# MATHS

## BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

## Progression, Related Inequalities and Series

### Exercise

**1.** General forms of progressions.



**2.** Classification of progression.

**3.** Relations between variables and facts about AP.

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**4.** Prove that square roots of three unequal prime numbers cannot be three terms of an AP.

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**5.** If x,y and z are positive real numbers diferent form 1 and $x^{18} - y^{21} = z^{28}$ show that $3\log_y x, 3\log_z y, 7\log_x z$ are in A.P.

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**6.** If x,y,z are real numbers satisfying the equation $25\left(9x^2 + y^2\right) + 9z^2 - 15(5xy + yz + 3zx) = 0$ then prove that x,y,z are in AP.

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**7.** At what values of parameter $a$ are there values of $x$ such that the numbers: $\left(5^{1+x}+5^{1-x}\right), \frac{a}{2}, \left(25^{x}+25^{-x}\right)$ form an $A.P.$ ?

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**8.** Prove that $(2n)^2$ where $n \in N$ can be expressed as the sum of $n$ terms of a series of integers in AP

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**9.** Suppose that $a_1, a_2, \ldots, a_n, \ldots$ is an A.P. Let $S_k = a_{(k-1)n+1} + a_{(k-1)n+2} + \ldots\ldots + a_{kn}$. Prove that $S_1, S_2, \ldots$are in A.P. having common difference equal to $n_2$ times the common differ-ence of the A.P. $a_1, a_2, \ldots$

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**10.** The ratio of the sums of m terms and n terms of an A.P. is $m^2 : n^2$.

Prove that the ratio of their mth and nth term will be (2m - 1) : (2n-1).

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**11.** Let $r$ be the common ratio of the GP $a_1, a_2, a_3, \ldots, a_n$. Show that

$$\frac{1}{a_1^m + a_2^m} + \frac{1}{a_2^m + a_3^m} + \ldots + \frac{1}{a_{n-1}^m + a_n^m} = \frac{1 - r^{(1-n)m}}{a_1^m (r^m - r^{-m})}.$$

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**12.** If $S = a + b + c$ then prove that $\frac{S}{S-a} + \frac{S}{S-b} + \frac{S}{S-c} > \frac{9}{2}$

where a,b & c are distinct positive reals.

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**13.** If a, b, x, y are positive natural numbers such that $\frac{1}{x} + \frac{1}{y} = 1$ then

prove that $\frac{a^x}{x} + \frac{b^y}{y} \geq ab.$

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**14.** If $a_i > 0, i = 1, 2, 3, \dots n$ and $(n-1)s = a_1 + a_2 + a_3 + \dots . + a_n$, prove that $a_1 \cdot a_2 \cdot a_3 \cdot \dots . a_n \geq (n-1)^n \cdot (s-a_1)(s-a_2)\dots(s-a_n)$.

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**15.** If $2a + b + 3c = 1$ and $a > 0, b > 0, c > 0$ , then the greatest value of $a^4 b^2 c^2$______.

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**16.** Given the perimeter of a triangle, prove that the triangle of the greatest area is equilateral.

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**17.** Find the sum of the series $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + 4 \cdot (n-3) + ... + (n-1) \cdot 2 + , \cdot 1$ also, find the coefficient of $x^{n-1}$ in th cxpansion of $\left(1 + 2x + 3x^2 + ....nx^{n-1}\right)^2$.

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**18.** Find the sum of first n terms of the series 0.7 + 0.77 + 0.777 + .....

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**19.** The sum of the series $\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + ....$ . upto n terms , is

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**20.** If $a_1, a_2, a_3, \ldots.., a_n$ are in AP where $a_i \neq k\pi$ for all $i$, prove that cosec $a_1 \cdot \text{cosec} a_2 + \text{cosec} a_2 \cdot \text{cosec} a_3 + \ldots + \text{cosec} a_{n-1} \cdot$ cosec $a_n = \dfrac{\cot a_1 - \cot a_n}{\sin(a_2 - a_1)}$.

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**21.** Find the sum of the series

$\cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21 + \cot^{-1} 31 + \ldots$ to n terms

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**22.** The coefficient of $x^{98}$ in the expression of $(x-1)(x-2)\ldots\ldots..(x-100)$ must be

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**23.** If m times the mth term of an AP is equal to n times its nth term, then show that (m + n)th term of an AP is zero.

