqrt{y} = a$ 

at the point 
$$\left(rac{a^2}{4}, rac{a^2}{4}
ight)$$
 a > 0

83. Find the equations of the tangent and the normal to the

curve  $y = x^2 + 4x + 1$  at the point whose abscissa is 3.

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84. Show that the tangents to the curve  $y = 2x^3 - 3$  at the

points where x = 2 and x = -2 are parallel.



**85.** Prove that the tangents to the curve  $y = x^2 - 5x + 6$  at the

points (2,0) and (3,0) are at right-angles.

**86.** Find the point of intersection of the tangetn lines to the curve  $y = 2x^2$  at the points (1,2) and (-1,2)





**89.** Find the equation of the tangent to the curve  $x^2 + 3y = 3$ , which is parallel to the line y - 4x + 5 = 0

**90.** Find the point on the curve  $y = 2x^3 - 15x^2 + 36x - 21$  at which the tangent is parallel to x-axis. Also, find the equation of tangents.

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**91.** Find points on the curve  $rac{x^2}{4}+rac{y^2}{25}=1$ at which the tangents

are parallel to x-axis.

**92.** Find points on the curve  $rac{x^2}{4}+rac{y^2}{25}=1$  at which the tangents

are parallel to y-axis.



93. Find the equations of the tangents to the curve:  $y = x^3 + 2x - 4$ . Which are perpendicular to line x + 14y + 3 = 0

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**94.** Find the equations of the tangents to the curve:  $y = x^3 + 2x - 4$ . Which are perpendicular to line x + 14y + 3 = 0

**95.** Find a point on the curve  $y = (x - 3)^2$  where the tangent is parallel to the chord joining (3,0) and (4,1)



**98.** Show that the curve  $xy = a^2$  and  $x^2 + y^2 = 2a^2$  touch each

other.



touch each other at the point (1,2)

101. Find the equations of normal(s) to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$  at the point(s) whose absscissa is 2.

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**102.** Find the coordinates of the point on the curve  $\sqrt{x} + \sqrt{y} = 4$  at which tangent is equally inclined to the axes.

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**103.** Show that the equations of the normal at any point  $\theta$  on the

curve  $x=3\cos heta-\cos^3 heta, y=3\sin heta-\sin^3 heta$  is

$$4ig(y\cos^3 heta-x\sin^3 hetaig)=3\sin4 heta$$

104. Find the angle of intersection of the curves  $y = x^2$  and  $y = 4 - x^2$ .

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105. Show that the line  $rac{x}{a}+rac{y}{b}=1$  touches the curve  $y=be^{x/a}$ 

at the point where the curve intersect the axis of y.

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106. Find the equation of the tangent to the curve  $y=\sqrt{3x-2}$ 

which is parallel to the line 4x - 2y + 5 = 0.



**107.** Prove that the curves  $x=y^2$  and xy=k cut at right angles

if 
$$8k^2=1.$$

108. Find the condition that the curves  $2x=y^2$  and 2 xy = k may

intersect orthogonally.

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109. For the curve  $y = 5x - 2x^3$ , if x increases at te rate of

2units/s, then how fast is the slope of curve changing when x =3?

**110.** In the following problems, use differentials to find the approximate values:

 $\sqrt{0.39}$ 

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**111.** In the following problems, use differentials to find the approximate values:

 $\sqrt{25.02}$ 

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**112.** In the following problems, use differentials to find the approximate values:

 $(28)^{1/3}$ 

**113.** In the following problems, use differentials to find the approximate values:

 $(80)^{1/4}$ 

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**114.** In the following problems, use differentials to find the approximate values:

 $f(2.01)where f(x) = x^3 - 4x + 7$ 



**115.** In the following problems, use differentials to find the approximate values:

 $\log_{10}(1.01)$ ~ where  $\log_{10}e$  = 0.4343.



**116.** In the following problems, use differentials to find the approximate values:

 $\sin 30.5^{\,\circ}$  , when  $\sin 30^{\,\circ}\,=0.5\,$  and  $\,\cos 30^{\,\circ}\,=0.866\,$ 

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117. If y = sin x and x change form 
$$\frac{\pi}{2} \rightarrow \frac{22}{14}$$
, what is the

approximate change in y?



**118.** A circular metal plate expands under heating so that its radius increases by 2%. Find the approximate increase in the area of he plate if the radius of the plate before heating is 10 cm.



**119.** If the side of a cube be increased by 0.1%. Find the corresponding increase in the volume of the cube.



120. Find the approximate change in the volume V of a cube of

side x meters caused by increasing the side by 2%



**121.** If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.



**122.** If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.



**123.** Find the maximum or minimum values of the functions.

$$f(x) = 16x^2 - 16x + 28$$

**124.** Find the maximum or minimum values of the functions.

$$f(x) = \sin x \cos x$$



125. Find the maximum or minimum values of the functions.

 $f(x)=\sin^4x+\cos^4x$ 

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126. Find the maximum or minimum values of the functions.

$$f(x)=\sin^4x+\cos^4x$$

127. Find the maximum or minimum values of the functions.

$$f(x) = \sin^6 x + \cos^6 x$$

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128. Find the maximum or minimum values of the functions.

 $f(x) = 3\sin x - 4\cos x$ 

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129. Find the maximum and the minimum values, if any, of the

following functions, without using the derivative:

$$x+rac{1}{x},x>0$$

**130.** Find the maximum and the minimum values, if any, of the following functions, without using the derivative:

 $11-7\sin x$ 



133. Determine the point of maximum of $f(x) = \sin x + \cos x \in 0 \leq x \leq rac{\pi}{2}$ 

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**134.** Prove that the following functions do not have maxima or minima.

 $e^{3x}$ 



135. Prove that the following functions do not have maxima or

minima.

 $2x^3 + 3x^2 + 6x + 1$ 



136. Prove that the following functions do not have maxima or

minima.

 $4x^3 - 18x^2 + 27x - 7$ 

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**137.** Prove that the following functions do not have maxima or minima.

