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## MATHS

## BOOKS - PRADEEP PUBLICATION

## APPLICATIONS OF DERIVATIVES

## Example

1. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2 x+1)$. Find the rate of change of its volume with respect to x .

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2. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long?

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3. The radius of a circle is increasing at the rate of $0.7 \mathrm{~cm} / \mathrm{s}$. What is the rate of increase of its circumference?

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4. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm , how fast is the enclosed area increasing?
5. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of $0.05 c \frac{m}{s}$. Find the rate at which its area is increasing when radius is 3.2 cm .

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6. Find the rate of change of the area of a circle per second with respect to its radius $r$ when $r=5 \mathrm{~cm}$.

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7. The volume of a balloon is changing at the rate of $25 \mathrm{~cm}^{3} / \mathrm{sec}$.

Find the rate of change of its surface area when its radius is 5 cm .
8. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

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9. If x and y are the sides of two squares such that $y=x-x^{2}$, find the rate of the change of the area of the second square with respect to the first square.

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10. Find the point on the curve $y^{2}=8 x$ for which the abscissa and ordiante change at the same rate.
11. A man 2 m high walks at a uniform speed of $6 \mathrm{k} / \mathrm{h}$ away from a lamp ost 6 metres high. Find the rate at which the length of his shadow increases.

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12. Sand is pouring from a pipe at the rate of 12 cubic $\mathrm{cm} . / \mathrm{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base.

At which rate is the height of the sand-cone increasing when the height is 4 cm . ?
13. An inverted conical vessel whose height is 10 cm and the radisu of whse base is 5 cm is being filled with water at the uniform rate of $1.5 \mathrm{cu} \mathrm{cm} / \mathrm{m}$. Find the rate at which the level of water in the vessel is rising when the depth is 4 cm .

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14. The length of a rectangle is increasing at the rate of 3.5 $\mathrm{cm} / \mathrm{sec}$ and its breadth is decreasing at the rate of $3 \mathrm{~cm} / \mathrm{sec}$. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm .

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15. The length ' $x$ ' of a rectangle is decreasing at the rate of 5 cm per minute and the width ' $y$ ' is increasing at the rate of 4 cm per
minute, when $x=8 \mathrm{~cm}$ and $y=6 \mathrm{~cm}$, find the rate of change of the perimeter of the rectangle.

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16. Without using the derivative, prove that $f(X)=2 x+5$ is increasing for all $x \in R$.

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17. Prove that the function $f(X)=a x+b$ is increasing iff $a>0$.

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18. Prove that $\mathrm{f}(\mathrm{X})=\frac{1}{x}$ is decreasing for all $\mathrm{x}>0$, without using the derviative.

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19. Prove that the exponential function $e^{x}$ is increasing for all x $\in \mathrm{R}$.

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20. Prove that the logarithmic function is increasing on $(0, \infty)$.

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21. Prove that $f(x)=\frac{3}{x}+7$ is strictly decreasing for $x \in R-\{0\}$.

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22. Show that the function $f(x)=x^{2}-6 x+3$ is increasing in the interval $[4,6]$

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23. Show that the function $x+\frac{1}{x}, x \geq 1$ is increasing.

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24. Let 1 be any interval disjoint form ( $-1,1$ ). Prove that the function $f(x)=x+\frac{1}{x}$ is strictly increasing on I .

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25. Prove that the function given by
$f(x)=x^{3}-3 x^{2}+3 x-100$ is increasing in R.

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26. Prove that the function $x-\cos x$ is increasing on $R$.

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27. Show that $y=\log (1+x)-2 \frac{x}{2+x}, x \succ 1$, is an increasing function of $x$. throughout its domain.

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28. Prove that the function $f(x)=\tan x-4 x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$
29. Prove that $y=4 \frac{\sin \theta}{2+\cos \theta}-\theta$, is an increasing function of $\theta$ in $\left[0, \frac{\pi}{2}\right]$.

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30. Prove that the function $f$ given by $f(x)=\log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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31. Prove that the function $x^{2}+x+1$ is neither increasing nor decreasing on (-1,0)

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32. Find the values of $x$ for which the function $f(x)=x^{2}-6 x+9$ is increasing or decreasing.

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33. Show that the following functions are increasing in the indicated intervals
$f(x)=x^{2}-8 x, x \geq 4$

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34. Show that the following functions are increasing in the indicated intervals

$$
f(x)=x^{3}-27 x, x<-3 \text { or } x>3
$$

35. Show that the following functions are increasing in the indicated intervals
$\mathrm{f}(\mathrm{x})=\frac{1}{1+x^{2}}, x \leq 0$

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36. Show that the following functions are increasing in the indicated intervals

$$
f(x)=-\frac{x}{2}+\sin x,-\frac{\pi}{3}<x<\frac{\pi}{3}
$$

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37. Show that the following functions are increasing in the indicated intervals

$$
f(x)=\frac{x}{\sin x}, 0<x<\frac{\pi}{2}
$$

38. Show that the following functions are increasing in the indicated intervals
$f(x)=1-\frac{1}{x}, x<0$

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39. Show that the following functions are increasing in the indicated intervals

$$
f(x)=x^{5}+20 x^{3}+25 x, x \in R
$$

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40. Show that the following functions are increasing in the indicated intervals
$x^{2} e^{-x}, 0 \leq x \leq 2$.

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41. Determine the intervals in which the following functions
$f(x)=x^{2}-4 x+6$ are strictly increasing or strictly decreasing.

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42. Find the interval in which the following function is decreasing
:

$$
f(x)=2 x^{3}-15 x^{2}+36 x+1
$$

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43. Find the intervals in which the following functions are increasing or decreasing :
$\frac{x}{2}+\frac{2}{x}$

## - Watch Video Solution

44. Find the intervals in which the following functions are increasing or decreasing :
$\frac{x}{x^{2}+1}$

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45. Find the intervals in which the following functions are increasing or decreasing :
$\sin 3 x, 0 \leq x \leq \frac{\pi}{2}$
46. Find the intervals in which the following functions are increasing or decreasing :
$5 x^{3 / 2}-3 x^{5 / 2}, x>0$

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47. Find the intervals in which the following functions are increasing or decreasing :
$x^{4}-\frac{1}{3} x^{3}$

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48. Find the intervals in which the following functions are strictly increasing or decreasing: $-2 x^{3}-9 x^{2}-12 x+1$
49. Find the intervals in which the following functions are increasing or decreasing :
$2 x^{3}-9 x^{2}+12 x-5$

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50. Find the intervals in which the following functions are increasing or decreasing :
$5 x^{3}-15 x^{2}-120 x+3$

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51. Find the intervals in which the following functions are strictly increasing or decreasing: $-2 x^{3}-9 x^{2}-12 x+1$

## - Watch Video Solution

52. Find the intervals in which the following functions are increasing or decreasing :
$x^{3}-4 x$

## - Watch Video Solution

53. Find the intervals in which the following functions are strictly increasing or strictly decreasing
$x^{3}-12 x^{2}+36 x+17$

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54. Find the intervals in which the following functions are increasing or decreasing :
$x^{3}-9 x^{2}+15 x+11$

## - Watch Video Solution

55. Find the intervals in which the following functions are increasing or decreasing :
$x^{3}-6 x^{2}+9 x+15$

## - Watch Video Solution

56. Find the intervals in which the following functions are increasing or decreasing :
$x^{4}-2 x^{2}$

## - Watch Video Solution

57. Find the intervals in which the following functions are increasing or decreasing :
$\cos ^{-1}\left(\frac{1-x^{2}}{1+x^{2}}\right)$

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58. Find the intervals in which the following functions are increasing or decreasing :
$(x+1)^{3}(x-3)^{3}$

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59. Find the intervals in which the following functions are increasing or decreasing :
$\frac{3}{10} x^{4}-\frac{4}{5} x^{3}-3 x^{2}+\frac{36}{5} x+11$
60. Find the intervals in which the following functions are increasing or decreasing :
$\sin x+\cos x, 0 \leq x \leq 2 \pi$

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61. Find the slope of the normal to the curve $y=x^{3}-x a t x=2$

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62. Find the equation of the tangent and normal to the curve
$\frac{X^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ at the point $(\sqrt{2} a, b)$

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63. Find the equations of the tangent line to the curve:
$y=\cot ^{2} x-2 \cot x+2 a t x=\frac{\pi}{4}$

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64. Find the points on the curve $4 x^{2}+9 y^{2}=1$,where the tangents are perpendicular to the line $2 y+x=0$

## - Watch Video Solution

65. Find the equation of the tangent line to the curve
$x=\theta+\sin \theta, y=1+\cos \theta$ at $\theta=\frac{\pi}{4}$

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66. Find the point on the curve $y=x^{2}$ where the slope of the tangent is equal to the $x$-coordinate of the point.

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67. Find the equation of the tangent to the curve $y=\frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x -axis.

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68. At what points on the curve $x^{2}+y^{2}-2 x-3=0$, the tangent is parallel to $y$-axis?
69. At what points on the curve $y=x^{2}-4 x+5=0$ is the tangent perpendicular to line $2 y+x=7$ ?

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70. Find the point (s) on the curve $\mathrm{y}=\{x(x-2)\}^{2}$ at which the tangent is parallel to x -axis.

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71. Find the equations of the normals to the curve $3 x^{2}-y^{2}=8$ which are parallel to the line $x+3 y=4$

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72. Find the equation of the normals to the curve $y=x^{3}+2 x+6$ which are parallel to the line $x+14 y+4=0$

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73. Find the equation of the tangent line to the curve $y=\sqrt{5 x-3}-2$ which is paralel to the line $4 x-2 y+3=0$

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74. Find the point at which the tangent to the curve
$y=\sqrt{4 x-3}-1$ has its slope $=\frac{2}{3}$

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75. Find the equation of tangents to the curve $y=\cos (x+y),-2 \pi \leq x \leq 2 \pi$ that are parallel to the line $x+2 y=0$.

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76. Find the co-ordinates of the points on the curvey $y=2 x^{2}+3 x+18$, the tangents at which pass through origin.

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77. For the curve $y=4 x^{3}-2 x^{5}$, find all the points at which the tangent passes through the origin.

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78. Find the condition that the line $\mathrm{x} \cos \alpha+y \sin \alpha=p$ may be a tangent to the curve $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$

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79. Find the equation of the normal to curve $x^{2}=4 y$ which passes through the point $(1,2)$.

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80. Show that the normal at any point $\theta$ to the curve $x=a \cos \theta+a \theta \sin \theta, y=a \sin \theta-a \theta \cos \theta$ is at a constant distance from the origin.
81. Show that the curves $y=a^{x}$ and $y=b^{x}, a>b>0$ intersect at an angle of $\tan ^{-1}\left(\left|\left(\frac{\log \left(\frac{a}{b}\right)}{1+\log a \log b}\right)\right|\right)$

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82. Find the angle of intersection of curves $y^{2}=4 a x$ and $x^{2}=4 b y, a b \neq 0$

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83. Show that the curves $x^{2}+y^{2}-2 x=0$ and $x^{2}+y^{2}-2 y=0$ cut orthogonally at the point $(0,0)$.

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84. Find the condition for the curves $\frac{X^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ and $x y=c^{2}$ to intersect orthogonally.

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85. Find the value of $p$ for which the curves $x^{2}=9 p(9-y)$ and $x^{2}=p(y+1)$ cut each other at right angles.

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86. Evaluate:
$\sqrt{25.3}$

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87. Evaluate:
$\sqrt{.037}$

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88. Evaluate:
$\sqrt{.082}$

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89. Evaluate:
$\sqrt{36.6}$ using differentials.

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90. Use differentials to approximate the cube root of 66 .

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91. Find the approximate value of $(1.999)^{5}$

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92. Find an approximate value of $(25)^{1 / 3}$ using differentials.

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93. Find the value of $\log _{10}$ (10.1), given that $\log _{10} e=0.4343$.

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94. Given $\sin 30^{\circ}=0.5$ and $\cos 30^{\circ}=0.866$, find an approximate value of $\sin 31^{\circ}$.

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95. If $y=x^{3}-4 x$ and $x$ chagnes form 2 to 1.99 , find the approximate change in the value of $y$.

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96. If $f(x)=3 x^{2}+5 x+3$, find an approximate value of (3.02)

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97. If the radius of a circle increases from 5 cm to 5.1 cm , find the increase in area.

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98. If the error committed in measuring the radius of a circle is
$0.01 \%$, find the corresponding error in calculating the area.

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99. Show that the relative error in the volue of a sphere due to an
error in measurement of radius is thhree times the relative error in radius.

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100. If in a $\Delta A B C$, the side c and the angle C remain constant, while the remaining elements are changed slightly. Using differential, show that $\frac{d a}{\cos A}+\frac{d b}{\cos B}=0$.

