MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Product of two Vectors

Example

1. if $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 4\vec{j} + 5\vec{k}$ then find a vector $\vec{a}$ such that $\vec{b} \cdot \vec{a} = 0$ and $\vec{c} \cdot \vec{a} = 0$. Also find the unit vector along $\vec{a}$.

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2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ prove that $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$.

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3. $A_1, A_2, \ldots, A_n$ are the vertices of a regular plane polygon with n sides and O as its centre. Show that $\sum_{i=1}^{n-1} \overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}} = (1-n)\left(\overrightarrow{OA_2} \times \overrightarrow{OA_1}\right)$

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4. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}$ is perpendicular to the plane $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, than value of $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is

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5. Let $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$. Determine a vector $\vec{R}$ satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$.

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6. The vectors $\overrightarrow{AB} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} + 2\hat{k}$ are the adjacent sides of a parallelogram ABCD then the angle between the diagonals is

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7. The position vectors of the points A and B are $\vec{i} + 2\vec{j} + 3\vec{k}$ and $2\vec{i} - \vec{j} - \vec{k}$ respectively. Find the projection of $\overrightarrow{AB}$ on the vector $\vec{i} + \vec{j} + \vec{k}$. Also find the resolved part of $\overrightarrow{AB}$ in that direction.

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8. Show that the altitude of $\triangle ABC$ through a vertex A is equal to $\frac{\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|}{\left|\vec{b} - \vec{c}\right|}$ where the position vectors of A,B, and C are respectively $\vec{a}$, $\vec{b}$ and $\vec{c}$.

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Exercise

1. For any vector $\vec{r}$, prove that $\vec{r} = \left(\vec{r}\,\hat{i}\right)\hat{i} + \left(\vec{r}\,\hat{j}\right)\hat{j} + \left(\vec{r}\,\hat{k}\right)\hat{k}$.

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2. If $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$ then show that $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

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3. Let $\hat{a}, \hat{b}$ be two unit vectors and θ be the angle between them.

What is $\sin\left(\frac{\theta}{2}\right)$ equal to ?

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4. Prove that $\left|\vec{a}+\vec{b}\right|=\sqrt{\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\vec{a}\cdot\vec{b}}$.

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5. Prove that ,

$$\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}.\vec{b}\right)^2=\left|\vec{a}\right|^2\left|\vec{b}\right|^2$$

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6. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$

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7. Find the angle between the vectors $\vec{a} = 2\vec{p} + 4\vec{q}$ and $\vec{b} = \vec{p} - \vec{q}$ where $\vec{p}$ and $\vec{q}$ are unit vectors forming an angle of 120°.

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8. If $\left|\vec{a}\right| = 1 = \left|\vec{b}\right|$ and $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$ then evaluate $\left(2\vec{a} - \vec{b}\right) \cdot \left(3\vec{a} + \vec{b}\right)$.

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9. Find λ such that the scalar product of the vector $\vec{i} + \vec{j} + \vec{k}$ with the unit vector parallel to the sum of

the vectors $2\vec{i} + 4\vec{j} - 5\vec{k}$ and $\lambda\vec{i} + 2\vec{j} + 3\vec{k}$ is equal to 1.

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10. The set of values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ acute and the angle between the vector $\vec{b}$ and the axis of ordinates is obtuse, is

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11. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of length $3, 4, 5$ respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is :

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12. Vectors $3\vec{a} - 5\vec{b} \; and \; 2\vec{a} = \vec{b}$ are mutually perpendicular. If $\vec{a} + 4\vec{b} \; and \; \vec{b} - \vec{a}$ are also mutually perpendicular, then the cosine of the angel between $a and b$ is $\frac{19}{5\sqrt{43}}$ b. $\frac{19}{3\sqrt{43}}$ c. $\frac{19}{2\sqrt{45}}$ d. $\frac{19}{6\sqrt{43}}$

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13. The pth, qth and rth terms of a GP are the positive numbers a,b and c respectively. Show that the vectors $\vec{i}\log_e a + \vec{j}\log_e b + \vec{k}\log_e c$ and $\vec{i}(q-r) + \vec{j}(r-p) + \vec{k}(p-q)$ are mutually perpendicular.

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14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitude, show the vectors $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

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15. A line makes angles $\alpha, \beta, \gamma and \delta$ with the diagonals of a cube. Show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=4/3$.

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16. $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of points A, B and C respectively, prove that :

$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is vector perpendicular to the plane of triangle ABC.