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**24.** Prove that the numbers $\sqrt{2}, \sqrt{3}, \sqrt{5}$ cannot be three terms (not necessarily consecutive) of an AP.

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**25.** If $a, b$ and $c$ are three terms of an $A.P.$ such that $a \neq b$, then $\frac{b-c}{a-b}$ may be equal to (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $1$ (D) $3$

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**26.** If $\log_l x, \log_m x, \log_n x$ are in AP where $x \neq 1$ and x is positive, prove that $n^2 = (\ln)^{\log_l m}$.

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**27.** If $\log_3 2, \log_3(2^x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in $A.P$, determine the value of $x$.

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**28.** If the triplets $\log a, \log b, \log c$ and $(\log a - \log 2b), (\log 2b - \log 3c), (\log 3c - \log a)$ are in arithmetic progression then

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**29.** Find three-digit numbers that are divisible by 5 as well as 9 and whose consecutive digits are in AP.

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**30.** 1000 ! Is divisible by $10^n$. Find the largest positive integral value of $n$.

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**31.** Find the number of common terms in the following sequences : 3, 7, 11, ... to 100 terms and 2, 5, 8, ... to 100 terms.

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**32.** Find the number of common terms in the following sequences : 1, 5, 9, ... to 100 terms and 4, 7, 10, ... to 100 terms.

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**33.** Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

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**34.** The sum of four whole numbers in AP is 24 and their product is 945, find the numbers.

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**35.** 4. (a) Divide 20 into 4 parts which are in A.P. and such that. the product of the first and fourth is to the product o the secondi.and third in the ratio 2:3.

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**36.** 22. There are two sets of numbers each consisting of 3 terms in A.P. and the sum of each set is 15. The common difference of the first set is greater by 1 than the common difference of the second set, and the product of the first set is to the product of the second set as 7 to 8; find the numbers

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**37.** Find four numbers between 4 and 40 so that the six numbers are consecutive terms of an AP.

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**38.** The sum of three numbers which are consecutive terms of an AP is 2 and the sum of their squares is $\frac{14}{9}$. Find the numbers.

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**39.** The sum of three numbers which are consecutive terms of an AP is 3 and the sum of their cubes is 4. Find the numbers.

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**40.**

If $a_1, a_2, a_3, \ldots., a_n$ are in AP, prove that $a_1 + a_n = a_r + a_{n-r+1}$

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41. If $a_1, a_2, a_3, \ldots .., a_n$ are in AP, prove that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \frac{1}{a_3a_4} + \ldots + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}$.

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42. If $a_1, a_2, a_3, \ldots \ldots \ldots a_n$ are in A.P, where $a_i.0$ for all i show that $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots \ldots . + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

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43. Let $a_1, a_2, a_3$ .... $a_n$ be in A.P. If $\frac{1}{a_1a_n} + \frac{1}{a_2a_{n-1}}$+... + $\frac{1}{a_na_1} = \frac{k}{a_1+a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots . \frac{1}{a_n}\right)$, then k is equal to :

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**44.** If the sequence $a_1, a_2, a_3, \ldots, a_n$ is an A.P., then prove that $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2n-1}^2 - a_{2n}^2 = \frac{n}{2n-1}\left(a_1^2 - a_{2n}^2\right)$

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**45.** If the nonzero numbers $a_1, a_2, a_3, \ldots., a_n$ are in AP, prove that $\frac{1}{a_1 a_2 a_3} + \frac{1}{a_2 a_3 a_4} + \ldots + \frac{1}{a_{n-2} a_{n-1} a_n} = \frac{1}{2(a_2 - a_1)}\left(\frac{1}{a_1 a_2} - \frac{1}{a_{n-1} a_n}\right)$

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**46.** If a,b,c are in A.P. prove that $b+c, c+a, a+b$ are also in A.P.

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**47.** If $ab + bc + ca \neq 0$ and $a, b, c$ are in A.P. prove that $a^2(b+c), b^2(c+a), c^2(a+b)$ are also in A.P.