## - Watch Video Solution

101. Find the maximum and minimum values of the function
$f(x)=3-2 \sin x$

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102. Find the maximum and minimum values of the function
$f(x)=a \sin x+b \cos x$

## - Watch Video Solution

103. Find the maximum and minimum values of the function

$$
f(x)=2+x-x^{2}
$$

## - Watch Video Solution

104. Find the maximum and minimum values of the function
$f(x)=\sin (\sin x)$

## - Watch Video Solution

105. Find the maximum and minimum values of the function

$$
f(x)=|\sin 4 x+3|
$$

## - Watch Video Solution

106. Find the local maximum and local minimum values of the function

$$
f(x)=\cos ^{4} x+\sin ^{4} x,(0, \mathrm{pi} / 2)
$$

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107. Find the maximum and minimum values of the function
$f(x)=x+\frac{4}{x}, x<0$

## - Watch Video Solution

108. Show that the function $f(x)=x^{3}+x^{2}+x+1$ has neither a maximum value nor a minimum value.

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109. Find the absolute maximum and minimum values of the function.
$f(x)=\frac{x}{x^{2}+1},-3 \leq x \leq 3$

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110. Find the absolute maximum and minimum values of the function.
$f(x)=12 x^{4 / 3}-6 x^{1 / 3} \in(-1,1)$

## - Watch Video Solution

111. Find the absolute maximum and minimum values of the function.

$$
f(x)=3 x^{4}-8 x^{3}+12 x^{2}-48 x+25 \in(0,3)
$$

112. Find the absolute maximum and minimum values of the function.
$2 x^{3}-15 x^{2}+36 x+1 \in(1,5)$

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113. Find the (absolute) maximum and minimum values of
$f(x)=-4 \sin x+2 x$ in $\left(0, \frac{\pi}{2}\right)$

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114. Find the absolute maximum and minimum values of
$f(x)=x+\sin 2 x o n(0,2 \pi)$ and also find the points of (absolute maximum and minimum.
115. Find the absolute maximum and minimum values of the function.

$$
f(x)=2 \cos 2 x-\cos 4 x \in(0, \pi)
$$

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116. Find the absolute maximum and minimum values of the function.
$f(x)=\sin ^{2} x-\cos x, x \in(0, \pi)$

## - Watch Video Solution

117. Find the maximum and minimum values of the function.

$$
f(x)=\sec x+\log \cos ^{2} x, \in(0,2 \pi)
$$

118. Find the points of local maxima and minima, if any, of he function :
$f(x)=-\left(\frac{3}{4}\right) x^{4}-8 x^{3}-\left(\frac{45}{2}\right) x^{2}+105$

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119. Find local minimum value of the function $f$ given by $f(x)=3+|x|, x \in R$

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120. Find the local maximum and minima of the function

$$
f(x)=2 x^{3}-21 x^{2}+36 x-20
$$

121. Find all points of local maxima and minima and the corresponding maximum and minimum values of the function given below:
$f(x)=-\frac{3}{4} x^{4}+2 x^{3}+\frac{9}{2} x^{2}+100$

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122. Prove that the function $f(x)=2 x^{3}-6 x^{2}+6 x+5$ has no maximum or minimum.

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123. Show that a local minimum value of $f(x)=x+\frac{1}{x}, x \neq 0$ is greater than a local maximum value.
124. Find the local maximum and local minimum of $f(x)=\sin 2 x-x,-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

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125. Find the local maximum and local minimum of $f(x)=2 \sin x-x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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126. Find the points of local maximum and local minimum, if any, of the function: $f(x)=\sin x-\cos x, 0<x<2 \pi$. Also find the local maximum and local minimum values.
127. Find the points of local maxima and minima of the function

$$
f(x)=3 x^{4}-4 x^{3}+5 \operatorname{in}(-1,2)
$$

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128. Discuss local maxima and minima of the function $f(x)=\left(4-x^{2}\right)^{2 / 3}$

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129. Discuss local maxima and minima of the function

$$
f(x)=x^{x}, x>0
$$

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130. If $y=a \log x+b x^{2}+x$ has its extreme values at $\mathrm{x}=-1$ and x
$=2$, then find $a$ and $b$.

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131. Discuss local maxima and minima of $f(x)=a x+\frac{b}{x}, x \neq 0$, where $a, b>0$.

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132. $y=\frac{a x-b}{(x-1)(x-4)}$ has a turning point $\mathrm{P}(2,-1)$. Find the values of ' $a$ ' and ' $b$ ' and show that $y$ is maximum at $P$.

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133. Prove that for $x>-c$, the minimum value of $f(x)=$ $(a+x) \frac{b+x}{c+x} \quad$ is $2 \sqrt{(c-a)(c-b)}+\mathrm{a}+\mathrm{b}-2 \mathrm{c}$ , $\operatorname{given}(c-a)(c-b)>0$.

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134. Find the points of local maxima and minima of the function
$f(x)=4 \sin x+\cos 2 x x \in(0,2 \pi)$ Also find absolute maxima and minimum values.

## D Watch Video Solution

135. Show that $\mathrm{f}(\mathrm{x})=x^{5}-5 x^{4}+5 x^{3}-1$ has a local maximum at $x=1$, a local minimum at $x=3$ and neither at $x=0$
136. How should we choose two numbers, each greater than or equal to -2 whose sum is $\frac{1}{2}$ so that the sum of square of the first and cube of the second is minimum?

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137. Find two positive numbers whose sum is 24 and whose product is maximum.

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138. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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139. Find two numbers whose sum is 15 and the square of one multiplied by the cube of the other is maximum.

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140. Show that of all rectangles with given perimeter square has maximum area

## - Watch Video Solution

141. Show that, of all the rectangles with a given area, the square has the smallest perimeter.

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142. Use the functio $f(x)=x^{1 / x}, \mathrm{x}>0$ to determine the greater of $e^{i}$ and $\pi^{e}$

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143. Find the area of the largest isosceles triangle whose perimeter is 36 untis.

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144. Two sides of a triangle are $a$ and $b$. Find the angle between them such that area shall be maximum

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145. Show that rectangle of maximum perimeter, which can be inscribed in a circle of a radius 'a' is a square of side $\sqrt{2} a$.

## - Watch Video Solution

146. Show that the rectangle of maximm area that can be inscribed in a circle of radius $r$ is a square of side $\sqrt{2} r$

## - Watch Video Solution

147. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

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148. A wire of length 28 cm is to be cut off into two pieces. One piece is to be made into a circle and other into a square. What should be the lengths of two pieces so that combined area of circle and square is minimum.

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149. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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150. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

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151. A figure consists of a semi-circle with a rectangle on its diameter. Given perimeter of the figure, find the dimensions in order that the area may be maximum.

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152. Show that the height of the circular cylinder of maximum volume that can be inscribed in a given right-circular cone of height $h$ is $\frac{1}{3} h$

## - Watch Video Solution

153. Show that the curved surface of a right circualr cylinder inscribed in a right circular cone is maximum when the radius of
its base is half that of the cone.

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154. A square piece of tin of side 24 cm is to be made into a box
without top by cutting a square from each comer and folding up
the flaps to form a box. What should be the side of square to be cut off so that the volume of box is maximum also find the volume?

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155. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre. rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.
156. A cylinderical can is to be made to hold 1 litre of oil. Find the dimensions which will minimize the cost of the metal to make the can.

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157. Assuming that the petrol burnt per hour in driving a motor boat varies as the cube of its velocity, show that the most economical speed when going against a current of $\mathrm{ckm} / \mathrm{hour}$ is $\frac{3 c}{2} \mathrm{~km} / \mathrm{hour}$.

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158. Find the area of the greatest isocels triangle that can be inscried in a given ellipse having its vertex coincident with one extremly of major axis.

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159. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$

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160. A telephone company in a town has 500 subsribers on its list and collects fixed charges of Rs. 300/- per year form each subsriber. The company proposes to increase the annual charges and it is believed that for every increases of Rs. $1 / 0$, one subsriber
will discontinue service. Find what increase will bring maximum revenue?

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161. Manufacturer can sell $x$ items at a price of rupees $R s\left(5-\left(\frac{x}{100}\right)\right)$ each. The cost price of x items is $R s\left(\left(\frac{x}{5}\right)+500\right)$. Find the number of items he should sell to earn maximum profit.

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162. $A B$ is a diameter of a circle and $C$ is any point on the cirle.

Show that the area of triangle $A B C$ is maximum, when it is isosceles.
163. The sum of the surface areas of a rectangualr parallelopiped with sides $x, 2 x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if $x$ is equal to three times the radius of the sphere. Also find the minimum value of the sum of their volumes.

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164. An Apache helicopter of enemy is flying along the curve given by $y=x^{2}+7$. A soldier, placed at (3,7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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1. The length $x$ of a rectangle is decreasign at the rate of 5 $\mathrm{cm} / \mathrm{min}$ and the width y is incresaing at the rate of $4 \mathrm{~cm} / \mathrm{min}$. when $x=8 \mathrm{~cm}$ and $y=6 \mathrm{~cm}$, find the rates of change of the area of the rectangle.

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2. The total cost $C(x)$ and the total revenue $R(X)$ associated with the production and sale of $x$ units of an item are given by

$$
C(x)=0.1 x^{2}+30 x+1000 \quad \text { and } \quad R(x)=0.2 x^{2}+36 x-100
$$

find the marginal cost and the marginal revenue when $x=20$.

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3. The total revenue in Rupees received from the sale of $x$ units of a product is givne by $R(x)=3 x^{2}+36 x+5$. Find the marginal revenue, when $x=6$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

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4. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm , how fast is the enclosed area increasing?

## - Watch Video Solution

5. Find the rate of change of the volume $V$ of a sphere of radius $r$ w.r.t change in the radius.

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6. The radius of a balloon is increasing at the rate of $10 \mathrm{~cm} / \mathrm{sec}$. At what rate is the surface area of the balloon increasing when its radius is 15 cm ?

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7. The radius of a spherical soap bubble is increasing at the rate of $0.2 \mathrm{~cm} / \mathrm{sec}$. Find the rate of increase of its volume when the radius is 4 cm .
8. A ballon, which always remains spherical has a variable diameter $\frac{3}{2}(2 x+3)$. Determine the rate of change of volume w.r.t.x.

## D Watch Video Solution

9. The volume of a spherical balloon is increasing at the rate of $20 c \frac{m^{3}}{\mathrm{sec}}$. Find the rate at which its surface area is increasing at the instant when its radius is 4 cm .

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10. An edge of a variable cube is increasing at the rate of 3 cm second. How fast is the volume of the cube increasing when the edge is 10 cm long?
11. A cone has a fixed radius $r$ and a variable height $h$. Find the rate of change of its curved surface S w.r.t change in the radius.

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12. A right-circular cone has a fixed radius $r$ and a variable height h. Find the rate of change of its curved surface $S$ w.r.t. change in the height.

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13. A solid right circular cylinder has its radius equal to half of its height. Find the rate of change of its total surface area w.r.t. a change in the radius $r$.

## - Watch Video Solution

14. From a cylindrical drum containing oil and kept vertical, the oil leaking at the rate of $10 \mathrm{~cm}^{3} / \mathrm{s}$. If the radius of the durm is 10 cm and height is 50 cm , then find the rate at which level of oil is changing when oil level is 20 cm .

## - Watch Video Solution

15. Two men $A$ and $B$ start with velocities $v$ at the same time from the junction of two roads inclined at $45^{\circ}$ to each other. If they travel by different roads, find the rate at which they are being separated.
16. A kite is moving horizontally at a height of 141.5 m . If the speed of kite is $10 \mathrm{~m} / \mathrm{s}$, how fast is the string being let out when the distance (direct) between the kite and the boy (who is flying it) is 250 m ? The height of the boy is 1.5 m .

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17. For what $x, \sin x$ increases half as fast as $x$ ?

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18. A particle moves along the curve $6 y=x^{3}+2$. Find the points on the curve at which the $y$-coordinate is changing 8 times as fast as the $x$-coordinate.
19. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of $1.1 \mathrm{~m} / \mathrm{sec}$. How fast is the length of his shadow increasing when he is 1 m away from the pole?

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20. A man, 2 m tall, walks at the rate of $1 \frac{2}{3} \mathrm{~m} / \mathrm{sec}$ towards a street light which is $5 \frac{1}{3} \mathrm{~m}$ above the ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is $3 \frac{1}{3} \mathrm{~m}$ from the light.

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21. A ladder 5 m long is leaning against a wall. The bottom of the
ladder is pulled along the ground, away from the wall, at the rate
of $3 \mathrm{~cm} / \mathrm{sec}$. How fast is its height on the wall decreasing when the foot of the ladder is 3 m away from the wall?

## - Watch Video Solution

22. Water is leaking from a conical funnel at a uniform rate of
$4 \mathrm{~cm}^{3} / \mathrm{sec}$ thorugh at small hole at the vertex. When the slant height of the water in the funnel is 3 cm , find the rate of decrease of the slant height of water given that semi vertical angle of the cone (funnel) is $30^{\circ}$.