 $\log(x+1).\,X>\,-1$ 

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**138.** Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$f(x) = x^3 - 3x^2 + 3x + 8 {\rm in}(0, 10)$$



**139.** Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$f(x)=(x-2)\sqrt{x-1}, 1\leq x\leq 9$$

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**140.** Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$f(x)=rac{x+1}{\sqrt{x^2+1}}, 0\leq x\leq 2$$

**141.** Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

 $f(x)=x+\sin x\in (0,2\pi)$ 



**142.** Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$f(x)=3+|x+1| ext{ in } (\,-2,3)$$

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**143.** Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:


145. At what point, the slope of the crve : $y = -x^3 + 3x^2 + 9x - 27$  is maximum? Also, find the

maximum slope.

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**146.** What is the maximum value of the function  $\sin x + \cos x$ ?

**147.** Prove that  $f(x) = \sin x + \sqrt{3} \cos x$  has maximum value at x

 $= \pi / 6$ 

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**148.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x)=(x-1)(x+2)^2$$



**149.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and

minimum values.

$$f(x) = (x-1)^3 (x+1)^2$$

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**150.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x)=\sin x(1+\cos x), 0< x<rac{\pi}{2}$$



**151.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x) = 18xe^{-x/3}$$

**152.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x) = (x-1)(x-2)(x-3)$$

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**153.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x)=rac{x^4}{x^4-1}, x
eq 1$$

**154.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x) = x \sqrt{32 - x^2} \in (\,-5,5)$$

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**155.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x) = rac{\log x}{x}$$

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**156.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and

minimum values.

$$f(x)=2\cos x+x\in (0,\pi)$$

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157. Find the maximum and minimum values.

$$f(x)=\sin^4x+\cos^4x, 0\leq x\leq rac{\pi}{2}$$

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**158.** Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$$f(X) = \ - \ rac{3}{4} x^4 - 8 x^3 - rac{45}{2} x^2 + 105$$

**159.** Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$f(x)=\sec x+\log\cos^2 x, 0\leq x\leq 2\pi, x
eq rac{\pi}{2}, rac{3\pi}{2}$$

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**160.** Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$



161. Show that a local minimum value of  $f(x) = x + rac{1}{x}, x 
eq 0$  is

greater than a local maximum value.

**162.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$ 

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**163.** Find the difference between the greatest and least values of the function  $f(x) = \sin 2x - x$ , on  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$ 

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164. It is given that at x = 1, the function  $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of



hypotenuse is maximum when the triangle is isosceles.

**168.** Find the area of the largest isosceles triangle having perimeter 18 metre.



**169.** A wire 10 meter long is cut into two parts. One part is bent into the shape of a circle and the other into the shape of an equilateral triangle. How should the wire be cut so that the combined area of the two figures is as small as possible?

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**170.** Show that a cylinder of given volume, open at the top, has minimum total surface area if its height is equal to radius of the

base.



that can be inscribed in a sphere of radius r.

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**173.** Show that the semi-vertical angle of the right-circular cone of

maximum volume and of given slant height is  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 

174. The combined resistance of two resistors  $R_1$  and  $R_{\circ}(R_1, R_2 > 0)$  is given by  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ . If  $R_1 + R_2 = C$ (a constant), show that maximum resistance R is obtained by choosing  $R_1 = R_2$ 



**175.** Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle  $\alpha$  is one-third that of the cone and the greatest volume of cylinder is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ 



176. Show that the surface area of a closed cuboid with surface

base and given volume is minimum when it is cube.



177. A metal box with a square box and vertical sides is to contain  $512c^3$  of volume. The material for the top and bottom costs Rs.  $5cm^2$  and that for the sides costs Rs.  $2.50cm^2$ . Find the least cost of the box.

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**178.** An isosceles triangle of vertical angle  $2\theta$  is inscribed in a circle of radius 'a' . Show that the area of triangle is maximum when  $\theta = \frac{\pi}{6}$ 

179. If the sum of the lengths of hypotenuse and a side of a rightangled triangle is given. Show that the area is maximum, when the angle between then is  $60^\circ$ 

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**180.** An open box with a square base is to be made out of a given iron sheet of area  $c^2$  square units. Show that the maximum volume of the box is  $\frac{c^3}{6\sqrt{3}}$  cubic units.



**181.** A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.Show that the minimum length of

the hypotenuse is  $\left(a^{rac{2}{3}}+b^{rac{2}{3}}
ight)^{rac{2}{3}}$ 



**182.** Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible. When revolved about one of its sides. Also find the maximum volume.

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**183.** If the sum of the surface areas of cube and a sphere is costnat, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum?

**184.** If the length of three sides of a trapezium other than base are equal to 10 cm, then find the area of trapezium when it is maximum.

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185. Find the shortest distance of the point (0,c) from the parabola  $y=x^2$  where  $0\leq c\leq 5$ 

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**186.** Let AP and BQ be two vertical poles at points A and B respectively. If AP=16 m , BQ=22 m and AB = 20m, then  $f \in dthedis \tan aceofap \oint \mathbb{R}$  on AB from the point A such that  $RP^2 + RQ^2$  is minimum.



**187.** A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

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**188.** Find the rate of change of area of a square when its side is increasing at the rate of 2 cm/min and the length of the side is 10 cm.

**189.** If the rate of change of volume of a sphere is numerically equal to that of the radius of the sphere, find the radius of the sphere.

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190. If the area of a circle increases at a uniform rate, then prove

that the perimeter varies inversely as the radius.

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**191.** Find a real number  $heta, 0 < heta < rac{\pi}{2}$ , which increases twice as

fast as its sine.



**192.** Find the rate of change of perimeter of a square when its side is increasing at the rate of 2 cm/min and the length of the side is 20 cm.



**193.** Find the rate of change of perimeter of a square when its side is increasing at the rate of 2 cm/min and the length of the side is 20 cm.



**194.** Find the rate of the change of surface area of a sphere when its radius is increasing at the rate of  $\frac{3}{\pi}$  cm/min and its radius is 1

cm.