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**48.** If $a^2, b^2, c^2$ are in A.P. prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

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**49.** If $a^2, b^2, c^2$ are in A.P, show that: $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

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**50.** If x, y, z are in AP, prove that $\frac{1}{\sqrt{y}+\sqrt{z}}, \frac{1}{\sqrt{z}+\sqrt{x}}, \frac{1}{\sqrt{x}+\sqrt{y}}$ will be in AP.

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**51.** If $S_n = an^2 + bn + c$ where $c \neq 0$ and $S_n$ denotes the sum to n terms of a series, verify whether the series is arithmetic.

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**52.** The sum of n terms of a series is $2n^2 + 3n$. Is the series arithmetic ? If so, find it.

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**53.** Find the sum of all positive integers less than 200 that are divisible by 3 and 5.

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**54.** Find the sum of all natural numbers $n \in [100, 300]$ that are neither divisible by 4 nor by 6.

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**55.** Find the sum of all 3-digit natural numbers which are of the form 3m+2, $m \in N$, i.e., leaves the remainder 2 when divided by 3.

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**56.** Sum the series to $n$ terms: $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \dots .$

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**57.** How many terms of $1+3+5+7+\dots$ amount to 1234321 ?

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**58.** The maximum sum of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} +$ is $310$ b. $300$ c. $0320$ d. none of these

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**59.** If $a_1, a_2..................., a_{19}$ are the first 19 term of an $AP$ and $a_1 + a_8 + a_{12} + a_{19} = 224.$ Then $\sum_{i=1}^{19} a_i$ is equal to :

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**60.** The sum of the first 100 terms common to the series

17, 21, 25, 29, 33, ...... ahd 16 , 21 ,26, 31 , 36 .... is

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**61.** in an $AP$, $S_p = q$, $S_q = p$ and $S_r$ denotes the sum of the first $r$ terms. Then $S_{p+q} =$

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**62.** Sum of the first p, q and r terms of an A.P are a, b and c, respectively.Prove that $\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$

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**63.** If $S_n$ is the sum of the first n terms of an A.P. then :

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**64.** If mth term of am A.P.is $\frac{1}{n}$ and nth term is $\frac{1}{m}$ find the sum of first $mn$ terms.

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**65.** The first term of an A.P. is $a$ and the sum of first $p$ terms is zero, show tht the sum of its next $q$ terms is $\frac{a(p+q)q}{p-1}$.

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**66.** Given an $A.P.$ whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its $4^th$ term is:

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**67.** If the sum of $m$ terms of an A.P. is equal to the sum of either the next $n$ terms or the next $p$ terms, then prove that $(m+n)\left(\frac{1}{m}-\frac{1}{p}\right)=(m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$.

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**68.** if $S_1, S_2, S_3.........., S_q$ are the sums of $n$ terms of $q$, $AP's$ whose first terms are $1, 2, 3, ..........q$ and common difference are $1, 3, 5, ..........,(2q-1)$ respectively ,show that $S_1+S_2+S_3.........+S_q=\frac{1}{2}nq(nq+1)$

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**69.** If $S_1$ be the sum of $(2n+1)$ term of an A.P. and $S_2$ be the sum of its odd terms then prove that $S_1 : S_2 = (2n+1) : (n+1)$.

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**70.** The number of terms in an AP is even. The sum of the odd terms is 24 while that of the even terms is 30. If the last term exceeds the first by 10.5 then find the number of terms.

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**71.** The ratio of the sums of $n$ terms of two Aps is $(3n-13) : (5n+21)$. Find the ratio of the 24th terms of the two progressions.

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**72.** Find the AP in which the ratio of the sum to $n$ terms to the sum of succeding $n$ terms is independent of $n$.

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**73.** If $x, 2x+2, 3x+3$ are the first three terms of a GP, find the fifth term of the sequence.

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**74.** In a GP, $t_4 = 27$ and $t_7 = 729$. Find $t_{11}$ where $t_n$ denotes the $nth$ term.

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**75.** In a GP, $t_4 = 27$ and $t_7 = 729$. Find $t_{11}$ where $t_n$ denotes the $nth$ term.

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**76.** In a GP, $t_{p+q} = a$ and $t_{p-q} = b$. Prove that $t_p = \sqrt{ab}$.

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**77.** If the pth, qth and rth terms of a G.P. are a,b and c, respectively. Prove that

$a^{q-r}b^{r-p}c^{p-q} = 1.$

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**78.** If $a$ be the first term, $b$ be the $nth$ term and P be the product of $n$ terms of a GP then prove that $P^2 = (ab)^n$.