## - Watch Video Solution

23. Water is leaking from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at a uniform rate of $2 \mathrm{~cm}^{2} / \mathrm{sec}$ in the surface area curved), through a tiny hole at the vertex of the bottom. When the slant
height of the cose is 4 cm , find the rate of decrease of the slant height of water.

## - Watch Video Solution

24. Find the marginal cost and the marginal revenue when $\mathrm{x}=10$, the total cost $\mathrm{C}(\mathrm{x})$ and the total revenue $\mathrm{R}(\mathrm{x})$ in Rupees, associated with production and sale of x units of an item are given by $C(x)=0.003 x^{3}+0.02 x^{2}+20 x+1200 R(x)=0.1 x^{2}+25 x-100$

## - Watch Video Solution

25. The total cost $C(x)$ in Rupees, associated with the production of $x$ units of an item is given by $C(x)=0.005 x^{3}-0.02 x^{2}+30 x+5000$ Find the marginal cost
when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

## - Watch Video Solution

26. The radius of a cylinder is increasing at the rate of $2 \mathrm{~m} / \mathrm{sec}$ and its height is decreasintg at the rate of $3 \mathrm{~m} / \mathrm{sec}$. Find the rate at which the volume of the cylinder is changing when the radius is 3 m and height is 5 m .

## - Watch Video Solution

27. An airforce plane is ascending vertically at the rate of $100 \frac{\mathrm{~km}}{\mathrm{~h}}$. If the radius of the earth is $r \mathrm{~km}$, how fast is the area of the earth, visible from the plane, increasing at 3 minutes after it started
ascending? Given that the visible area $A$ at height $h$ is given by $A=2 \pi r^{2} \frac{h}{r+h}$

## - Watch Video Solution

28. A spherical ball of salt is dissovlging water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radis is decreasing at a constant rate.

## - Watch Video Solution

29. If the area of a circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.
30. A swimming pool is to be drained for cleaning. If $L$ represents the number of litres of water in the pool $t$ seconds after the pool has been plugged off to drain and $L=200(10-t)^{2}$. How fast is the water running cost at the end of 5 seconds? What is the average rate at which the water flows out during the first 5 seconds?

## - Watch Video Solution

31. Prove that the function $f(X)=a x+b$ is increasing iff $a>0$.

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32. Show that the function $f$ given by $f(X)=7 x-3$ is strictly increasing on R .
33. Show that the function $f(x)=\frac{x}{1+|x|}$ is a strict increasing function.

## - Watch Video Solution

34. Show that the function $f(x)=x-\frac{1}{x}$ is a strict increasing function fox $x>0$

## - Watch Video Solution

35. Show that $f(x)=2 x+\cot ^{-1} x+\log \left(\sqrt{1+x^{2}}-x\right)$ is increasing in R.

## D Watch Video Solution

36. Show that for a $\geq 1 f(x)=\sqrt{3} \sin x-\cos x-2 a x+b$ is decreasing in R .

## - Watch Video Solution

37. Show that the function $f$ given by
$f(x)=x^{3}-3 x^{2}+4 x, x \in R$ is strictly increasing on R.

## - Watch Video Solution

38. Show that $f(X)=x^{3}-6 x^{2}+18 x+5$ is an increasing function for all $x \in R$.

- Watch Video Solution

39. Show that the function $\mathrm{f}(\mathrm{X})=\left(x^{2}-2 x+2\right) e^{x}+3$ is an increasing function for all $x \in R$.

## - Watch Video Solution

40. Show that the function $x^{2}-x+1$ is neither increasing nor decreasing on ( 0,1 ).

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41. Indicate the interval in which the function $f(X)$
$=\cos x, 0 \leq x \leq 2 \pi$, decreases.

## - Watch Video Solution

42. Prove that the function $f(X)=\cos x$ is strictly decreasing $(0, \pi)$ and strictly increasing on $(\pi, 2 \pi)$

## - Watch Video Solution

43. Determine whether the following function is increasing or
decreasing in the given interval
$f(X)=\cos \left(2 x+\frac{\pi}{4}\right), \frac{3 \pi}{8} \leq x \leq \frac{5 \pi}{8}$.

## - Watch Video Solution

44. Prove that the function $f(x)=\tan x-4 x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

## - Watch Video Solution

45. Find for which values of ' $x$ ', the functions: $y=x^{4}-\frac{4 x^{3}}{3}$ is increasing and for which values, it is decreasing.

## D Watch Video Solution

46. Find the intervals in which the function $f$ given by $f(x)=4 x^{3}-6 x^{2}-72 x+30$ is decreasing.

## - Watch Video Solution

47. Determine the values of x for which the function $x+\frac{1}{x}$ is strictly increasing or decreasing

## - Watch Video Solution

48. Determine the values of $x$ for which the function $2 x^{3}-24 x+5$ is increasing or decreasing.

## - Watch Video Solution

49. Determine the intervals in which the function $f(x)=x^{3}+5 x^{2}-1$ is increasing or decreasing.

## - Watch Video Solution

50. Find the interval in which the function $f(X)=3 x^{4}-4 x^{3}-12 x^{2}+5$ is increasing or decreasing.
51. Determine the intervals in which the function $f(x)=x^{4}-8 x^{3}+22 x^{2}-24 x+21 \quad$ is decreasing or increasing.

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52. Find the intervals in which the function $f(X)=$
$(x-1)^{3}(x-2)^{2}$ is increasing or decreasing.

## - Watch Video Solution

53. Find the intervals in which the function ' $f$ ' given by:
$f(x)=\sin x-\cos x, 0 \leq x \leq 2 \pi$ is strictly increasing or decreasing.
54. Show that the function $f(x)=(\sin 3 x)$ is strict decreasing on $\left(0, \frac{\pi}{2}\right)$

## - Watch Video Solution

55. Show that the function $f$ given by
$f(x)=\tan ^{-1}(\sin x+\cos x), x>0$ is always an increasing function in $f,\left(0, \frac{\pi}{4}\right)$

## - Watch Video Solution

56. Find the intervals of increase and decrease of the function
$f(x)=\frac{\log x}{x}$.

## - Watch Video Solution

57. Find the intervals in which the function is increasing or decreasing:

$$
f(x)=\log (1+x)-\frac{2 x}{2+x}
$$

## - Watch Video Solution

58. Prove that $f(X)=x-[x]$ is increasing in $(0,1)$

## - Watch Video Solution

59. Find the interval of monotonocity of the function $f(X)=2 x^{2}-\log |x|, x \neq 0$ `

## D Watch Video Solution

60. Integrate the function: $\sin ^{-1}\left(\frac{2 x}{1+x^{2}}\right)$

## D Watch Video Solution

61. Find all a $\in R$ s.t. the function $f(X)=$ $(a+2) x^{3}-3 a x^{2}+9 a x-1$ decrease for all $\mathrm{x} \in \mathrm{R}$.

## - Watch Video Solution

62. Find the equation of the tangent to the curve $y=-5 x^{2}+6 x+7$ at the point $\left(\frac{1}{2}, \frac{35}{4}\right)$

## - Watch Video Solution

63. Find the equations of the tangent and the normal to each of the following curves at the given point
$y=x^{3} a t(2,8)$
64. Find the equations of the tangent and the normal to each of the following curves at the given point
$y=x^{3} a t(1,1)$

## - Watch Video Solution

65. Find the equations of the tangent and the normal to each of the following curves at the given point

$$
y=x^{4}-6 x^{3}+13 x^{2}-10 x+5 \text { at }(0,5)
$$

## - Watch Video Solution

66. Find the equations of the tangent and the normal to each of the following curves at the given point
$y=x^{2} a t(0,0)$

## - Watch Video Solution

67. Find the equations of the tangent and the normal to each of the following curves at the given point
$y=x^{4}-6 x^{3}+13 x^{2}-10 x+5$ at $(1,3)$

## - Watch Video Solution

68. Find the equations of the tangent and the normal to the following curves at the given point:

$$
20 x^{4}+y^{4}=36 \text { at }(-1,-2)
$$

## - Watch Video Solution

69. Find the equations of the tangent and the normal to the following curves at the given point:
$9 x^{2}+16 y^{2}=144 a t\left(x_{1} \cdot y_{1}\right)$ where $y_{1}=2, x_{1}>0$

## - Watch Video Solution

70. Find the equations of the tangent and the normal to the following curves at the given point:
$y=\sin ^{2} x$ at $x=\frac{\pi}{2}$

## - Watch Video Solution

71. Find the equations of the tangent and the normal to the following curves at the given point:
$y=\frac{x^{3}}{4-x} a t(2,4)$
72. Find the equations of the tangent and the normal to the following curves at the given point:
$x=\cos t, y=\sin t$ at $t=\frac{\pi}{4}$

## - Watch Video Solution

73. Find the equations of the tangent and the normal to the following curves at the given point:
$x=a \cos t, y=b \sin t$ at $t=\frac{\pi}{2}$

## - Watch Video Solution

74. Find the equations of the tangent and the normal to the following curves at the given point:
$x=a(t+\sin t), y=a(1-\cos t)$ at $t=\frac{\pi}{2}$

## - Watch Video Solution

75. Find the equations of the tangent and the normal to the following curves at the given point:
$x=1+\cos \theta, y=\theta+\sin \theta$ at $\theta=\pi / 4$

## - Watch Video Solution

76. Find the equations of the tangent and the normal to the following curves at the given point:
$x=1-\cos \theta, y=\theta-\sin \theta a t \theta=\frac{\pi}{2}$

## - Watch Video Solution

77. Find the equations of the tangent and the normal to the following curves at the given point:
$\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1 a t\left(x_{0}, y_{0}\right)$

## D Watch Video Solution

78. Find the equations of the tangent and the normal to the following curves at the given point:
$x=\sin 3 t, y=\cos 2 t$ at $\mathrm{t}=\frac{\pi}{4}$

## - Watch Video Solution

79. Find the equations of the tangent and the normal to the following curves at the given point:

$$
x^{2 / 3}+y^{2 / 3}=2 a t(1,1)
$$

80. Find the equations of the tangent to the following curves at the given point:
$x=a \sin ^{3} t, y=b \cos ^{3} t a t t=\frac{\pi}{4}$

## - Watch Video Solution

81. Find the equation of the tangent to the curve
$\frac{X^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1 a t\left(x_{0} . y_{0}\right)$

## - Watch Video Solution

82. Find the equation of the tangent to the curve $\sqrt{x}+\sqrt{y}=a$ at the point $\left(\frac{a^{2}}{4}, \frac{a^{2}}{4}\right) \mathrm{a}>0$
83. Find the equations of the tangent and the normal to the curve $y=x^{2}+4 x+1$ at the point whose abscissa is 3 .

## - Watch Video Solution

84. Show that the tangents to the curve $y=2 x^{3}-3$ at the points where $\mathrm{x}=2$ and $\mathrm{x}=-2$ are parallel.

## - Watch Video Solution

85. Prove that the tangents to the curve $y=x^{2}-5 x+6$ at the points $(2,0)$ and $(3,0)$ are at right-angles.
86. Find the point of intersection of the tangetn lines to the curve $y=2 x^{2}$ at the points ( 1,2 ) and ( $-1,2$ )

## - Watch Video Solution

87. Find the points on the curve $4 y=x^{3}$ where the slope of the tangent is $\frac{16}{3}$

## - Watch Video Solution

88. Find the point(s) on the curve: $y=3 x^{2}-12 x+6$ at which the tangent is parallel to $x$-axis.

## - Watch Video Solution

89. Find the equation of the tangent to the curve $x^{2}+3 y=3$, which is parallel to the line $y-4 x+5=0$

## - Watch Video Solution

90. Find the point on the curve $y=2 x^{3}-15 x^{2}+36 x-21$ at which the tangent is parallel to $x$-axis. Also, find the equation of tangents.

## - Watch Video Solution

91. Find points on the curve $\frac{x^{2}}{4}+\frac{y^{2}}{25}=1$ at which the tangents are parallel to x -axis.
92. Find points on the curve $\frac{x^{2}}{4}+\frac{y^{2}}{25}=1$ at which the tangents are parallel to $y$-axis.

## ( Watch Video Solution

93. Find the equations of the tangents to the curve:
$y=x^{3}+2 x-4$. Which are perpendicular to line $x+14 y+3=0$

## - Watch Video Solution

94. Find the equations of the tangents to the curve:
$y=x^{3}+2 x-4$. Which are perpendicular to line $\mathrm{x}+14 \mathrm{y}+3=0$

- Watch Video Solution

95. Find a point on the curve $y=(x-3)^{2}$ where the tangent is parallel to the chord joining $(3,0)$ and $(4,1)$

## - Watch Video Solution

96. At what point of the curve $y=x^{2}$ does the tangent make an angle of $45^{\circ} \mathrm{C}$ with the x -axis?

## - Watch Video Solution

97. Find the equations of the tangents to the curve $3 x^{2}-y^{2}=2$
which are perpendicular to the line $x+3 y=2$.

## - Watch Video Solution

98. Show that the curve $x y=a^{2}$ and $x^{2}+y^{2}=2 a^{2}$ touch each other.

## - Watch Video Solution

99. Prove that the curves $x y=4$ and $x^{2}+y^{2}=8$ touch each other.

## - Watch Video Solution

100. Prove that the curves $y^{2}=4 x$ and $x^{2}+y^{2}-6 x+1=0$ touch each other at the point $(1,2)$

## - Watch Video Solution

101. Find the equations of normal(s) to the curve $x^{2}+2 y^{2}-4 x-6 y+8=0$ at the point(s) whose absscissa is 2.

## - Watch Video Solution

102. Find the coordinates of the point on the curve $\sqrt{x}+\sqrt{y}=4$ at which tangent is equally inclined to the axes.