**195.** The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

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**196.** A particle is moving in a straight line such that its displacement s at an instant of time t is given by the relation  $s = 3t^2 + 4t + 5$  Find the initial velocity of the particle.

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197. The rate of change of the volume of a sphere w.r.t. its surface

area, when the radius is 2cm, is

**198.** Prove that the function  $f(x) = \tan x - 4x$  is strictly decreasing on

$$\Big(-rac{\pi}{3},rac{\pi}{3}\Big)$$

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**199.** Find the interval in which the function  $f(X) = x^4 - rac{4x^3}{3}$  is

strictly increasing.

200. Show that the function f given by 
$$f(x)= an^{-1}(\sin x+\cos x), x>0$$
 is always an increasing function in  $f,\left(0,rac{\pi}{4}
ight)$ 

**201.** State whether the function  $f(x) = x^3$  is increasing or decreasing.

**202.** State whether the function 
$$f(x) = e^x$$
 is increasing or decreasing.

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**203.** Is the function  $y = \cot x$  decreasing on the interval  $(0, \pi)$ ?



**204.** Find the interval of increase of the function  $f(x) = \tan^{-1} x$ .



**207.** Find the slope of the tangent to the curve  $y = 3x^2$  at the

point 
$$\left(\frac{1}{6}, \frac{1}{12}\right)$$

**208.** For the curve  $y = 5x - 2x^3$ , if x increases at te rate of 2units/s, then how fast is the slope of curve changing when x =3? Watch Video Solution **209.** Find the slope of the normal to the curve  $y=x^3$  at the point whose abscissa is 2. Watch Video Solution

**210.** At what point on the curve  $y = x^2 - 2x$ , the tangent is parallel to x-axis.



**211.** At what point on the curve  $y = 2x^2 - 4x + 3$ , the normal is

parallel to Y-axis.



**214.** Find the change in the value of  $y=x^2$  when x = 10 and  $\Delta x=0.01$ 



**215.** Find the approximate volume of metal in a hollow spherical shel, whose internal and external radii are 3 cm and 3.0005 cm respectively.



**216.** Find the maximum value of sin x cos x.



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**219.** Write down the maximum value of a sin x + b cos x,  $x \in R$ .



**220.** Does the function  $f(x) = 3^x$  have any extreme point?

**221.** Using differentials, find the approximate value of  $\sqrt{0.082}$ 



**224.** Find the maximum value of  $f(X) = 2x^2 - x + 1$ 

**225.** If f(X) attains a local minimum value at x = c and f(c) > 0, then what can you say about f'(c)? Watch Video Solution **226.** If a function f(x) has a local extremum at a point x = c, then what can be said about f' (c)? Watch Video Solution **227.** If x>0, what is the minimum value of  $x + \frac{4}{7}$ Watch Video Solution

**228.** If x<0, what is the maximum value of  $x + rac{1}{x}$ 



**231.** Write down the minimum value of  $\sqrt{3}$  sin x + cos x`



232. Discuss the local maximum or minimum of  $f(x) = x^3 a t x = 0$ 

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**233.** The amount of pollution content added in air in a city due to x-d i e s e l vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$ . Find the arginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

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234. Find the rate of change of the area of a circle with respect to

its radius r when r = 3 cm.

235. Find the rate of change of the area of a circle with respect to

its radius r when r=4cm

Watch Video Solution

**236.** The volume of a cube is increasing at the rate of  $8c \frac{m^3}{s}$ . How

fast is the surface area increasing when the length of an edge is

12 cm?



**237.** The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

**238.** An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

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**239.** A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing?



**240.** The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?



**241.** The length 'x' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when x = 8 cm and y = 6 cm, find the rate of change of the perimeter of the rectangle.

### > Watch Video Solution

**242.** The length x of a rectangle is decreasign at the rate of 5 cm/min and the width y is incresaing at the rate of 4cm/min. when x = 8 cm and y = 6 cm, find the rates of change of the area of the rectangle.

**243.** A balloon which always remains spherical is being inflated by .pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.



**244.** A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.



**245.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

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**246.** A particle moves along the curve  $6y = x^3 + 2$ . Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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**247.** The radius of an air bubble is increasing at the rate of  $\frac{1}{2}c\frac{m}{s}$ . At what rate is the volume of the bubble increasing when the radius is 1 cm?
**248.** A balloon, which always remains spherical, has a variable diameter  $\frac{3}{2}(2x + 1)$ . Find the rate of change of its volume with respect to x.



**249.** Sand is pouring from a pipe at the rate of 12 cubic cm./sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm. ?



250. The total cost C(x) in Rupees associated with the production of x units of an item is given by  $C(x) = 0.007x^3 + 0.003x^2 + 15x + 4000$  Find the marginal cost when 17 units are produced.

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**251.** The total revenue in Rupees received from the sale of x units of a product is given by  $R(x) = 13x^2 + 26x + 15$  Find the marginal revenue when x = 7.

A. Choose the correct answer in the exercises 17 and 18.

Β.

C.

D.

Answer:



252. Find the rate of change of the area of a circle with respect to

its radius r at r = 6 cm

A.  $10\pi$ 

B.  $12\pi$ 

C.  $8\pi$ 

D.  $11\pi$ 

### Answer:

# Watch Video Solution

253. The total revenue in Rupees received from its sale of x units of a product is given by  $R(X)=3x^2+36x+5$ . Find the

## marginal revenue, when x = 15

A. 116

B.96

C. 90

D. 126

### Answer:

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**254.** Show that the function given by f(x) = 3x + 17 is increasing on R.



**255.** Show that the function given by  $f(x) = e^2 x$  is increasing on

R.



increasing nor decreasing in  $(0, \pi)$ .



**259.** Find the intervals in which the function f given by f(x)

 $=2x^2-3x$  is

A. strictly increasing

B. strictly decreasing.

C.

D.

### Answer:



260. Find the intervals in which the function f, given by  $f(x)=2x^3-3x^2-36x+7$  is strictly increasing and strictly

decreasing.

A. strictly increasing

B. strictly decreasing.

C.

D.