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17. Given the vectors $\vec{A}, \vec{B}, and \vec{C}$ form a triangle such that $\vec{A} = \vec{B} + \vec{C}$. find $a, b, c, and d$ such that the area of the triangle is 56 where $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ $\vec{B} = d\hat{i} + 3\hat{j} + 4\hat{k}\, \vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$

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18. If AC and BD are the diagonals of a quadrilateral ABCD, prove that its area is equal to $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$.

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19. $A, B, CandD$ are any four points in the space, then prove that $\left|\overrightarrow{A}\,B \times \overrightarrow{C}\,D + \overrightarrow{B}\,C \times \overrightarrow{A}\,D + \overrightarrow{C}\,A \times \overrightarrow{B}\,D\right| = 4$ (area of ABC .)

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20. If the angle between the vectors $x\overrightarrow{i} + \overrightarrow{j} - \overrightarrow{k}$ and $\overrightarrow{i} + x\overrightarrow{j} + \overrightarrow{k}$ is equal to $\frac{\pi}{3}$ then x=_____.

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21. If the vector $\overrightarrow{a} = x\overrightarrow{i} + y\overrightarrow{j} + 2\overrightarrow{k}$ is perpendicular to the vector $\overrightarrow{b} = \overrightarrow{i} - \overrightarrow{j} + \overrightarrow{k}$ and the scalar product of $\overrightarrow{a}$

and $\vec{c} = \vec{i} + 2\vec{j}$ is equal to 4 then x= _____ and y=_____.

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22. If $\vec{a}$ is a vector of magnitude 50 and parallel to $\vec{b} = 6\vec{i} - 8\vec{j} - \frac{15}{2}\vec{k}$ and makes an acute angle with the z-axis then $\vec{a}$ = _____.

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23. If $\vec{a}, \vec{b}, and \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $\vec{a}\dot{\vec{b}} + \vec{b}\dot{\vec{c}} + \dot{\vec{c}}\vec{a}$.

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24. Prove that ,

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) + \vec{b} \times \left(\vec{c} + \vec{a}\right) + \vec{c} \times \left(\vec{a} + \vec{b}\right) = \vec{0}$$

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25. If $\vec{a}$ and $\vec{b}$ are vectors, each of magnitude 3 then $\left|\vec{a} + \vec{b}\right|^2 + \left|\vec{a} - \vec{b}\right|^2 =$ _____.

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26. If $\vec{A} = (1, 1, 1), \vec{C} = (0, 1, -1)$ are given vectors, then prove that a vector $\vec{B}$ satisfying the equations $\vec{A} \times \vec{B} = \vec{C}$ and $\vec{A} \cdot \vec{B} = 3$ is $\left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right)$.

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27. Find a unit vector perpendicular to the plane determined by the points $(1, -1, 2), (2, 0, -1) and (0, 2, 1)$.

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28. Let $\beta = 4\hat{i} + 3\hat{j}$ and $\vec{\gamma}$ be two vectors perpendicular to each other in the XY plane. Find all the vectors in the same plane having the projections 1 and 2 along $\vec{\beta}$ and $\vec{\gamma}$ respectively.

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29. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 5, \left|\vec{c}\right| = 7$ and $\vec{a} + \vec{b} + \vec{c} = 0$ then angle between $\vec{a}$ and $\vec{b}$ is

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30. The conditions under which the vector $\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$ will be at right angles to each other is

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31. Let $\vec{OA} - \vec{a}, \hat{OB} = 10\vec{a} + 2\vec{b} and \vec{OC} = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB$, and let q denote the area of

parallelogram with $OAandOC$ as adjacent sides. If $p = kq$, then find k.

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32. Vector $\overrightarrow{O}A = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\frac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$.

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33. A vector a has components a_1, a_2, a_3 in a right handed rectangular cartesian coordinate system $OXYZ$ the coordinate axis is rotated about z axis through an angle $\frac{\pi}{2}$. The components of a in the new system

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34. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ then

A. $\vec{a} = 0$

B. $\vec{b} = \vec{c}$

C. $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$

D. all of these

Answer: D

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35. If $\vec{A} + \vec{B} + \vec{C} = 0$ then $\vec{A} \times \vec{B}$ is

A. $\vec{c} \times \vec{a}$

B. $\vec{c}$

C. $\vec{b} \times \vec{c}$

D. $\vec{a} \times \vec{c}$

Answer:

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36. If $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \neq \vec{0}$ then which one is true where k is a suitable scalar?