## - Watch Video Solution

103. Show that the equations of the normal at any point $\theta$ on the
curve

$$
\begin{equation*}
x=3 \cos \theta-\cos ^{3} \theta, y=3 \sin \theta-\sin ^{3} \theta \tag{is}
\end{equation*}
$$

$4\left(y \cos ^{3} \theta-x \sin ^{3} \theta\right)=3 \sin 4 \theta$

## - Watch Video Solution

104. Find the angle of intersection of the curves $y=x^{2}$ and $y=4-x^{2}$.

## - Watch Video Solution

105. Show that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=b e^{x / a}$ at the point where the curve intersect the axis of y .

## - Watch Video Solution

106. Find the equation of the tangent to the curve $y=\sqrt{3 x-2}$ which is parallel to the line $4 x-2 y+5=0$.

## - Watch Video Solution

107. Prove that the curves $x=y^{2}$ and $x y=k$ cut at right angles if $8 k^{2}=1$.

## D Watch Video Solution

108. Find the condition that the curves $2 x=y^{2}$ and $2 \mathrm{xy}=\mathrm{k}$ may intersect orthogonally.

## - Watch Video Solution

109. For the curve $y=5 x-2 x^{3}$, if x increases at te rate of 2units/s, then how fast is the slope of curve changing when $x=3$ ?

## - Watch Video Solution

110. In the following problems, use differentials to find the approximate values:
$\sqrt{0.39}$

## - Watch Video Solution

111. In the following problems, use differentials to find the approximate values:
$\sqrt{25.02}$

## D Watch Video Solution

112. In the following problems, use differentials to find the approximate values:
$(28)^{1 / 3}$
113. In the following problems, use differentials to find the approximate values:
$(80)^{1 / 4}$

## - Watch Video Solution

114. In the following problems, use differentials to find the approximate values:
$f(2.01) w h e r e f(x)=x^{3}-4 x+7$

## - Watch Video Solution

115. In the following problems, use differentials to find the approximate values:
$\log _{10}(1.01) \sim$ where $\log _{10} e=0.4343$.

## - Watch Video Solution

116. In the following problems, use differentials to find the approximate values:
$\sin 30.5^{\circ}$, when $\sin 30^{\circ}=0.5$ and $\cos 30^{\circ}=0.866$

## - Watch Video Solution

117. If $y=\sin x$ and $x$ change form $\frac{\pi}{2} \rightarrow \frac{22}{14}$, what is the approximate change in y ?

## - Watch Video Solution

118. A circular metal plate expands under heating so that its radius increases by $2 \%$. Find the approximate increase in the area of he plate if the radius of the plate before heating is 10 cm .

## - Watch Video Solution

119. If the side of a cube be increased by $0.1 \%$. Find the corresponding increase in the volume of the cube.

## - Watch Video Solution

120. Find the approximate change in the volume $V$ of a cube of side x meters caused by increasing the side by $2 \%$

## - Watch Video Solution

121. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm , then find the approximate error in calculating its volume.

## - Watch Video Solution

122. If the radius of a sphere is measured as 9 m with an error of 0.03 m , then find the approximate error in calculating its surface area.

## - Watch Video Solution

123. Find the maximum or minimum values of the functions.

$$
f(x)=16 x^{2}-16 x+28
$$

124. Find the maximum or minimum values of the functions.
$f(x)=\sin x \cos x$

## - Watch Video Solution

125. Find the maximum or minimum values of the functions.
$f(x)=\sin ^{4} x+\cos ^{4} x$

## - Watch Video Solution

126. Find the maximum or minimum values of the functions.

$$
f(x)=\sin ^{4} x+\cos ^{4} x
$$

## - Watch Video Solution

127. Find the maximum or minimum values of the functions.

$$
f(x)=\sin ^{6} x+\cos ^{6} x
$$

## ( Watch Video Solution

128. Find the maximum or minimum values of the functions.
$f(x)=3 \sin x-4 \cos x$

## - Watch Video Solution

129. Find the maximum and the minimum values, if any, of the following functions, without using the derivative:
$x+\frac{1}{x}, x>0$

## - Watch Video Solution

130. Find the maximum and the minimum values, if any, of the following functions, without using the derivative:
$11-7 \sin x$

## - Watch Video Solution

131. Find the minimum value of $f(X)=x^{3}-3 x \in(0,2)$

## - Watch Video Solution

132. Determine the maximum value of $f(x)=x+\sin 2 x \in$ [0,2pi]

## - Watch Video Solution

133. Determine the point of maximum of
$f(x)=\sin x+\cos x \in 0 \leq x \leq \frac{\pi}{2}$

## - Watch Video Solution

134. Prove that the following functions do not have maxima or
minima.
$e^{3 x}$

## - Watch Video Solution

135. Prove that the following functions do not have maxima or
minima.
$2 x^{3}+3 x^{2}+6 x+1$
136. Prove that the following functions do not have maxima or minima.
$4 x^{3}-18 x^{2}+27 x-7$

## D Watch Video Solution

137. Prove that the following functions do not have maxima or minima.
$\log (x+1) \cdot X>-1$

## - Watch Video Solution

138. Find the (absolute) maximum and minimum values of the
following functions in the indicated intervals also find points of
(absolute) maxima and minima.

$$
f(x)=x^{3}-3 x^{2}+3 x+8 \operatorname{in}(0,10)
$$

## D Watch Video Solution

139. Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$
f(x)=(x-2) \sqrt{x-1}, 1 \leq x \leq 9
$$

## - Watch Video Solution

140. Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$
f(x)=\frac{x+1}{\sqrt{x^{2}+1}}, 0 \leq x \leq 2
$$

## - Watch Video Solution

141. Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$
f(x)=x+\sin x \in(0,2 \pi)
$$

## - Watch Video Solution

142. Find the (absolute) maximum and minimum values of the following functions in the indicated intervals also find points of (absolute) maxima and minima.

$$
f(x)=3+|x+1| \text { in }(-2,3)
$$

## D Watch Video Solution

143. Find the absolute maximum value and the absolute minimum
value of the following functions in the given intervals:

$$
f(x)=\sin x+\cos x, x \in[0, \pi]
$$

## - Watch Video Solution

144. At what points in the interval $[0,2 \pi]$, does the function sin $2 x$ attain its maximum value?

## - Watch Video Solution

145. At what point, the slope of the crve : $y=-x^{3}+3 x^{2}+9 x-27$ is maximum? Also, find the maximum slope.

## - Watch Video Solution

146. What is the maximum value of the function $\sin x+\cos x$ ?

## (D) Watch Video Solution

147. Prove that $f(x)=\sin x+\sqrt{3} \cos \mathrm{x}$ has maximum value at x
$=\pi / 6$

## D Watch Video Solution

148. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$
f(x)=(x-1)(x+2)^{2}
$$

## - Watch Video Solution

149. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and
minimum values.

$$
f(x)=(x-1)^{3}(x+1)^{2}
$$

## D Watch Video Solution

150. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$
f(x)=\sin x(1+\cos x), 0<x<\frac{\pi}{2}
$$

## - Watch Video Solution

151. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$
f(x)=18 x e^{-x / 3}
$$

152. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.
$f(x)=(x-1)(x-2)(x-3)$

## - Watch Video Solution

153. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.
$f(x)=\frac{x^{4}}{x^{4}-1}, x \neq 1$

## - Watch Video Solution

154. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.

$$
f(x)=x \sqrt{32-x^{2}} \in(-5,5)
$$

## - Watch Video Solution

155. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.
$f(x)=\frac{\log x}{x}$

## - Watch Video Solution

156. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and
minimum values.

$$
f(x)=2 \cos x+x \in(0, \pi)
$$

## D Watch Video Solution

157. Find the maximum and minimum values.
$f(x)=\sin ^{4} x+\cos ^{4} x, 0 \leq x \leq \frac{\pi}{2}$

## - Watch Video Solution

158. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$$
f(X)=-\frac{3}{4} x^{4}-8 x^{3}-\frac{45}{2} x^{2}+105
$$

159. Find the points of local maxima and minima (if any) of each of the following functions. Find also the local maximum and minimum values.
$f(x)=\sec x+\log \cos ^{2} x, 0 \leq x \leq 2 \pi, x \neq \frac{\pi}{2}, \frac{3 \pi}{2}$

## D Watch Video Solution

160. Find the points of local maxima and local minima, if any, of the following functions. Find also the local maximm and local minimum values:

$$
f(x)=3 x^{4}+4 x^{3}-12 x^{2}+12
$$

## - Watch Video Solution

161. Show that a local minimum value of $f(x)=x+\frac{1}{x}, x \neq 0$ is greater than a local maximum value.

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162. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x)=\frac{x}{2}+\frac{2}{x}, x>0$

## - Watch Video Solution

163. Find the difference between the greatest and least values of the function $\mathrm{f}(\mathrm{x})=\sin 2 \mathrm{x}-\mathrm{x}$, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

## D Watch Video Solution

164. It is given that at $x=1$, the function $x^{4}-62 x^{2}+a x+9$ attains its maximum value, on the interval $[0,2]$. Find the value of
a.

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165. Amongst all pairs of positive numbers with product 64 find those whose sum is least

## - Watch Video Solution

166. Of all rectangles, each of which has perimeter: 40 cm . Find the one having maximum area. Also find that area.

## - Watch Video Solution

167. Prove that the area of right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
168. Find the area of the largest isosceles triangle having perimeter 18 metre.

## - Watch Video Solution

169. A wire 10 meter long is cut into two parts. One part is bent into the shape of a circle and the other into the shape of an equilateral triangle. How should the wire be cut so that the combined area of the two figures is as small as possible?

## D Watch Video Solution

170. Show that a cylinder of given volume, open at the top, has minimum total surface area if its height is equal to radius of the base.

## - Watch Video Solution

171. Find the altitude of the circular cylinder of maximum volume that can be inscribed in a sphere of radius a.

## - Watch Video Solution

172. Find the altitude of the circular cone of maximum volume that can be inscribed in a sphere of radius $r$.

## - Watch Video Solution

173. Show that the semi-vertical angle of the right-circular cone of maximum volume and of given slant height is $\cos ^{-1}\left(\frac{1}{\sqrt{3}}\right)$
174. The combined resistance of two resistors
$R_{1}$ and $R_{\circ}\left(R_{1}, R_{2}>0\right)$ is given by $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$. If $R_{1}+R_{2}=C$ (a constant), show that maximum resistance R is obtained by choosing $R_{1}=R_{2}$

## - Watch Video Solution

175. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height $h$ and semi vertical angle $\alpha$ is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27} \pi h^{3} \tan ^{2} \alpha$

## - Watch Video Solution

176. Show that the surface area of a closed cuboid with surface base and given volume is minimum when it is cube.

## - Watch Video Solution

177. A metal box with a square box and vertical sides is to contain $512 c^{3}$ of volume. The material for the top and bottom costs Rs. $5 \mathrm{~cm}^{2}$ and that for the sides costs Rs. $2.50 \mathrm{~cm}^{2}$. Find the least cost of the box.

## - Watch Video Solution

178. An isosceles triangle of vertical angle $2 \theta$ is inscribed in a circle of radius 'a' . Show that the area of triangle is maximum when $\theta=\frac{\pi}{6}$
179. If the sum of the lengths of hypotenuse and a side of a rightangled triangle is given. Show that the area is maximum, when the angle between then is $60^{\circ}$

## - Watch Video Solution

180. An open box with a square base is to be made out of a given iron sheet of area $c^{2}$ square units. Show that the maximum volume of the box is $\frac{c^{3}}{6 \sqrt{3}}$ cubic units.

## - Watch Video Solution

181. A point on the hypotenuse of a triangle is at distance $a$ and $b$ from the sides of the triangle.Show that the minimum length of
the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{2}{3}}$

## - Watch Video Solution

182. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible. When revolved about one of its sides. Also find the maximum volume.

## - Watch Video Solution

183. If the sum of the surface areas of cube and a sphere is costnat, what is the ratio of an edge of the cube to the diameter of the sphere, when the sum of their volumes is minimum?

## - Watch Video Solution

184. If the length of three sides of a trapezium other than base are equal to 10 cm , then find the area of trapezium when it is maximum.