#### Answer:

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261. Find the intervals in which the following functions are strictly

increasing or decreasing:  $x^2 + 2x - 5$ 

**262.** Find the intervals in which the following functions are strictly increasing or decreasing:  $10 - 6x - 2x^2$ 



strictly increasing or decreasing:  $-2x^3 - 9x^2 - 12x + 1$ 

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**264.** Find the intervals in which the following functions are strictly increasing or decreasing:  $-2x^3 - 9x^2 - 12x + 1$ 

**265.** Find the intervals in which the following functions are strictly increasing or decreasing

 $(x+!)^3(x-3)^3$ 

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**266.** Show that 
$$y=\log(1+x)-2rac{x}{2+x}, x\succ 1$$
, is an

increasing function of x. throughout its domain.



**267.** Find the values of x for which  $y = \left[x(x-2)
ight]^2$  is an

increasing function.





270. Prove that the function f given by  $f(x) = x^2 - x + 1$  is

neither strictly increasing nor decreasing on (-1, 1).

271. Which of the following functions are strictly decreasing on

$$\left(0,\frac{\pi}{2}\right)$$
 ?

A. cos x

B. cox 2x

C. cos 3 x

D. tan x

Answer:

**Watch Video Solution** 

272. On which of the following intervals is the function f given by

 $f(x) = x^{100} + \sin x - 1$ decreasing ?

A. (0,1)

 $\mathsf{B}.\left(\frac{\pi}{2},\pi\right)$  $\mathsf{C}.\,0,\frac{\pi}{2}\right)$ 

D. none of these

### Answer:



**273.** Find the least value of a such that the function f given by

 $f(X)=x^2+ax+1$  is strictly increasing on (1,2)



**274.** Let 1 be any interval disjoint form (-1,1). Prove that the function  $f(x) = x + \frac{1}{x}$  is strictly increasing on I.

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**275.** Prove that the function f given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$ .



**278.** The interval in which  $y = x^2 e^{-x}$  is increasing is:

A. 
$$(\,-\infty,\infty)$$

B. (-2,0)

 $\mathsf{C}.\left(2,\infty
ight)$ 

D. (0,2)

Answer:

Watch Video Solution

**279.** Find the slope of the tangent to the curve  $y = 3x^4 - 4x$  at x

= 4.

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**280.** Find the slope of the tangent to the curve  $y = rac{x-1}{x-2}$  at x =

10.



**281.** Find the slope of the tangent to curve  $y = x^3 - x + 1$  at the

point whose x-coordinate is 2.



**284.** Find the slope of the nromal to the curve  $x = 1 + a \sin t, y = -b \cos^2 t a t t = \frac{\pi}{2}$ 

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**285.** Find points at which the tangent to the curve  $y = x^3 - 3x^2 - 9x + 7$  is parallel to the x-axis.

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**286.** Find a point on the curve  $y = \left(x-2
ight)^2$  at which the tangent

is parallel to the chord joining the points (2,0) and (4,4)

**287.** Find the point on the curve  $y = x^3 - 11x + 5$  at which the

tangent is y = x - 11

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288. Find the equation of all lines having slope – 1 that are tangents to the curve 
$$y = \frac{1}{x-1}, x \neq -1$$

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**289.** Find the equation of all lines having slope 2 which are tangent to the curve  $y = \frac{1}{x-3}, x \neq 3$ .



293. Find the equations of the tangent and normal to the given

curves at the indicated points:
$$y = x^4 - 6x^3 + 13x^2 - 10x + 5at(0,5)$$

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294. Find the equations of the tangent and normal to the given

curves at the indicated points: $y=x^4-6x^3+13x^2-10x+5at(1,3)$ 

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295. Find the equations of the tangent and normal to the given

curves at the points given  $y=x^3at(1,1)$ 

**296.** Find the equations of the tangent and normal to the given curves at the points given  $y = x^2 at(0,0)$ 



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**298.** Find the equation of the tangent line to the curve

 $y=x^2-2x+7$ , which is parallel to the line 2x-y+9=0.

299. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$  which is perpendicular to the line 5y - 15x = 13

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**300.** Show that the tangents to the curve  $y=7x^3+11$  at the points x = 2 and x =-2 are parallel.



**301.** Show that the tangents to the curve  $y = 2x^3 - 3$  at the

points where x = 2 and x = -2 are parallel.

**302.** Find the points on the curve  $y = x^3$  at which the slope of

the tangent is equal to the y-coordinate of the point.



**303.** For the curve  $y = 4x^3 - 2x^5$ , find all the points at which the

tangent passes through the origin.

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**304.** Find the points on the curve  $x^2 + y^2 - 2x - 3 = 0$  at which

the tangents are parallel to the x-axis.



**305.** Find the equation of the normal at the point  $\left(am^2, am^3
ight)$ 

for the curve  $ay^2=x^3$ 

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**306.** Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line x + 14y + 4 = 0

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**307.** Find the equations of the tangent and normal to the parabola  $y^2 = 4ax$  at the point  $\left(at^2, 2at
ight)$ 

**308.** Prove that the curves  $x = y^2$  and xy = k cut at right angles

if 
$$8k^2=1$$
.

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**309.** Find the equations of the tangent and normal to the hyperbola  $rac{x^2}{a^2}-rac{y^2}{b^2}=1$  at the point  $(x_0,y_0)$ 

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**310.** Find the equation of the tangent to the curve  $y=\sqrt{3}x-2$  ,

which is parallel to the line 4x - 2y + 5 = 0.

**311.** The slope of the normal to the curve  $y = 2x^2 + 3\sin x$  at x = 0 is:

A. 3 B.  $\frac{1}{3}$ C. -3 D.  $-\frac{1}{3}$ 

#### Answer:



**312.** The line y = x + 1, is a tangent to the curve  $y^2 = 4x$  at the

point :

A. (1,2)

B. (2,1)

C. (1,-2)

D. (-1,2)

### Answer:



313. Using differentials, find the approximate value of each of the

following up to 3 places of decimal:  $\sqrt{25.3}$ 

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314. Using differentials, find the approximate value of each of the

following up to 3 places of decimal:  $\sqrt{49.5}$ 

315. Using differentials, find the approximate value of each of the

following up to 3 places of decimal:  $\sqrt{0.6}$ 



316. Using differentials, find the approximate value of each of the

following up to 3 places of decimal.