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**79.** If a,b,c are in GP, prove that $(a^2 - b^2)(b^2 + c^2) = (b^2 - c^2)(a^2 + b^2)$.

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**80.** If a, b, c, d are in G.P., then prove that:

$$(b-c)^2+(c-a)^2+(d-b)^2=(a-d)^2$$

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**81.** If a,b,c,d are in GP then prove that $a+b, b+c, c+d$ are in GP.

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**82.** If a,b,c,d are in GP then prove that $a(b-c)^3=d(a-b)^3$.

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**83.** If a,b,c,d are in GP then prove that $ax^3+bx^2+cx+d$ has a factor $ax^2+c$.

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**84.** Let $a_1, a_2, a_3, \ldots., a_n$ be in GP whose common ratio is r. Show that

$$\sum_{k=1}^{n-1} \frac{1}{a_k^2 - a_{k+1}^2} = \frac{1 - r^{2(n-1)}}{a_1^2 \cdot r^{2(n-2)} \cdot (1 - r^2)^2}.$$

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**85.** Does there exist a GP containing 27,8 and 12 as threee of its terms ? If it exist, how many such progressions are possible ?

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**86.** Prove that no GP can have three of its terms (not necessarily consecutive) as three consecutive nonzero integers.

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**87.** The product of three numbers in GP is 216 and their sum is 19. Find the numbers.

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**88.** Divide 63 into three parts that are in GP and the product of the first and the second part is $\frac{3}{4}$ times of the third part.

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**89.** The first term of a GP is unity. For what value of the common ratio of the progression the sum of 4 times the second term and 5 times the third term will be minimum ?

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**90.** For the series $\sum t_n, S_n = \sum_{n=1}^{n} t_n = 2t_n - 1$. Is the series geometric ? If so, find the sum to $n$ terms of the series.

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**91.** If $S_1, S_2, S_3$ are the sums to n, 2n, 3n terms respectively for a GP then prove that $S_1, S_2 - S_1, S_3 - S_2$ are in GP.

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**92.** If $S_1, S_2, S_3$ be respectively the sums of $n, 2n, 3n$ terms of a G.P., then prove that $S12 + S22 = S_1(S_2 + S_3)$ .

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**93.** Find $\sum_{n=1}^{n} u_n$ if $u_n = \sum_{n=0}^{n} \frac{1}{2^n}$.

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**94.** STATEMENT -1: (666...n digit)^2+(888... ndigit)=(444....2ndigits) STATEMENT- 2 (111....1) 12 times is a prime number. (A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is a correct explanation for (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for (C) STATEMENT-1 is True, STATEMENT-2 s False .n digits+ (888. digits) (44 .2n digits) 2 is a prime number 12 times n H.P., wherea STATEMENT-1 STATEMENT-1 STATEMENT-1 is False, STATEMENT-2 is True (D)

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**95.** Let $a_1, a_2, a_3, \ldots.$ are in GP. If $a_n > a_m$ when $n > m$ and $a_1 + a_n = 66$ while $a_2 \cdot a_{n-1} = 128$ and $\sum_{i=1}^{n} a_i = 126$, find the value of $n$.

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**96.** Find the sum of $2n$ terms of the series whose every even term is $'a'$ times the term before it and every od term is $'c'$ times the term before it, the first term being unity.

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**97.** If the sum of an infinite G.P is 32 and the some of its first two terms is 24, find the series.

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**98.** If $S_n = 1 + r^n + r^{2n} + r^{3n} + \ldots.$ to $\infty$ and $s_n = 1 - r^n + r^{2n} - r^{3n} + \ldots$ to $\infty$ then prove that $S_n + s_n = 2S_{2n}$.