## - Watch Video Solution

185. Find the shortest distance of the point $(0, c)$ from the parabola $y=x^{2}$ where $0 \leq c \leq 5$

## - Watch Video Solution

186. Let $A P$ and $B Q$ be two vertical poles at points $A$ and $B$
respectively. If $\mathrm{AP}=16 \mathrm{~m}, \quad \mathrm{BQ}=22$
$m$ and $A B=20 m$, then $f \in d$ thedis $\tan$ aceofap $\oint_{R} \quad$ on $\quad A B$ from the point A such that $R P^{2}+R Q^{2}$ is minimum.
187. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m , find the dimensions of the rectangle that will produce the largest area of the window.

## - Watch Video Solution

188. Find the rate of change of area of a square when its side is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$ and the length of the side is 10 cm.

## - Watch Video Solution

189. If the rate of change of volume of a sphere is numerically equal to that of the radius of the sphere, find the radius of the sphere.

## - Watch Video Solution

190. If the area of a circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

## - Watch Video Solution

191. Find a real number $\theta, 0<\theta<\frac{\pi}{2}$, which increases twice as fast as its sine.

## - Watch Video Solution

192. Find the rate of change of perimeter of a square when its side is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$ and the length of the side is 20 cm .

## - Watch Video Solution

193. Find the rate of change of perimeter of a square when its side is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$ and the length of the side is 20 cm .

## - Watch Video Solution

194. Find the rate of the change of surface area of a sphere when
its radius is increasing at the rate of $\frac{3}{\pi} \mathrm{~cm} / \mathrm{min}$ and its radius is 1 cm.
195. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.

## D Watch Video Solution

196. A particle is moving in a straight line such that its displacement $s$ at an instant of time $t$ is given by the relation $s=3 t^{2}+4 t+5$ Find the initial velocity of the particle.

## - Watch Video Solution

197. The rate of change of the volume of a sphere w.r.t. its surface area, when the radius is 2 cm , is
198. Prove that the function $f(x)=\tan x-4 x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

## - Watch Video Solution

199. Find the interval in which the function $f(X)=x^{4}-\frac{4 x^{3}}{3}$ is strictly increasing.

## - Watch Video Solution

200. Show that the function $f$ given by
$f(x)=\tan ^{-1}(\sin x+\cos x), x>0$ is always an increasing function in $f,\left(0, \frac{\pi}{4}\right)$
201. State whether the function $f(x)=x^{3}$ is increasing or decreasing.

## - Watch Video Solution

202. State whether the function $f(x)=e^{x}$ is increasing or decreasing.

## - Watch Video Solution

203. Is the function $\mathrm{y}=\cot \mathrm{x}$ decreasing on the interval $(0, \pi)$ ?

## - Watch Video Solution

204. Find the interval of increase of the function $\mathrm{f}(\mathrm{x})=\tan ^{-1} x$.

## D Watch Video Solution

205. Find the interval of decrease of the function $f(X)=\sin x-2 x$

## - Watch Video Solution

206. Find the interval of increase of the function $f(x)=3 x+\cos x$

## - Watch Video Solution

207. Find the slope of the tangent to the curve $y=3 x^{2}$ at the point $\left(\frac{1}{6}, \frac{1}{12}\right)$
208. For the curve $y=5 x-2 x^{3}$, if x increases at te rate of 2units/s, then how fast is the slope of curve changing when $x=3$ ?

## - Watch Video Solution

209. Find the slope of the normal to the curve $y=x^{3}$ at the point whose abscissa is 2.

## - Watch Video Solution

210. At what point on the curve $y=x^{2}-2 x$, the tangent is parallel to $x$-axis.

## - Watch Video Solution

211. At what point on the curve $y=2 x^{2}-4 x+3$, the normal is parallel to $Y$-axis.

## - Watch Video Solution

212. Find the slope of the normal to the curve $y=\frac{1}{x}$ at the point $\left(\frac{1}{3}, 3\right)$

## - Watch Video Solution

213. Find the equation of the tangent to the curve $y=x^{2}-x+2$ where it crosses the $y$-axis.

## - Watch Video Solution

214. Find the change in the value of $y=x^{2}$ when $\mathrm{x}=10$ and $\Delta x=0.01$

## - Watch Video Solution

215. Find the approximate volume of metal in a hollow spherical shel, whose internal and external radii are 3 cm and 3.0005 cm respectively.

## - Watch Video Solution

216. Find the maximum value of $\sin x \cos x$.
217. Find the maximum vlaue $\frac{1}{x^{2}+x+1}, \mathrm{x} \in \mathrm{R}$

## - Watch Video Solution

218. Prove that $f(x)=\sin x+\sqrt{3} \cos \mathrm{x}$ has maximum value at x
$=\pi / 6$

## - Watch Video Solution

219. Write down the maximum value of $a \sin x+b \cos x, x \in R$.

## - Watch Video Solution

220. Does the function $\mathrm{f}(\mathrm{x})=3^{x}$ have any extreme point?
221. Using differentials, find the approximate value of $\sqrt{0.082}$

## - Watch Video Solution

222. Find the minimum value of $f(x)=x-x^{2}$

## D Watch Video Solution

223. What is the minimum value of $x^{x}, x>0$ ?

## - Watch Video Solution

224. Find the maximum value of $\mathrm{f}(X)=2 x^{2}-x+1$

## - Watch Video Solution

225. If $f(X)$ attains a local minimum value at $x=c$ and $f(c)>0$ , then what can you say about $f^{\prime}(c)$ ?

## - Watch Video Solution

226. If a function $f(x)$ has a local extremum at a point $x=c$, then what can be said about $\mathrm{f}^{\prime}(\mathrm{c})$ ?

## - Watch Video Solution

227. If $\mathrm{x}>0$, what is the minimum value of $x+\frac{4}{x}$

## - Watch Video Solution

228. If $\mathrm{x}<0$, what is the maximum value of $x+\frac{1}{x}$

## - Watch Video Solution

229. For what value of x , does the function $f(x)=x^{2 / 3}$ have a local minimum value?

## - Watch Video Solution

230. Is every critical point, a point of local maximum or local minimum.

## - Watch Video Solution

231. Write down the minimum value of $\sqrt{3} \sin x+\cos x$

## - Watch Video Solution

232. Discuss the local maximum or minimum of $f(x)=x^{3} a t x=0$

## - Watch Video Solution

233. The amount of pollution content added in air in a city due to x -d i es el vehicles is given by $P(x)=0.005 x^{3}+0.02 x^{2}+30 x$.

Find the arginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

## - Watch Video Solution

234. Find the rate of change of the area of a circle with respect to its radius $r$ when $r=3 \mathrm{~cm}$.
235. Find the rate of change of the area of a circle with respect to its radius $r$ when $r=4 c m$

## D Watch Video Solution

236. The volume of a cube is increasing at the rate of $8 c \frac{m^{3}}{s}$. How fast is the surface area increasing when the length of an edge is 12 cm ?

## - Watch Video Solution

237. The radius of a circle is increasing uniformly at the rate of 3
$\mathrm{cm} / \mathrm{s}$. Find the rate at which the area of the circle is increasing when the radius is 10 cm .
238. An edge of a variable cube is increasing at the rate of $3 \mathrm{~cm} / \mathrm{s}$. How fast is the volume of the cube increasing when the edge is 10 cm long?

## - Watch Video Solution

239. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 7.5 cm , how fast is the enclosed area increasing?

## - Watch Video Solution

240. The radius of a circle is increasing at the rate of $0.7 \mathrm{~cm} / \mathrm{s}$.

What is the rate of increase of its circumference?

## - Watch Video Solution

241. The length ' $x$ ' of a rectangle is decreasing at the rate of 5 cm per minute and the width 'y' is increasing at the rate of 4 cm per minute, when $x=8 \mathrm{~cm}$ and $y=6 \mathrm{~cm}$, find the rate of change of the perimeter of the rectangle.

## D Watch Video Solution

242. The length $x$ of a rectangle is decreasign at the rate of 5 $\mathrm{cm} / \mathrm{min}$ and the width y is incresaing at the rate of $4 \mathrm{~cm} / \mathrm{min}$. when $x=8 \mathrm{~cm}$ and $\mathrm{y}=6 \mathrm{~cm}$, find the rates of change of the area of the rectangle.

## D Watch Video Solution

243. A balloon which always remains spherical is being inflated by .pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm .

## - Watch Video Solution

244. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm .

## - Watch Video Solution

245. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of $2 \mathrm{~cm} / \mathrm{s}$. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

## - Watch Video Solution

246. A particle moves along the curve $6 y=x^{3}+2$. Find the points on the curve at which the $y$-coordinate is changing 8 times as fast as the $x$-coordinate.

## - Watch Video Solution

247. The radius of an air bubble is increasing at the rate of $\frac{1}{2} c \frac{m}{s}$. At what rate is the volume of the bubble increasing when the radius is 1 cm ?

## - Watch Video Solution

248. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2 x+1)$. Find the rate of change of its volume with respect to x .

## - Watch Video Solution

249. Sand is pouring from a pipe at the rate of 12 cubic $\mathrm{cm} . / \mathrm{sec}$.

The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. At which rate is the height of the sand-cone increasing when the height is 4 cm . ?
250. The total cost $C(x)$ in Rupees associated with the production of $x$ units of an item is given by $C(x)=0.007 x^{3}+0.003 x^{2}+15 x+4000$ Find the marginal cost when 17 units are produced.

## - Watch Video Solution

251. The total revenue in Rupees received from the sale of $x$ units of a product is given by $R(x)=13 x^{2}+26 x+15$ Find the marginal revenue when $x=7$.
A. Choose the correct answer in the exercises 17 and 18.
B.
C.
D.

## Watch Video Solution

252. Find the rate of change of the area of a circle with respect to its radius $r$ at $r=6 \mathrm{~cm}$
A. $10 \pi$
B. $12 \pi$
C. $8 \pi$
D. $11 \pi$

## Answer:

## - Watch Video Solution

253. The total revenue in Rupees received from its sale of $x$ units of a product is given by $R(X)=3 x^{2}+36 x+5$. Find the
marginal revenue, when $x=15$
A. 116
B. 96
C. 90
D. 126

## Answer:

## - Watch Video Solution

254. Show that the function given by $f(x)=3 x+17$ is increasing on R .

## - Watch Video Solution

255. Show that the function given by $f(x)=e^{2} x$ is increasing on R.

## - Watch Video Solution

256. $f(x)=\sin x$ is a strictly increasing in $\left(0, \frac{\pi}{2}\right)$

## - Watch Video Solution

257. $f(x)=\sin x$ is a strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

## - Watch Video Solution

258. Show that the function given by $f(x)=\sin x$ is neither increasing nor decreasing in $(0, \pi)$.
259. Find the intervals in which the function $f$ given by $f(x)$ $=2 x^{2}-3 x$ is
A. strictly increasing
B. strictly decreasing.
C.
D.

## Answer:

## - Watch Video Solution

260. Find the intervals in which the function f, given by $f(x)=2 x^{3}-3 x^{2}-36 x+7$ is strictly increasing and strictly

## decreasing.

A. strictly increasing
B. strictly decreasing.
C.
D.

## Answer:

## - Watch Video Solution

261. Find the intervals in which the following functions are strictly increasing or decreasing: $x^{2}+2 x-5$

## - Watch Video Solution

262. Find the intervals in which the following functions are strictly increasing or decreasing: $10-6 x-2 x^{2}$

## - Watch Video Solution

263. Find the intervals in which the following functions are strictly increasing or decreasing: $-2 x^{3}-9 x^{2}-12 x+1$

## - Watch Video Solution

264. Find the intervals in which the following functions are strictly increasing or decreasing: $-2 x^{3}-9 x^{2}-12 x+1$

## - Watch Video Solution

265. Find the intervals in which the following functions are strictly increasing or decreasing
$(x+!)^{3}(x-3)^{3}$

## - Watch Video Solution

266. Show that $y=\log (1+x)-2 \frac{x}{2+x}, x \succ 1$, is an increasing function of $x$.throughout its domain.

## - Watch Video Solution

267. Find the values of x for which $y=[x(x-2)]^{2}$ is an increasing function.

## - Watch Video Solution

268. Prove that $y=4 \frac{\sin \theta}{2+\cos \theta}-\theta$, is an increasing function of $\theta$ in $\left[0, \frac{\pi}{2}\right]$.

## - Watch Video Solution

269. Prove that the logarithmic function is increasing on $(0, \infty)$.

## - Watch Video Solution

270. Prove that the function f given by $f(x)=x^{2}-x+1$ is neither strictly increasing nor decreasing on $(-1,1)$.

## D Watch Video Solution

271. Which of the following functions are strictly decreasing on
$\left(0, \frac{\pi}{2}\right) ?$
A. $\cos x$
B. $\operatorname{cox} 2 x$
C. $\cos 3 x$
D. $\tan x$

## Answer:

## - Watch Video Solution

272. On which of the following intervals is the function $f$ given by $f(x)=x^{100}+\sin x-1$ decreasing ?
A. $(0,1)$
B. $\left(\frac{\pi}{2}, \pi\right)$
C. $0, \frac{\pi}{2}$ )
D. none of these

## Answer:

## - Watch Video Solution

273. Find the least value of a such that the function $f$ given by $f(X)=x^{2}+a x+1$ is strictly increasing on (1,2)

## - Watch Video Solution

274. Let 1 be any interval disjoint form ( $-1,1$ ). Prove that the function $f(x)=x+\frac{1}{x}$ is strictly increasing on $I$.