 $(0.009)^{1/3}$ 

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317. Using differentials, find the approximate value of each of the

following up to 3 places of decimal.

 $(0.999)^{1\,/\,10}$ 

318. Using differentials, find the approximate value of each of the

following up to 3 places of decimal.

 $(15)^{1/4}$ 

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**319.** Using differntials, find the approximate value of each of the

folloiwng upto 3 places of decimal

 $(0.007)^{\frac{1}{3}}$ 



320. Using differentials, find the approximate value of each of the

following up to 3 places of decimal:  $\frac{(255)^1}{4}$ 



 $(255)^{\frac{1}{4}}$ 



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322. Using differentials, find the approximate value of each of the

following up to 3 places of decimal.

 $(401)^{1/2}$ 



**323.** Using differentials, find the approximate value of each of the

following up to 3 places of decimal:  $(0.0037)^{rac{1}{2}}$ 



326. Using differentials, find the approximate value of each of the

following up to 3 places of decimal: `(3.968)^ 3/2

327. Using differentials, find the approximate value of each of the

following up to 3 places of decimal.

 $(32.15)^{1\,/\,5}$ 



330. Find the approximate change in the surface area of a cube of

side x metres caused by decreasing the side by 1~% .



332. If the radius of a sphere is measured as 9 m with an error of

0.03 m, then find the approximate error in calculating its surface

area.



**333.** If  $f(x) = 3x^2 + 15x + 5$ , then the approximate value of f (3.02) is :

A. 47.66

B. 57.66

C. 67.66

D. 77.66

### Answer:



**334.** The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

A.  $0.06x^3m^3$ 

 $\mathsf{B}.\,0.6x^3m^3$ 

 $C.0.09x^3m^3$ 

 $\mathsf{D}.\,0.9x^3m^3$ 

Answer:

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335. Find the maximum and minimum values, if any, of the

following functions given by:  $f(x) = \left(2x-1
ight)^2 + 3$ 

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**336.** Find the maximum and minimum values, if any, of the following functions given by:  $f(x) = 9x^2 + 12x + 2$ 

**337.** Find the maximum and minimum values, if any, of the following functions given by:  $f(x) = -(x-1)^2 + 10$ 



**338.** Find the maximum and minimum values, if any, of the following functions given by:  $g(x) = x^3 + 1$ 

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**339.** Find the maximum and minimum values, if any, of the following functions given by: f(x) = |x+2| - 1

**340.** Find the maximum and minimum values, if any, of the following functions given by: g(x) = -|x+1| + 3



**341.** Find the maximum and minimum values, if any, of the following functions given by:  $h(x) = \sin(2x) + 5$ 

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342. Find the maximum and minimum values, if any, of the

following functions given by:  $f(x) = |\sin(4x) + 3|$
**343.** Find the maximum and minimum values, if any, of the following functions given by:  $h(x) = x + 1, x \in (-1, 1)$ 

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**344.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$f(x) = x^2$$

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**345.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$g(x) = x^3 - 27x$$



**346.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$k(x) = \sin x + \cos x, 0 < x < \mathsf{pi/2}$$
`



**347.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$f(x)=\sin x-\cos x, 0< x<2\pi$$

348. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = x^3 - 6x^2 + 9x + 15$ 

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**349.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$ 

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**350.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = \frac{1}{x^2 + 2}$ 

**351.** Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = x\sqrt{1-x}$ 

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**352.** Prove that the following functions do not have maxima or minima:  $f(x) = e^x$ 

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**353.** Prove that the following functions do not have maxima or minima:  $g(x) = \log x$ 

354. Prove that the following functions do not have maxima or

minima:  $h(x) = x^3 + x^2 + x + 1$ 



355. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:  $f(x) = x^3, x \in [-2, 2]$ 

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**356.** Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:  $f(x) = \sin x + \cos x, x \in [0, \pi]$ 

**357.** Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:  $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$ 

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**358.** Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:  $f(x) = (x-1)^2 + 3, x \in [-3,1]$ 

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**359.** Find both the maximum value and the minimum value of  $3x^4 - 8x^3 + 12x^2 - 48x + 25$  on the interval [0, 3]





**363.** It is given that at x = 1, the function  $x^4 - 62x^2 + ax + 9$  attains its maximum value, on the interval [0, 2]. Find the value of a.

**364.** Find the maximum and minimum values of  $x + \sin 2x$  on  $[0, 2\pi]$ 



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365. . Find two numbers whose sum is 24 and whose product is as

large as possible.



**366.** Find two positive numbers x and y such that x + y = 60 and

 $xy^3$  is maximum.

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**367.** Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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**368.** A square piece of tin of side 12 cm is to be made into a box without a top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of square to be cut off'. so that the volume of box is maximum and also find the volume of box.

**369.** A rectangular sheet of tin 45 cm x 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.



**370.** Show that of all rectangles inscribed in a given circle the square has maximum area.



**371.** Show that the rectangle of maximm area that can be inscribed in a circle of radius r is a square of side  $\sqrt{2}r$ 

**372.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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**373.** If all the closed cylindrical cans (right circular), which enclose a given volume of 100 cubic centimeters. Find the dimensions of the can which has the minimum surface area.



**374.** A wire of length 28 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle.

What should be the length of two pieces so that the combined

area of the square and the circle is minimum?



375. Prove that volume of largest cone, which can be inscribed in

a sphere, is  $\left(rac{8}{27}
ight)^{th}$  part of volume of sphere.

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376. Show that the right circular cone of least curved surface and

given volume has an altitude equal to  $\sqrt{2}$  time the radius of the

base.