A. $\vec{a} + \vec{b} = k\vec{c}$

B. $\vec{a} + \vec{c} = k\vec{b}$

C. $\vec{b} + \vec{c} = k\vec{a}$

D. none of these

Answer: B

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37. A, B, C and D are four points in a plane with position vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$, respectively, such that $\left(\vec{a}-\vec{d}\right).\left(\vec{b}-\vec{c}\right)=\left(\vec{b}-\vec{d}\right).\left(\vec{c}-\vec{a}\right)=0$

Then point D is the of triangle ABC

A. incentre

B. circumcentre

C. orthocentre

D. centroid

Answer:

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38. If $\left|\vec{a}\right| = \left|\vec{b}\right| = 1$ and $\left|\vec{a} \times \vec{b}\right| = 1$ then

A. $\vec{a} \mid\mid \vec{b}$

B. $\vec{a} \perp \vec{b}$

C. $\vec{a} \cdot \vec{b} = 1$

D. none of these

Answer: B

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39. The vector $\vec{b} = 3j + 4k$ is to be written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = i + j$, and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1$, equals

A. $\frac{3}{2}\left(\vec{i} + \vec{j}\right)$

B. $\frac{2}{3}\left(\vec{i} + \vec{j}\right)$

C. $\left(\frac{1}{2}\left(\vec{i} + \vec{j}\right)\right.$

D. none of these

Answer:

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40. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$

be three vectors . A vector in the plane of $\vec{b}$ and $\vec{c}$ whose length of projection on $\vec{a}$ is of $\sqrt{\frac{2}{3}}$ is

A. $2\vec{i} + 3\vec{j} - 3\vec{k}$

B. $2\vec{i} + 3\vec{j} + 3\vec{k}$

C. $-2\vec{i} - \vec{j} + 5\vec{k}$

D. $2\vec{i} + j + 5\vec{k}$

Answer:

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41. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 5$ and $\left|\vec{c}\right| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is 60^0.

A. $\frac{\pi}{3}$

B. $5\frac{\pi}{3}$

C. $2\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer:

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42. If $\vec{a} . \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$ then which one of the following is correct ?

A. $\vec{a} \mid \mid \vec{b}$

B. $\vec{a} \perp \vec{b}$)

C. $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$

D. none of these

Answer:

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43. If $\vec{a} = 4\hat{i} + 6\hat{j} and \vec{b} = 3\hat{j} + 4\hat{k}$, then find the component of $\vec{a} and \vec{b}$.

A. $\left(\frac{9}{5\sqrt{3}}\left(3\vec{j} + 4\vec{k}\right)\right.$

B. $\frac{18}{25}$

C. $\frac{18}{25}\left(3\vec{j} + 4\vec{k}\right)$

D. $\frac{18}{\sqrt{13}}\left(3\vec{j} + 4\vec{k}\right)$

Answer:

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44. In each of the following, fill in the blank so that the resulting statement is correct. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} = \lambda \vec{b} + \mu \vec{c}$ an $\vec{a} \cdot \vec{b} = 0, \vec{a} \cdot \vec{c} = \frac{1}{2}$ then $\left| \vec{b} \cdot \vec{c} \right| =$ _____.

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45. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between $A \to B$ is

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46. Find a unit vector $\vec{c}$ if $-\vec{i} + \vec{j} - \vec{k}\, bisects the angle between \vec{c}$ and 3

$\hat{i} + 4\hat{j}$.

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47. If θ is the angle between the unit vectors $\hat{a}$ and $\hat{b}$ then $\cos\left(\frac{\theta}{2}\right)$ is equal to

A. $\left|\hat{a} + \hat{b}\right|$

B. $\frac{1}{2}\left|\hat{a} - \hat{b}\right|$

C. $\frac{1}{2}\left|\hat{a} \cdot \hat{b}\right|$

D. $\frac{1}{2}\left|\hat{a} + \hat{b}\right|$

Answer: D

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48. In each of the following, one or more options are correct. Choose the correct option(s). If $\left|\vec{a}\right| = \left|\vec{b}\right|$ then $\vec{a} + \vec{b}$ is perpendicular to the vector

A. $\left(\vec{a} \cdot \vec{b}\right)\vec{a}$

B. $\vec{a} - \vec{b}$

C. $\vec{a} \times \vec{b}$

D. $\left(\vec{a} \cdot \vec{b}\right)\vec{b}$

Answer: B, C

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