## - Watch Video Solution

275. Prove that the function $f$ given by $f(x)=\log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

## - Watch Video Solution

276. Prove that the function $f$ given by $f(x)=\log (\cos x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

## - Watch Video Solution

277. Prove that the function given by $f(x)=x^{3}-3 x^{2}+3 x-100$ is increasing in R.

## - Watch Video Solution

278. The interval in which $y=x^{2} e^{-x}$ is increasing is:
A. $(-\infty, \infty)$
B. $(-2,0)$
C. $(2, \infty)$
D. $(0,2)$

## Answer:

## D Watch Video Solution

279. Find the slope of the tangent to the curve $y=3 x^{4}-4 x$ at x
$=4$.

## - Watch Video Solution

280. Find the slope of the tangent to the curve $y=\frac{x-1}{x-2}$ at $\mathrm{x}=$ 10.
281. Find the slope of the tangent to curve $y=x^{3}-x+1$ at the point whose $x$-coordinate is 2 .

## - Watch Video Solution

282. Find the slope of the tangent to the curve $y=x^{3}-3 x+2$
at the point whose x -coordinate is 3 .

## - Watch Video Solution

283. Find the slope of the normal to the curve $x=$ $a \cos ^{3} 0, y=a \sin ^{3} 0 a t 0=\frac{\pi}{2}$

## - Watch Video Solution

284. Find the slope of the nromal to the curve $x=1+a \sin t, y=-b \cos ^{2} t a t t=\frac{\pi}{2}$

## - Watch Video Solution

285. Find points at which the tangent to the curve $y=x^{3}-3 x^{2}-9 x+7$ is parallel to the x -axis.

## - Watch Video Solution

286. Find a point on the curve $y=(x-2)^{2}$ at which the tangent is parallel to the chord joining the points $(2,0)$ and $(4,4)$

## - Watch Video Solution

287. Find the point on the curve $y=x^{3}-11 x+5$ at which the tangent is $y=x-11$

## D Watch Video Solution

288. Find the equation of all lines having slope - 1 that are tangents to the curve $y=\frac{1}{x-1}, x \neq-1$

## - Watch Video Solution

289. Find the equation of all lines having slope 2 which are tangent to the curve $y=\frac{1}{x-3}, x \neq 3$.

## - Watch Video Solution

290. Find the equations of all lines having slope 0 which are tangent to the curve $y=\frac{1}{x 2-2 x+3}$

## - Watch Video Solution

291. Find points on the curve $\frac{x^{2}}{9}+\frac{y^{2}}{16}=1$ at which the tangents are parallel to $x$-axis

## - Watch Video Solution

292. Find points on the curve $\frac{x^{2}}{9}+\frac{y^{2}}{16}=1$ at which the tangents are parallel to $y$-axis

## - Watch Video Solution

293. Find the equations of the tangent and normal to the given
curves at the indicated points:
$y=x^{4}-6 x^{3}+13 x^{2}-10 x+5 a t(0,5)$

## - Watch Video Solution

294. Find the equations of the tangent and normal to the given indicated points:
$y=x^{4}-6 x^{3}+13 x^{2}-10 x+5 a t(1,3)$

## D Watch Video Solution

295. Find the equations of the tangent and normal to the given curves at the points given $y=x^{3} a t(1,1)$
296. Find the equations of the tangent and normal to the given curves at the points given $y=x^{2} a t(0,0)$

## - Watch Video Solution

297. Find the equations of the tangent and normal to the given curves at the points given $x=\cos t, y=\sin t a t \mathrm{t}=(\mathrm{pi}) /(4)^{\prime}$

## - Watch Video Solution

298. Find the equation of the tangent line to the curve $y=x^{2}-2 x+7$, which is parallel to the line $2 x-y+9=0$.

## - Watch Video Solution

299. Find the equation of the tangent line to the curve $y=x^{2}-2 x+7$ which is perpendicular to the line $5 y-15 x=13$

## - Watch Video Solution

300. Show that the tangents to the curve $\mathrm{y}=7 x^{3}+11$ at the points $x=2$ and $x=-2$ are parallel.

## - Watch Video Solution

301. Show that the tangents to the curve $y=2 x^{3}-3$ at the points where $x=2$ and $x=-2$ are parallel.

## - Watch Video Solution

302. Find the points on the curve $y=x^{3}$ at which the slope of the tangent is equal to the $y$-coordinate of the point.

## - Watch Video Solution

303. For the curve $y=4 x^{3}-2 x^{5}$, find all the points at which the tangent passes through the origin.

## - Watch Video Solution

304. Find the points on the curve $x^{2}+y^{2}-2 x-3=0$ at which the tangents are parallel to the x -axis.

## - Watch Video Solution

305. Find the equation of the normal at the point $\left(a m^{2}, a m^{3}\right)$ for the curve $a y^{2}=x^{3}$

## - Watch Video Solution

306. Find the equation of the normals to the curve $y=x^{3}+2 x+6$ which are parallel to the line $x+14 y+4=0$

## - Watch Video Solution

307. Find the equations of the tangent and normal to the parabola $y^{2}=4 a x$ at the point $\left(a t^{2}, 2 a t\right)$
308. Prove that the curves $x=y^{2}$ and $x y=k$ cut at right angles if $8 k^{2}=1$.

## - Watch Video Solution

309. Find the equations of the tangent and normal to the hyperbola $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ at the point $\left(x_{0}, y_{0}\right)$

## - Watch Video Solution

310. Find the equation of the tangent to the curve $y=\sqrt{3} x-2$, which is parallel to the line $4 x-2 y+5=0$.

## - Watch Video Solution

311. The slope of the normal to the curve $y=2 x^{2}+3 \sin x$ at $x=0$ is:
A. 3
B. $\frac{1}{3}$
C. -3
D. $-\frac{1}{3}$

## Answer:

## - Watch Video Solution

312. The line $y=x+1$, is a tangent to the curve $y^{2}=4 x$ at the point:
A. $(1,2)$
B. $(2,1)$
C. $(1,-2)$
D. $(-1,2)$

## Answer:

## - Watch Video Solution

313. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{25.3}$

## - Watch Video Solution

314. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{49.5}$
315. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\sqrt{0.6}$

## - Watch Video Solution

316. Using differentials, find the approximate value of each of the following up to 3 places of decimal.
$(0.009)^{1 / 3}$

## - Watch Video Solution

317. Using differentials, find the approximate value of each of the following up to 3 places of decimal.
$(0.999)^{1 / 10}$
318. Using differentials, find the approximate value of each of the following up to 3 places of decimal.
$(15)^{1 / 4}$

## - Watch Video Solution

319. Using differntials, find the approximate value of each of the folloiwng upto 3 places of decimal
$(0.007)^{\frac{1}{3}}$

## - Watch Video Solution

320. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $\frac{(255)^{1}}{4}$
321. Using differntials, find the approximate value of each of the folloiwng upto 3 places of decimal $(255)^{\frac{1}{4}}$

## D Watch Video Solution

322. Using differentials, find the approximate value of each of the following up to 3 places of decimal.
$(401)^{1 / 2}$

## - Watch Video Solution

323. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(0.0037)^{\frac{1}{2}}$

## - Watch Video Solution

324. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(26.57)^{\frac{1}{3}}$

## - Watch Video Solution

325. Using differentials, find the approximate value of each of the following up to 3 places of decimal: $(81.5)^{\frac{1}{4}}$

## - Watch Video Solution

326. Using differentials, find the approximate value of each of the following up to 3 places of decimal: `(3.968)^ $3 / 2$
327. Using differentials, find the approximate value of each of the following up to 3 places of decimal.
$(32.15)^{1 / 5}$

## - Watch Video Solution

328. Find the approximate value of $f(2.01)$ where

$$
f(x)=4 x^{2}+5 x+2 .
$$

## - Watch Video Solution

329. Find the approximate value of $f(5 \cdot 001)$, where $f(x)=x^{3}-7 x^{2}+15$.
330. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by $1 \%$.

## - Watch Video Solution

331. If the radius of a sphere is measured as 7 m with an error of 0.02 m , then find the approximate error in calculating its volume.

## - Watch Video Solution

332. If the radius of a sphere is measured as 9 m with an error of 0.03 m , then find the approximate error in calculating its surface area.
333. If $f(x)=3 x^{2}+15 x+5$, then the approximate value of f (3.02) is :
A. 47.66
B. 57.66
C. 67.66
D. 77.66

## Answer:

## - Watch Video Solution

334. The approximate change in the volume of a cube of side $x$ metres caused by increasing the side by $3 \%$ is
A. $0.06 x^{3} m^{3}$
B. $0.6 x^{3} m^{3}$
C. $0.09 x^{3} m^{3}$
D. $0.9 x^{3} m^{3}$

## Answer:

## - Watch Video Solution

335. Find the maximum and minimum values, if any, of the following functions given by: $f(x)=(2 x-1)^{2}+3$

## - Watch Video Solution

336. Find the maximum and minimum values, if any, of the following functions given by: $f(x)=9 x^{2}+12 x+2$
337. Find the maximum and minimum values, if any, of the following functions given by: $f(x)=-(x-1)^{2}+10$

## - Watch Video Solution

338. Find the maximum and minimum values, if any, of the following functions given by: $g(x)=x^{3}+1$

## - Watch Video Solution

339. Find the maximum and minimum values, if any, of the following functions given by: $f(x)=|x+2|-1$

## - Watch Video Solution

340. Find the maximum and minimum values, if any, of the following functions given by: $g(x)=-|x+1|+3$

## - Watch Video Solution

341. Find the maximum and minimum values, if any, of the following functions given by: $h(x)=\sin (2 x)+5$

## - Watch Video Solution

342. Find the maximum and minimum values, if any, of the following functions given by: $f(x)=|\sin (4 x)+3|$

## - Watch Video Solution

343. Find the maximum and minimum values, if any, of the following functions given by: $h(x)=x+1, x \in(-1,1)$

## D Watch Video Solution

344. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$
f(x)=x^{2}
$$

## D Watch Video Solution

345. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$
g(x)=x^{3}-27 x
$$

## - Watch Video Solution

346. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$
k(x)=\sin x+\cos x, 0<x<\mathrm{pi} / 2^{`}
$$

## - Watch Video Solution

347. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be

$$
f(x)=\sin x-\cos x, 0<x<2 \pi
$$

## - Watch Video Solution

348. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x)=x^{3}-6 x^{2}+9 x+15$

## - Watch Video Solution

349. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x)=\frac{x}{2}+\frac{2}{x}, x>0$

## - Watch Video Solution

350. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x)=\frac{1}{x^{2}+2}$
351. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be: $f(x)=x \sqrt{1-x}$

## - Watch Video Solution

352. Prove that the following functions do not have maxima or minima: $f(x)=e^{x}$

## - Watch Video Solution

353. Prove that the following functions do not have maxima or minima: $g(x)=\log x$
354. Prove that the following functions do not have maxima or minima: $h(x)=x^{3}+x^{2}+x+1$

## - Watch Video Solution

355. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:
$f(x)=x^{3}, x \in[-2,2]$

## - Watch Video Solution

356. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:
$f(x)=\sin x+\cos x, x \in[0, \pi]$
357. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals: $f(x)=4 x-\frac{1}{2} x^{2}, x \in\left[-2, \frac{9}{2}\right]$

## - Watch Video Solution

358. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:
$f(x)=(x-1)^{2}+3, x \in[-3,1]$

## - Watch Video Solution

359. Find both the maximum value and the minimum value of $3 x^{4}-8 x^{3}+12 x^{2}-48 x+25$ on the interval $[0,3]$
360. At what points in the interval $[0,2 \pi]$, does the function sin $2 x$ attain its maximum value?

## - Watch Video Solution

361. What is the maximum value of the function $\sin x+\cos x$ ?

## D Watch Video Solution

362. Find the maximum value of $2 x^{3}-24 x+107$ in the interval
$[1,3]$. Find the maximum value of the same function in
$[-3,-1]$.
363. It is given that at $x=1$, the function $x^{4}-62 x^{2}+a x+9$ attains its maximum value, on the interval $[0,2]$. Find the value of
a.

## - Watch Video Solution

364. Find the maximum and minimum values of $x+\sin 2 x$ on $[0,2 \pi]$

## - Watch Video Solution

365. . Find two numbers whose sum is 24 and whose product is as large as possible.