**377.** Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is  $\tan^{-1}\sqrt{2}$ .



**379.** The point on the curve  $x^2 = 2y$  which is nearest to the point (0, 5) is:

A.  $(2\sqrt{2}, 4)$ B.  $(2\sqrt{2}, 0)$  C. (0,0)

D. (2,2)

Answer:

**D** Watch Video Solution

**380.** For all real values of x, the minimum value of  $\frac{1-x+x^2}{1+x+x^2}$  is:

A. 0

B. 1

C. 3

 $\mathsf{D}.\,\frac{1}{3}$ 

Answer:

**381.** The maximum value of  $[x(x-1)+1]^{rac{1}{3}}, 0\leq x\leq 1$  is:

A. 
$$\left(\frac{1}{3}\right)^1$$
  
B.  $\frac{1}{2}$   
C. 1

/3

D. 0

### Answer:



382. Using differentials, find an approximate value of each of the

following:

$$\left(\frac{17}{81}\right)^{\frac{1}{4}}$$

## 383. Using differentials, find an approximate value of each of the

following:

 $(66)^{\frac{1}{3}}$ 

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<b>384.</b> Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$
<b>Vatch Video Solution</b>

**385.** The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

**386.** Find the slope of the normal to curve xy = 6 at point (1, 6)



**387.** Show that the normal at any point  $\theta$  to the curve  $x = a\cos\theta + a\theta\sin\theta, y = a\sin\theta - a\theta\cos\theta$  is at a constant

distance from the origin.



**388.** Find the intervals in which the function f given by  $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$  is increasing.

**389.** Find the intervals in which the function f given by  $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$  is increasing.

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**390.** Find the intervals in which the function f given by  $f(x) = x^3 + \left(\frac{1}{x^3}\right), x \neq 0$  is increasing.

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**391.** Find the intervals in which the function f given by  $f(x) = x^3 + \left(\frac{1}{x^3}\right), x \neq 0$  is decreasing.

392. Find the maximum area of an isosceles triangle inscribed in

the ellipse  $rac{x^2}{a^2}+rac{y^2}{b^2}=1$  with its vertex at one end of the major axis.

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**393.** A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is  $\frac{8}{m^3}$ . If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?



394. The sum of the perimeter of a circle and square is k, where k

is some constant. Prove that the sum of their areas is least when

the side of square is double the radius of the circle.



**395.** A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 20 m. Find the dimensions of the window to admit maximum light through the whole opening.

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**396.** A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle.Show that the minimum length of the hypotenuse is  $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{2}{3}}$ 

**397.** Find the points at which the function f given by  $f(x) = (x-2)^4 (x+1)^3$  has local maxima.

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**398.** Find the points at which the function f given by  $f(x) = (x-2)^4 (x+1)^3$  has local minima.

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**399.** Find the points at which the function f given by  $f(x) = (x-2)^4 (x+1)^3$  has point of inflexion.

**400.** Find the absolute maximum and minimum values of the function f given by  $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$ 



**402.** Let f be a function defined on [a, b] such that f'(x) > 0 for all  $x \in (a, b)$ . Then prove that f is an increasing function on (a, b).



**403.** Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is  $2\frac{R}{\sqrt{3}}$ . Also find the maximum volume.

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**404.** Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle  $\alpha$  is one-third that of the cone and the greatest volume of cylinder is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ 

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**405.** A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:

A. 1m/h

B. 0.1 m/h

C. 1.1 m/h

D. 0.5 m/h

**Answer:** 





A. 
$$\frac{22}{7}$$
  
B. 
$$\left(\frac{6}{7}\right)$$
  
C. 
$$\frac{7}{6}$$
  
D. 
$$-\frac{6}{7}$$

### Answer:



**407.** The line y = mx + 1, is a tangent to the curve  $y^2 = 4x$  if

the value of m is:

A. 1

B. 2

C. 3

 $\mathsf{D}.\,\frac{1}{2}$ 

#### Answer:

**408.** The normal at the point (1,1) on the curve  $2y + x^2 = 3$  is:

A. x+y =0 B. x - y = 0 C. x + y + 1=0

D. x-y + 1 = 0

#### Answer:

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**409.** The normal to the curve  $x^2 = 4y$  passing (1,2) is:

A. x+y = 3

B. x-y = 3

C. x+y = 1

D. x - y = 1

#### Answer:

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**410.** The points on the curve  $9y^2 = x^3$ , where the normal to the curve makes equal intercepts with the axes are:

A. 
$$\left(4 \pm \frac{8}{3}\right)$$
  
B.  $\left(4, \frac{-8}{3}\right)$   
C.  $\left(4, \pm \frac{3}{8}\right)$   
D.  $\left(\pm 4, \frac{3}{8}\right)$ 

#### Answer:

411. The rate of change of the volume of a sphere w.r.t. its surface

area, when the radius is 2cm, is



The slope of the tangent to the curve  $y^2=x^3$  at the origin is



When 0 la < 1, x > 0, the function y =  $\log_a$  x is a \_\_\_\_\_

function.



**419.** The values of a for which  $y = x^2 + ax + 25$  touches the axis

of x are ........

The function  $f(x) = x + rac{9}{x}$  has a local \_\_\_\_\_ value at x = 3.



The function f(x) = 4x - 3 is strict \_\_\_\_ function for all x in R.



Maximum value of the function  $f(X) = rac{1}{4x^2+2x+1}$  is \_\_\_\_\_.



For x> 0, the function  $f(x) = \log_{1/3} x$  is a strict \_\_\_\_\_

function.



•

The equation of normal to the curve y = an xat(0,0) is



433. Fill in the blanks:

The curves  $y = 4x^2 + 2x - 8$  and  $y = x^3 - x + 10$  touch each

other at the point.....

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**434.** Fill in the blanks:

At (0,0) the curves  $y = x^2$  and  $y^2 = x$ 

The function  $f(X) = rac{2x^2-1}{x^4}$ . X> 0 is a decreaisng function in

the interval.


**438.** Fill in the blanks:

The equation of the tangent to the curve  $y=e^{2x}$  at the point

(0,1) is\_\_\_\_\_.

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**439.** Is the function  $f(x) = x^2, \xi n R$  increasing?

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440. True or False statements:

The rate of change of sin x w.r.t. cos x in  $\left(0, \frac{\pi}{2}\right)$  is tan x.