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**99.** If $x=\sum_{n=0}^{\infty} a^n, y=\sum_{n=0}^{\infty} b^n$ where $|a|<1, |b|<1$ then prove that $\sum_{n=0}^{\infty} (ab)^n=\frac{xy}{x+y-1}$

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**100.** If exp. $\{(\sin^2 x+\sin^4 x+\sin^6 x+\ldots \in f.)In2\}$ satisfies the equation $x^2-9x+8=0$ find the value off $\frac{\cos x}{\cos x+\sin x}, 0<x<\frac{\pi}{2}$

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**101.** If $S_1, S_2, S_3, S_4, \ldots., S_p$ denotes the sums of infinite geometric series whose first terms are 1, 2, 3,...p respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots. \frac{.1}{p+1}$ respectively .then $S_1+S_2+S_3+\ldots.+S_p$ = kp(p+3), where k =?

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**102.** If $S_1, S_2, S_3, S_4, \ldots .., S_p$ denotes the sums of infinite geometric series whose first terms are 1, 2, 3,...p respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots . \frac{.1}{p+1}$ respectively .then $S_1 + S_2 + S_3 + \ldots . + S_p$ = kp(p+3), where k =?

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**103.** If $S_1, S_2, S_3, S_4, \ldots .., S_p$ denotes the sums of infinite geometric series whose first terms are 1, 2, 3,...p respectively and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots . \frac{.1}{p+1}$ respectively .then $S_1 + S_2 + S_3 + \ldots . + S_p$ = kp(p+3), where k =?

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**104.** If a,b,c are in G.P., then show that : $\log a, \log b, \log c$ are in A.P.

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**105.** If $a, b, c$ are in A.P. and $x, y, z$ in G.P., prove that $x^{b-c}. y^{c-a}. z^{a-b} = 1$

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**106.** If $a, b, c$ are in $A. P$ and $a, b, d$ are in $G. P$, prove that $a, a-b, d-c$ are in $G. P$.

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**107.** 29. If $a, b, c$ are in $G. P.$ and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ prove that $x, y, z$ are in $A. P.$

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**108.** If a, x, b as well as c, x, d are in GP while $a^2, y, b^2$ as well as $c^2, y, d^2$ are in AP then prove that $a^n + b^n = c^n + d^n$ where $n$ is an even integer or $a^n + b^n + c^n + d^n = 0$ where $n$ is an odd integer.

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**109.** If pth, qth and rth terms of an A.P. and G.JP. Both be a,b and c respectively, show that $a^{b-c}.\, b^{c-a}.\, c^{a-b}=1$

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**110.** If pth, qth , rth and sth terms of an AP are in GP then show that (p-q), (q-r), (r-s) are also in GP

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**111.** If second third and sixth terms of an A.P. are consecutive terms o a G.P. write the common ratio of the G.P.

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**112.** Find an AP of distinct terms whose first term is unity such that the second, tenth and thirty-fourth terms form a GP.

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**113.** In a set of four number, the first three are in GP & the last three are in A.P. with common difference 6. If the first number is the same as the fourth, find the four numbers.

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**114.** In a set of four numbers, the first three are in GP and the last three are in AP with a common difference of 6. If the number is same as the fourth, then find the four numbers.

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**115.** Prove that three unequal numbers cannot be in GP when each number increased (or decreased) by the same quantity are in AP.

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**116.** Three numbers whose sum is 15 are in A.P. If 1,4,19 be added to them respectively the resulting numbers re in G.P. Find the numbers.

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**117.** The sum of three numbers in G.P. is 14. If the first to terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. find the numbers.

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**118.** The sum of three numbers in G.P. is 70. If each of the two extremes be multiplied by 4 and the mean by 5, the products are in A.P. Find the numbers.

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**119.** Three non-zero real numbers from an A.P. and the squares of these numbers taken in same order from a G.P. Then, the number of all possible value of common ratio of the G.P. is

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**120.** Find three numbers a, b,c between 2 & 18 such that; O their sum is 25 @ the numbers 2, a, b are consecutive terms of an AP & Q.3 the numbers b?c?18 are consecutive terms ofa GP

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**121.** Given four numbers, the first three of which are three successive terms of a G.P. and the last three are successive terms of an A.P. The sum of the extreme numbers is 32 and that of the middle numbers is 24. If the numbers are in ascending order, then the first number is

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**122.** The first term of an AP is the same as that of a GP. The common difference of the AP and the common ratio of the GP are each equal to 2. If the sum of the first five terms of each series be the same, find the sixth term of each series.

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**123.** The sum oif the first ten terms of an A.P. is equal to 155, and the sum of the first two terms of a G.P. is 9. Find these progressionsif the first term of the A.P. equals the common ratio of the G.P. and the 1st term of G.P. equals the common difference of A.P.