## - Watch Video Solution

366. Find two positive numbers x and y such that $x+y=60$ and $x y^{3}$ is maximum.

## - Watch Video Solution

367. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

## - Watch Video Solution

368. A square piece of tin of side 12 cm is to be made into a box without a top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of square to be cut off'. so that the volume of box is maximum and also find the volume of box.
369. A rectangular sheet of tin $45 \mathrm{~cm} \times 24 \mathrm{~cm}$ is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

## - Watch Video Solution

370. Show that of all rectangles inscribed in a given circle the square has maximum area.

## - Watch Video Solution

371. Show that the rectangle of maximm area that can be inscribed in a circle of radius $r$ is a square of side $\sqrt{2} r$
372. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

## - Watch Video Solution

373. If all the closed cylindrical cans (right circular), which enclose a given volume of 100 cubic centimeters. Find the dimensions of the can which has the minimum surface area.

## - Watch Video Solution

374. A wire of length 28 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle.

What should be the length of two pieces so that the combined area of the square and the circle is minimum?

## - Watch Video Solution

375. Prove that volume of largest cone, which can be inscribed in a sphere, is $\left(\frac{8}{27}\right)^{t h}$ part of volume of sphere.

## - Watch Video Solution

376. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.

## - Watch Video Solution

377. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan ^{-1} \sqrt{2}$.

## - Watch Video Solution

378. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin ^{-1}\left(\frac{1}{3}\right)$.

## - Watch Video Solution

379. The point on the curve $x^{2}=2 y$ which is nearest to the point $(0,5)$ is:
A. $(2 \sqrt{2}, 4)$
B. $(2 \sqrt{2}, 0)$
C. $(0,0)$
D. $(2,2)$

## Answer:

## D Watch Video Solution

380. For all real values of x , the minimum value of $\frac{1-x+x^{2}}{1+x+x^{2}}$ is:
A. 0
B. 1
C. 3
D. $\frac{1}{3}$

## Answer:

381. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$ is:
A. $\left(\frac{1}{3}\right)^{1 / 3}$
B. $\frac{1}{2}$
C. 1
D. 0

## Answer:

## - Watch Video Solution

382. Using differentials, find an approximate value of each of the
following:
$\left(\frac{17}{81}\right)^{\frac{1}{4}}$
383. Using differentials, find an approximate value of each of the following:


## - Watch Video Solution

384. Show that the function given by $f(x)=\frac{\log x}{x}$ has maximum at $x=e$

## - Watch Video Solution

385. The two equal sides of an isosceles triangle with fixed base $b$ are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?
386. Find the slope of the normal to curve $x y=6$ at point $(1,6)$

## - Watch Video Solution

387. Show that the normal at any point $\theta$ to the curve $x=a \cos \theta+a \theta \sin \theta, y=a \sin \theta-a \theta \cos \theta$ is at a constant distance from the origin.

## - Watch Video Solution

388. Find the intervals in which the function $f$ given by $f(x)=\frac{4 \sin x-2 x-x \cos x}{2+\cos x}$ is increasing.
389. Find the intervals in which the function $f$ given by $f(x)=\frac{4 \sin x-2 x-x \cos x}{2+\cos x}$ is increasing.

## - Watch Video Solution

390. Find the intervals in which the function $f$ given by $f(x)=x^{3}+\left(\frac{1}{x^{3}}\right), x \neq 0$ is increasing.

## D Watch Video Solution

391. Find the intervals in which the function $f$ given by $f(x)=x^{3}+\left(\frac{1}{x^{3}}\right), x \neq 0$ is decreasing.

## - Watch Video Solution

392. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ with its vertex at one end of the major axis.

## - Watch Video Solution

393. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $\frac{8}{m^{3}}$. If building of tank costs Rs 70 per sq metres for the base and Rs 45 per square metre for sides. What is the cost of least expensive tank?

## D Watch Video Solution

394. The sum of the perimeter of a circle and square is $k$, where $k$ is some constant. Prove that the sum of their areas is least when
the side of square is double the radius of the circle.

## - Watch Video Solution

395. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 20 m .

Find the dimensions of the window to admit maximum light through the whole opening.

## - Watch Video Solution

396. A point on the hypotenuse of a triangle is at distance a and
$b$ from the sides of the triangle.Show that the minimum length of
the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{2}{3}}$

## - Watch Video Solution

397. Find the points at which the function $f$ given by $f(x)=(x-2)^{4}(x+1)^{3}$ has local maxima.

## - Watch Video Solution

398. Find the points at which the function $f$ given by $f(x)=(x-2)^{4}(x+1)^{3}$ has local minima.

## D Watch Video Solution

399. Find the points at which the function $f$ given by $f(x)=(x-2)^{4}(x+1)^{3}$ has point of inflexion.

## - Watch Video Solution

400. Find the absolute maximum and minimum values of the function f given by $f(x)=\cos ^{2} x+\sin x, x \in[0, \pi]$

## - Watch Video Solution

401. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius $r$ is $4 \frac{r}{3}$.

## - Watch Video Solution

402. Let f be a function defined on $[a, b]$ such that $f^{\prime}(x)>0$ for all $x \in(a, b)$. Then prove that f is an increasing function on $(a, b)$.

## - Watch Video Solution

403. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $2 \frac{R}{\sqrt{3}}$. Also find the maximum volume.

## - Watch Video Solution

404. Show that height of the cylinder of greatest volume which
can be inscribed in a right circular cone of height $h$ and semi vertical angle $\alpha$ is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27} \pi h^{3} \tan ^{2} \alpha$

## Watch Video Solution

405. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of:
A. $1 \mathrm{~m} / \mathrm{h}$
B. $0.1 \mathrm{~m} / \mathrm{h}$
C. $1.1 \mathrm{~m} / \mathrm{h}$
D. $0.5 \mathrm{~m} / \mathrm{h}$

## Answer:

## - Watch Video Solution

406. The slope of the tangent to the curve $x=t^{2}+3 t-8, y=2 t^{2}-2 t-5$ at the point (2,-1) is:
A. $\frac{22}{7}$
B. $\left(\frac{6}{7}\right)$
C. $\frac{7}{6}$
D. $-\frac{6}{7}$

## Answer:

## - Watch Video Solution

407. The line $y=m x+1$, is a tangent to the curve $y^{2}=4 x$ if the value of $m$ is:
A. 1
B. 2
C. 3
D. $\frac{1}{2}$

## Answer:

- Watch Video Solution

408. The normal at the point $(1,1)$ on the curve $2 y+x^{2}=3$ is:
A. $x+y=0$
B. $x-y=0$
C. $x+y+1=0$
D. $x-y+1=0$

## Answer:

- Watch Video Solution

409. The normal to the curve $x^{2}=4 y$ passing (1,2) is:
A. $x+y=3$
B. $x-y=3$
C. $x+y=1$
D. $x-y=1$

## Answer:

## - Watch Video Solution

410. The points on the curve $9 y^{2}=x^{3}$, where the normal to the curve makes equal intercepts with the axes are:
A. $\left(4 \pm \frac{8}{3}\right)$
B. $\left(4, \frac{-8}{3}\right)$
C. $\left(4, \pm \frac{3}{8}\right)$
D. $\left( \pm 4, \frac{3}{8}\right)$

## Answer:

411. The rate of change of the volume of a sphere w.r.t. its surface area, when the radius is 2 cm , is

## - Watch Video Solution

412. Fill in the blanks:

Derivative of $\sin \mathrm{x}$ w.r.t $\cos \mathrm{x}$ is $\qquad$

## - Watch Video Solution

413. Fill in the blanks:

The side (Variable) of a square is $x$, then the rate of change of area of the square w.r.t. its perieter is $\qquad$ .

## - Watch Video Solution

414. Fill in the blanks:

The slope of the tangent to the curve $y^{2}=x^{3}$ at the origin is

## - Watch Video Solution

415. Fill in the blanks:

The function $f(x)=x^{3}$ is a strict $\qquad$ function.

## - Watch Video Solution

416. Fill in the blanks:

The tangent to the curve $y=x^{3}$ at the point $(4,8)$ is $\qquad$ .

- Watch Video Solution

417. Fill in the blanks:

When $0 l a<1, x>0$, the function $\mathrm{y}=\log _{a} \mathrm{x}$ is a function.

## - Watch Video Solution

418. At $x=0$, the function $f(x)=|x|$ is

## - Watch Video Solution

419. The values of a for which $y=x^{2}+a x+25$ touches the axis of $x$ are

- Watch Video Solution

420. Fill in the blanks:

The function $f(x)=x+\frac{9}{x}$ has a local___ value at $\mathrm{x}=3$.

## - Watch Video Solution

421. Fill in the blanks:

An approximate value of $\sqrt{66}$ is $\qquad$ .

## - Watch Video Solution

422. Fill in the blanks:

An approximate value of $\sqrt[3]{28}$ is $\qquad$ .

## - Watch Video Solution

423. Fill in the blanks:

The function $f(x)=4 x-3$ is strict $\qquad$ function for all x in R .

## D Watch Video Solution

424. If $\mathrm{x}<0$, what is the maximum value of $x+\frac{1}{x}$

## - Watch Video Solution

425. Fill in the blanks:

At $\mathrm{x}=\mathrm{e}$, the function $\mathrm{f}(\mathrm{X})=\frac{\log x}{x}$ has the ___ value.

- Watch Video Solution

426. Fill in the blanks:

Maximum value of the function $f(X)=\frac{1}{4 x^{2}+2 x+1}$ is

## - Watch Video Solution

427. Fill in the blanks:

If a function $f$ is the differentiable at $c$ such that $f^{\prime}(c)=0$ and $f^{\prime \prime}(c$
) < 0 , then c is a point of $\qquad$ .

## - Watch Video Solution

428. Fill in the blanks:

If a function $f$ has a local maximum at $x=c$ and $f^{\prime \prime}(c)<0$, then $f^{\prime}$
(c) is equal to $\qquad$ .
429. Fill in the blanks:

For $\mathrm{x}>0$, the function $f(x)=\log _{1 / 3} x$ is a strict function.

## - Watch Video Solution

430. Minimum value of f if $f(x)=\sin x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is

## - Watch Video Solution

431. Fill in the blanks:

The maximum value of $\sin x+\cos x, x$ in $^{`} \mathrm{R}$, is $\qquad$ .

- Watch Video Solution

432. Fill in the blanks:

The equation of normal to the curve $y=\tan \operatorname{xat}(0,0)$ is

## - Watch Video Solution

433. Fill in the blanks:

The curves $y=4 x^{2}+2 x-8$ and $y=x^{3}-x+10$ touch each
other at the point.

## D Watch Video Solution

434. Fill in the blanks:

At $(0,0)$ the curves $y=x^{2}$ and $y^{2}=x$
435. Fill in the blanks:

The function $f(X)=\frac{2 x^{2}-1}{x^{4}} . \mathrm{X}>0$ is a decreaisng function in the interval.

## D Watch Video Solution

436. The least of the function
$f(x)=a x+\frac{b}{x}(a>0, b>0, x>0)$ is .............. .

## - Watch Video Solution

437. The values of a for which the function

$$
f(x)=\sin x-a x+b \text { increases on } \mathrm{R} \text { are }
$$

$\qquad$

## - Watch Video Solution

438. Fill in the blanks:

The equation of the tangent to the curve $y=e^{2 x}$ at the point $(0,1)$ is $\qquad$ .

## - Watch Video Solution

439. Is the function $f(x)=x^{2}, \xi n R$ increasing?

## - Watch Video Solution

440. True or False statements:

The rate of change of $\sin \mathrm{x}$ w.r.t. $\cos \mathrm{x}$ in $\left(0, \frac{\pi}{2}\right)$ is $\tan \mathrm{x}$.

## - Watch Video Solution

441. True or False statements:

The function $f(X)=x|x|$ is strictly increasing on $R$.

## D Watch Video Solution

442. True or False statements:

The function $f(X)=\log _{1 / 4} x$ is strict increasing on $(0, \infty)$

## - Watch Video Solution

443. True or False statements:

If a function $f$ has an extreme point at $x=c$, then $f^{\prime}(c)=0$

## - Watch Video Solution

444. True or False statements:

Fermat Theorem If a function $f$ has a local extremum at $x=c$, then
$f^{\prime \prime}(C)=0$.

## - Watch Video Solution

445. True or False statements:

Tangent to the curve $y=x^{2} a t x=0$ is parallel to the $x$-axis.

## - Watch Video Solution

446. True or False statements:

The function $f(X)=x|x|$ is strictly increasing on $R$.

## - Watch Video Solution

447. True or False statements:

A strict monotone function is always one-one.

## - Watch Video Solution

448. True or False statements:

A one-one function is always an increasing function.

## - Watch Video Solution

449. True or False statements:

The function $f(x)=\frac{1}{3} x^{3}-x$ is strictly decreasing on $(-1,1)$

## - Watch Video Solution

450. True or False statements:

The function $f(x)=x^{\frac{1}{3}}$ is a strict increasing function on R .