The function f(X) = x|x| is strictly increasing on R.



If a function f has an extreme point at x = c, then f' (c) = 0

Fermat Theorem If a function f has a local extremum at x = c, then

f" (C) = 0.



Tangent to the curve  $y=x^2atx=0$  is parallel to the x-axis.



**446.** True or False statements:

The function f(X) = x|x| is strictly increasing on R.



A strict monotone function is always one-one.

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<b>448.</b> True or False statements:
A one-one function is always an increasing function.

449. True or False statements:

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The function  $f(x) = rac{1}{3}x^3 - x$  is strictly decreasing on (-1,1)

The function  $f(x) = x^{rac{1}{3}}$  is a strict increasing function on R.



**451.** True or False statements:

If a function f is twice differentiable at x = c,f' (c) = 0 and f" (c) >

0, then f has a local minimam value at x = c.



**452.** True or False statements:

For any function f a local maximum value is always greater than a

local minimum value.



If a function f is such that f' (c) = 0, then at the point, c the

function f has either a local maximum or a local minimum.

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<b>454.</b> True or False statements:
$e^{\pi} < \pi^e$
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<b>455.</b> The function $f(x)=x^2, x\in R$ has no maximum value.
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**457.** True or False statements:

Maximum value of the function  $f(x) = \sin^4 x + \cos^4 x is 2$ .



458. True or False statements:

The angle between the tangents to the curves  $y = x^2$  and  $x = y^2$  at the point (0,0) is  $rac{\pi}{2}$ 

For 
$$x \in \left(0, rac{\pi}{2}
ight)$$
,  $\sin x < x < an x.$ 

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460. True or False statements:

There is a unique real number x > 0 such that  $\left(rac{1}{4}
ight)^x = x = \log_{1/4^x}$ 

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461. True or False statements :

If and B are invertible matrices such that AB = BA, then  $(AB)^{-1} = A^{-1}B^{-1}$ 

**462.** Statement I :Among all the rectangles of the given perimeter, the square has the largest area. Also among all the rectangles of given area, the square has the least perimeter. Statement II :For x > 0, y > 0, if x + y= constant, then xy will be maximum for y=x and if xy=constant, then x+y will be minimum

for y=x.

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463. True or False statements:

Of all rectangles of a given area, square has the largest perimeter.



**464.** Find x+y+z





**465.** Find the rate of change of the area of a circle per second with respect to its radius r when r = 5 cm.

A.  $10\pi$ 

B.  $10\pi cm^2/cm$ 

C.  $\frac{220}{7}$ 

D. none of these

#### Answer:



**466.** The volume of a cube is increasing at the rate of  $9cm^3/\sec$ . The rate of increase of its surface area when its edge (side) is 6 cm is

A. 3.6 cm/sec

B. 3.6

C. 6 cm /sec

 $\mathsf{D.}\,6cm^2\,/\,\mathrm{sec}$ 



**467.** A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 10 cm, the rate of increase of the area enclosed is

A.  $80\pi$ 

B.  $\frac{1760}{7}$ C.  $80\pi c \frac{m^2}{\text{sec}}$ 

D. none of these



**468.** The length x of a rectangle is decreasing at the rate of 3 cm/sec and its breadth y is increasing at the rate of 2 cm/sec. At the instant, when x = 100 cm and y = 60 cm, the rate of change of its area is

A.  $20cm^2/\sec$ 

 $\mathsf{B.}-20cm/\sec^2$ 

C. 20/sec

D. none of these

# Answer:



469. The total revenue Rs. R received form the sale of x units of a

product is given by  $R(x) = 3x^2 + 36x + 5$ . The marginal

revenue when x = 5 is (marginal revenue is the rate of change of total revenue w.r.t. number of items sold at an instant)

A. 66

B. Rs. 66

C. 69

D. none of these

# Answer:



**470.** The sides of an equilateral triangle are increasing at the rate

of 2 cm/sec, then the rate at which the area increases, when side

is 10 cm is

A.  $10cm^2/\sec$ 

B. 
$$\sqrt{3}cm^2/\sec$$

$$\mathsf{C.}\,10\sqrt{3}cm^2\,/\,\mathrm{sec}$$

D. 
$$\frac{10}{3} cm^2/\sec$$

#### Answer:



**471.** A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

A. 
$$\frac{1}{10}$$
 radian / sec  
B.  $\frac{1}{20}$  radian / sec

C. 20 radian/sec

D. 10 radian / sec

### Answer:



**472.** On R, the function f(x) = 7x - 3 is

A. strict decreasing

B. decreasing

C. increasing

D. strict increasing



**473.** The function f (x) = 2-3 x is

A. decreasing

B. increasing

C. strict decreasing

D. neither increasing nor decreasing.

### **Answer:**

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**474.**  $y = x(x-3)^2$  decreases for the values of x given by :

A. 1ltxlt3

B. x lt O

C. xgt 0

$$\mathsf{D}.\, 0 < x < \frac{3}{2}$$

# Answer:



**475.** The function tan x - x , 
$$-rac{\pi}{2} < x < rac{\pi}{2}$$

A. always increases

B. always decreases

C. never increases

D. sometimes increases and sometimes decreases.



**476.** The function  $f(x) = x^2$ , for all real x is

A. decreasing

B. increasing

C. none of these

D. neither increasing nor decreasing.

### Answer:

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**477.** The function  $f(x) = x^2 - 4x + 6$  is

A. strictly decreasing on  $(2,\infty)$ 

B. strictly increasing on  $(-\infty,2)$ 

C. strictly increasing on R

D. strictly increasing on  $(2,\infty)$ 

#### Answer:

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**478.** Find the intervals in which the function given by  $f(x) = \sin 3x$ , x in  $[0, \frac{\pi}{2}]$  is decreasing.

A. 
$$\left(0, \frac{\pi}{6}\right)$$
  
B.  $\left(0, \frac{\pi}{4}\right)$   
C.  $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$   
D.  $\left(0, \frac{\pi}{2}\right)$ 

### Answer:

**479.** The function  $f(x) = 4 \sin^3 x - 6 \sin^2 x + 12 \sin x + 100$  is strictly

A. increasing in 
$$\left(\pi, 3\frac{\pi}{2}\right)$$
  
B. decreasing in  $\left(\frac{\pi}{2}, \pi\right)$   
C. decreasin  $g \in (-pi/2, pi/2)$ `

D. decreasing in 
$$\left(0,\,rac{\pi}{2}
ight)$$

### Answer:

**480.** Which of the following functions is decreasing on 
$$\left(0, \frac{\pi}{2}\right)$$

A. sin 2 x

C. cos x

D. cos 3 x

Answer:

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**481.** On the interval  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ , the function  $\tan^{-1}(\sin x + \cos x)$ 

is

A. increasing

B. non-decreasing

C. decreasing

D. neither increasing nor decreasing.

**482.** Find the slope of the tangent to the curve  $y = x^3 - x$  at x = 2A. 6 B. 0 C. 11 D. none of these.

# Answer:



**483.** The point on the curv3e  $y = \sqrt{4x - 3}$ -1, at which the slope of the tangent is  $\frac{2}{3}$ , is

A. (3,2)

B. (2,3)

C. (1,0)

D. none of these

Answer:

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**484.** The point on the curve  $y = x^2 - 4x + 4$  at which the tangent is parallel to x-axis is

A. (0,2)

B. (2,0)

C. (0,0)

D. none of these

# Answer:





C. 
$$\frac{1}{3}$$

D. 
$$\frac{\pi}{2}$$

### Answer:

**486.** The tangent to the curve  $y = e^{2x}$  at the point (0, 1) meets x-

axis at :

A. (0,1) B.  $\left(-\frac{1}{2},0\right)$ C. (2,0)

# Answer:



**487.** The points at which the tangents to the curve  $y = x^3 - 12x + 18$  are parallel to x-axis are :

A. (2,-2), (-2, -34)

B. (2,34), (-2,0)

C. (0,34),(-2,0)

D. (2,2), (-2,34)

### Answer:



**488.** The point at which the curves  $x^2 = y$  and  $y^2 = x$  cut orthogonally is

A. (0,0)

B. (1,1)

C. (2,2)

D. none of these

# Answer:



**489.** The equation of tangent to the curve  $y(1+x^2) = 2-x$ ,

where it crosses x-axis is :

A. x+5y = 2

B. x-5 y = 2

C. 5x - y =2

D. 5x + y = 2

#### Answer:

**490.** If the curve at  $ay + x^2 = 7$  and  $x^3 = y$ , cut orthogonally at

(1, 1), then the value of a is :

A. -6

B. 6

C. 1

D. none of these

Answer:



**491.** The function f(x) = x has

A. only one maximum

B. only one minimum

C. one maximum and one minimum value

D. no extreme value

### Answer:

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**492.** The function  $f(x) = 2x^3 - 6x^2 + 6x + 5$  has

A. a local maximum at x = 1

B. a local minimum at x = 1

C. neither maximum nor minimum at x = 1

D. none of these



**493.** Let 
$$f(x) = egin{cases} |x|f ext{ or } 0 < |x| \le 1 \ 1f ext{ or } x = 0 \end{cases}$$
 then

A. f has a local minimum at 0

B. f has a local maximum at 0

C. f has a point of inflextion at 0

D. none of these

### Answer:

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**494.** The abscissa of the point on the curve  $3y = 6x - 5x^3$ , the normal at which passes through origin is

A. 
$$\frac{1}{2}$$

$$\mathsf{B.}\,\frac{1}{3}$$

C. 2

D. 1

### Answer:



**495.** At 
$$x = 5\frac{\pi}{6}$$
, f(x) = 2 sin 3x + 3 cos 3 x is

A. maximum

B. minimum

C. zero

D. neither maximum nor minimum



**496.** The two curves  $x^3 - 3xy^2 + 2 = 0$  and  $3x^2y - y^3 - 2 = 0$ 

intersect at an angle of

A. 
$$\frac{\pi}{4}$$
  
B.  $\frac{\pi}{3}$   
C.  $\frac{\pi}{2}$   
D.  $\frac{\pi}{6}$ 



**497.** The function 
$$f(x) = \log_e \left(x^3 + \sqrt{x^6 + 1}
ight)$$
 is

- A. even and increasing
- B. even and decreasing
- C. odd and increasing
- D. odd and decreasing.

### **Answer:**



**498.** Let  $f(X) = an^{-1} g(x)$ , where g (x) is monotonically increasing for 0

A. increasing on 
$$\left(0, \frac{\pi}{2}\right)$$
  
B. decreasing on  $\left(0, \frac{\pi}{2}\right)$   
C. increasing on  $\left(0, \frac{\pi}{4}\right)$  and decreasing on  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$   
D. none of these

# Answer:



**499.** Function  $f(X) = \cos x - 2 \lambda x$  is monotonic decreasing when

A.  $\lambda \geq rac{1}{2}$ B.  $\lambda \leq rac{1}{2}$ C.  $\lambda \leq 2$ D.  $\lambda \geq 2$ .



500.

$$g(x)=2f\Big(rac{x}{2}\Big)+f(2-x) ext{ and } f'\,'(x)<0\,orall\,x\in(0,2).$$
 If  $g(x)$  increases in  $(a,b)$  and decreases in  $(c,d),$  then the value of

$$a+b+c+d-rac{2}{3}$$
 is

A. increase on (0,a)

B. decreases on (0,a)

C. increases on (-a,0)

D. decreases on (a,2a)

Answer:



**501.**  $f(x) = \log_A x$  is increasing on R, if
A. 0 < a < 1B. a > 1C. a < 1

D. a > 0

**Answer:** 



**502.** The function  $f(x) = a^x$  is strict decreasing on R, If

A. a > 0

 $\mathsf{B.}\,a>1$ 

 ${\sf C}.\,0 < a < 1$ 

 $\mathsf{D}.\,a<0$ 

## Answer:

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