## - Watch Video Solution

451. True or False statements:

If a function $f$ is twice differentiable at $x=c, f^{\prime}(c)=0$ and $f^{\prime \prime}(c)>$

0 , then f has a local minimam value at $\mathrm{x}=\mathrm{c}$.

## - Watch Video Solution

452. True or False statements:

For any function $f$ a local maximum value is always greater than a local minimum value.
453. True or False statements:

If a function $f$ is such that $f^{\prime}(c)=0$, then at the point, $c$ the function $f$ has either a local maximum or a local minimum.

## D Watch Video Solution

454. True or False statements:

$$
e^{\pi}<\pi^{e}
$$

## - Watch Video Solution

455. The function $f(x)=x^{2}, x \in R$ has no maximum value.

## - Watch Video Solution

456. True or False statements:

Maximum
value
of
the
function
$f(x)=3 \sin x+4 \cos x, x \in R i s 5$.

## D Watch Video Solution

457. True or False statements:

Maximum value of the function $f(x)=\sin ^{4} x+\cos ^{4} x i s 2$.

## - Watch Video Solution

458. True or False statements:

The angle between the tangents to the curves
$y=x^{2}$ and $x=y^{2}$ at the point $(0,0)$ is $\frac{\pi}{2}$
459. True or False statements:

For $x \in\left(0, \frac{\pi}{2}\right), \sin x<x<\tan x$.

## - Watch Video Solution

460. True or False statements:

There is a unique real number $\mathrm{x}>0$ such that
$\left(\frac{1}{4}\right)^{x}=x=\log _{1 / 4^{x}}$

## - Watch Video Solution

461. True or False statements :

If and $B$ are invertible matrices such that $A B=B A$, then $(A B)^{-1}=A^{-1} B^{-1}$
462. Statement I :Among all the rectangles of the given perimeter, the square has the largest area. Also among all the rectangles of given area, the square has the least perimeter.

Statement II :For $x>0, y>0, \quad$ if $x+y=$ constant, then xy will be maximum for $\mathrm{y}=\mathrm{x}$ and if $\mathrm{xy}=$ constant, then $\mathrm{x}+\mathrm{y}$ will be minimum for $\mathrm{y}=\mathrm{x}$.

## - Watch Video Solution

463. True or False statements:

Of all rectangles of a given area, square has the largest perimeter.

## - Watch Video Solution

464. Find $x+y+z$


## - Watch Video Solution

465. Find the rate of change of the area of a circle per second with respect to its radius $r$ when $r=5 \mathrm{~cm}$.
A. $10 \pi$
B. $10 \pi \mathrm{~cm}^{2} / \mathrm{cm}$
C. $\frac{220}{7}$

## Answer:

## - Watch Video Solution

466. The volume of a cube is increasing at the rate of $9 \mathrm{~cm}^{3} / \mathrm{sec}$.

The rate of increase of its surface area when its edge (side) is 6 cm is
A. $3.6 \mathrm{~cm} / \mathrm{sec}$
B. 3.6
C. $6 \mathrm{~cm} / \mathrm{sec}$
D. $6 \mathrm{~cm}^{2} / \mathrm{sec}$

## Answer:

467. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when te radius of the circular wave is 10 cm , the rate of increase of the area enclosed is
A. $80 \pi$
B. $\frac{1760}{7}$
C. $80 \pi c \frac{m^{2}}{\mathrm{sec}}$
D. none of these

## Answer:

468. The length $x$ of a rectangle is decreasing at the rate of 3 $\mathrm{cm} / \mathrm{sec}$ and its breadth y is increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. At the instant, when $x=100 \mathrm{~cm}$ and $\mathrm{y}=60 \mathrm{~cm}$, the rate of change of its area is
A. $20 \mathrm{~cm}^{2} / \mathrm{sec}$
B. $-20 \mathrm{~cm} / \mathrm{sec}^{2}$
C. 20/sec
D. none of these

## Answer:

## - Watch Video Solution

469. The total revenue Rs. $R$ received form the sale of $x$ units of a product is given by $R(x)=3 x^{2}+36 x+5$. The marginal
revenue when $x=5$ is (marginal revenue is the rate of change of total revenue w.r.t. number of items sold at an instant)
A. 66
B. Rs. 66
C. 69
D. none of these

## Answer:

## - Watch Video Solution

470. The sides of an equilateral triangle are increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$, then the rate at which the area increases, when side is 10 cm is
A. $10 \mathrm{~cm}^{2} / \mathrm{sec}$
B. $\sqrt{3} \mathrm{~cm}^{2} / \mathrm{sec}$
C. $10 \sqrt{3} \mathrm{~cm}^{2} / \mathrm{sec}$
D. $\frac{10}{3} \mathrm{~cm}^{2} / \mathrm{sec}$

## Answer:

## - Watch Video Solution

471. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of $2 \mathrm{~cm} / \mathrm{s}$. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?
A. $\frac{1}{10}$ radian $/ \mathrm{sec}$
B. $\frac{1}{20}$ radian $/ \mathrm{sec}$
C. 20radian $/ \mathrm{sec}$
D. $10 \mathrm{radian} / \mathrm{sec}$

## Answer:

## - Watch Video Solution

472. On $R$, the function $f(x)=7 x-3$ is
A. strict decreasing
B. decreasing
C. increasing
D. strict increasing

## Answer:

473. The function $f(x)=2-3 x$ is
A. decreasing
B. increasing
C. strict decreasing
D. neither increasing nor decreasing.

## Answer:

## - Watch Video Solution

474. $y=x(x-3)^{2}$ decreases for the values of x given by:

## A. 1ltx|t3

B. $x$ It 0
C. xgt 0
D. $0<x<\frac{3}{2}$

## Answer:

## - Watch Video Solution

475. The function $\tan \mathrm{x}-\mathrm{x},-\frac{\pi}{2}<x<\frac{\pi}{2}$
A. always increases
B. always decreases
C. never increases
D. sometimes increases and sometimes decreases.

## Answer:

- Watch Video Solution

476. The function $\mathrm{f}(\mathrm{x})=x^{2}$, for all real x is
A. decreasing
B. increasing
C. none of these
D. neither increasing nor decreasing.

## Answer:

## - Watch Video Solution

477. The function $\mathrm{f}(\mathrm{x})=x^{2}-4 x+6$ is
A. strictly decreasing on $(2, \infty)$
B. strictly increasing on $(-\infty, 2)$
C. strictly increasing on $R$
D. strictly increasing on $(2, \infty)$

## Answer:

## - Watch Video Solution

478. Find the intervals in which the function given by $f(x)=\sin 3 x, \mathrm{x}$ in $\left[0, \frac{\pi}{2}\right]$ is decreasing.
A. $\left(0, \frac{\pi}{6}\right)$
B. $\left(0, \frac{\pi}{4}\right)$
C. $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$
D. $\left(0, \frac{\pi}{2}\right)$

## Answer:

479. The function $f(x)=4 \sin ^{3} x-6 \sin ^{2} x+12 \sin x+100$ is strictly
A. increasing in $\left(\pi, 3 \frac{\pi}{2}\right)$
B. decreasing in $\left(\frac{\pi}{2}, \pi\right)$
C. decrea $\sin g \in(-\mathrm{pi} / 2, \mathrm{pi} / 2)^{\prime}$
D. decreasing in $\left(0, \frac{\pi}{2}\right)$

## Answer:

## D Watch Video Solution

480. Which of the following functions is decreasing on $\left(0, \frac{\pi}{2}\right)$
A. $\sin 2 x$
B. $\tan x$
C. $\cos x$
D. $\cos 3 x$

## Answer:

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481. On the interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the function $\tan ^{-1}(\sin x+\cos x)$ is
A. increasing
B. non-decreasing
C. decreasing
D. neither increasing nor decreasing.

## Answer:

482. Find the slope of the tangent to the curve $y=x^{3}-x$ at $x=2$
A. 6
B. 0
C. 11
D. none of these.

## Answer:

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483. The point on the curv3e $y=\sqrt{4 x-3}-1$, at which the slope of the tangent is $\frac{2}{3}$, is
A. $(3,2)$
B. $(2,3)$
C. $(1,0)$
D. none of these

## Answer:

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484. The point on the curve $y=x^{2}-4 x+4$ at which the tangent is parallel to $x$-axis is
A. $(0,2)$
B. $(2,0)$
C. $(0,0)$
D. none of these

## Answer:

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485. The tangent to the curve given by:
$x=e^{t} \cos t, y=e^{t} \sin t a t t=\frac{\pi}{4}$ makes with x-axis an angle
A. 0
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

## Answer:

- Watch Video Solution

486. The tangent to the curve $y=e^{2 x}$ at the point $(0,1)$ meets $x$ axis at:
A. $(0,1)$
B. $\left(-\frac{1}{2}, 0\right)$
C. $(2,0)$
D. $(0,2)$

## Answer:

## D Watch Video Solution

487. The points at which the tangents to the curve $y=x^{3}-12 x+18$ are parallel to $x$-axis are :
A. $(2,-2),(-2,-34)$
B. $(2,34),(-2,0)$
C. (0,34),(-2,0)
D. $(2,2),(-2,34)$

## Answer:

## - Watch Video Solution

488. The point at which the curves $x^{2}=y$ and $y^{2}=x$ cut orthogonally is
A. $(0,0)$
B. $(1,1)$
C. $(2,2)$
D. none of these

## Answer:

## - Watch Video Solution

489. The equation of tangent to the curve $y\left(1+x^{2}\right)=2-x$, where it crosses $x$-axis is :
A. $x+5 y=2$
B. $x-5 y=2$
C. $5 x-y=2$
D. $5 x+y=2$

## Answer:

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490. If the curve at $a y+x^{2}=7$ and $x^{3}=y$, cut orthogonally at
$(1,1)$, then the value of $a$ is :
A. -6
B. 6
C. 1
D. none of these

## Answer:

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491. The function $f(x)=x$ has
A. only one maximum
B. only one minimum
C. one maximum and one minimum value
D. no extreme value

## Answer:

## - Watch Video Solution

492. The function $f(x)=2 x^{3}-6 x^{2}+6 x+5$ has
A. a local maximum at $x=1$
B. a local minimum at $x=1$
C. neither maximum nor minimum at $x=1$
D. none of these

## Answer:

493. Let $f(x)=\left\{\begin{array}{l}|x| f \text { or } 0<|x| \leq 1 \\ 1 f \text { or } x=0\end{array}\right.$ then
A. $f$ has a local minimum at 0
B. f has a local maximum at 0
C. $f$ has a point of inflextion at 0
D. none of these

## Answer:

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494. The abscissa of the point on the curve $3 y=6 x-5 x^{3}$, the normal at which passes through origin is
A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. 2
D. 1

## Answer:

## - Watch Video Solution

495. At $x=5 \frac{\pi}{6}, \mathrm{f}(\mathrm{x})=2 \sin 3 \mathrm{x}+3 \cos 3 \mathrm{x}$ is
A. maximum
B. minimum
C. zero
D. neither maximum nor minimum

## Answer:

496. The two curves $x^{3}-3 x y^{2}+2=0$ and $3 x^{2} y-y^{3}-2=0$
intersect at an angle of
A. $\frac{\pi}{4}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{2}$
D. $\frac{\pi}{6}$

## Answer:

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497. The function $f(x)=\log _{e}\left(x^{3}+\sqrt{x^{6}+1}\right)$ is
A. even and increasing
B. even and decreasing
C. odd and increasing
D. odd and decreasing.

## Answer:

## - Watch Video Solution

498. Let $f(X)=\tan ^{-1} g(x)$, where $g(\mathrm{x})$ is monotonically increasing for 0
A. increasing on $\left(0, \frac{\pi}{2}\right)$
B. decreasing on $\left(0, \frac{\pi}{2}\right)$
C. increasing on ( $0, \frac{\pi}{4}$ ) and decreasing on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
D. none of these

## Answer:

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499. Function $f(X)=\cos x-2 \lambda x$ is monotonic decreasing when
A. $\lambda \geq \frac{1}{2}$
B. $\lambda \leq \frac{1}{2}$
C. $\lambda \leq 2$
D. $\lambda \geq 2$.

## Answer:

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$g(x)=2 f\left(\frac{x}{2}\right)+f(2-x)$ and $f^{\prime \prime}(x)<0 \forall x \in(0,2) . \quad$ If $g(x)$ increases in $(a, b)$ and decreases in $(c, d)$, then the value of $a+b+c+d-\frac{2}{3}$ is
A. increase on ( $0, a$ )
B. decreases on (0,a)
C. increases on ( $-\mathrm{a}, 0$ )
D. decreases on (a,2a)


## Answer:

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501. $\mathrm{f}(\mathrm{x})=\log _{A} x$ is increasing on R , if
A. $0<a<1$
B. $a>1$
C. $a<1$
D. $a>0$

## Answer:

## D Watch Video Solution

502. The function $\mathrm{f}(\mathrm{x})=a^{x}$ is strict decreasing on R , If
A. $a>0$
B. $a>1$
C. $0<a<1$
D. $a<0$

## Answer:

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