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**124.** The three digit number whose digits are in G.P. and the digits of the number obtained from it by subtracting 400 form an A.P. is equal to.

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**125.** Find a three digit numberwhose consecutive digits form a GP. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an AP.

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**126.** If the number $\frac{1}{k+2}, \frac{1}{k^2}$ and $\frac{1}{4k+5}$ are three consecutiv terms of an HP then find k.

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**127.** If in an HP, $t_m = n$ and $t_n = m$ then prove that $t_{m+n} = \frac{mn}{m+n}$.

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**128.** The sum of three consecutive terms in H.P is 37 and the sum of their reciprocal is $\frac{1}{4}$. Find the numbers.

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**129.** If pth, qth and rth terms of a HP be respectively$a, b$ and $c$, has prove that $(q-r)bc + (r-p)ca + (p-q)ab = 0$.

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**130.** If $x, y, z$ are real and $4x^2 + 9y^2 + 16z^2 - 6xy - 12yz - 8zx = 0, thenx, y, z$ are in a. A.P. b. G.P. c. H.P. d. none of these

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**131.** (i) a , b, c are in H.P. , show that $\frac{b+a}{b-a}+\frac{b+c}{b-c}=2$

(ii) If $a^2, b^2, c^2$ are A.P. then b + c , c + a , a + b are in H.P. .

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**132.** If $b$ is the harmonic mean between $a$ and $c$, then prove that

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}.$$

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**133.** If a, b, c are in HP then prove that

$$b^2(a-c)^2=2\left\{c^2(b-a)^2+a^2(c-b)^2\right\}.$$

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134. If $\log(a+c)+\log(a+c-2b)=2\log(a-c)$ then

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135. If a, b, c, d be in H.P. then prove that $ab+bc+cd=3ad$

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136. If $x_1, x_2, x_3 \ldots., x_n$ are in H.P. prove that $x_1x_2+x_2x_3+x_3x_4+\ldots\ldots\ldots+x_{n-1}x_n=(n-1)x_1x_n$

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137. If $(b+c), (c+a), (a+b)$ are in H. P. then prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P

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**138.** If $\frac{x+y}{2}, y, \frac{y+z}{2}$ are in HP, then x,y, z are in

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**139.** If a, b, c, d be four numbers of which the first three are in AP and the last three are in HP then prove that ad=bc.

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**140.** Suppose a is a fixed real number such that $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$

If p,q,r, are in A.P., then prove that x,y,z are in H.P.

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**141.** If a,b,c,d be in G.P. and $a^x=b^y=c^z=d^w, provet\frac{\hat{1}}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{w}$ are in A.P.

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**142.** If three numbers are in G.P., then the numbers obtained by adding the middle number to each of these numbers are in

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**143.** If $a, b, c,$ are both in $G.P.$ and $H.P.$, $T_p = 0, T_q = b, T_r = C,$ then $a(b-c)\log a + b(C-a)\log b + c(a-b)\log c$

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**144.** If a,b,c,d,e be 5 numbers such that a,b,c are in A.P; b,c,d are in GP & c,d,e are in HP then prove that a,c,e are in GP

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**145.** If x,y and z are in A.P ax,by and cz in G.P and a, b, c in H.P then prove that $\frac{x}{z} + \frac{z}{x} = \frac{a}{c} + \frac{c}{a}$

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**146.** a, b, x are in AP, a, b, y are in GP and a, b, z are in HP, then prove that

$4z(x-y)(y-z) = y(x-z)^2$.

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**147.** If $a, b, c,$ are in $AP, a^2, b^2, c^2$ are in HP, then prove that either $a = b = c$ or $a, b, \ -\frac{c}{2}$ from a GP (2003, 4M)

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**148.** If $x, 1, andz$ are in A.P. and $x, 2, andz$ are in G.P., then prove that $x, and4, z$ are in H.P.

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**149.** If a, b, c, d, e be 5 numbers such that a, b, c are in AP, b, c, d, are in GP & c, d, e are in HP then

(i) Prove that a,c,e are in GP.

(ii) Prove that $e = (2b - a)^2 / a$.

(iii) If $a = 2 \& e = 18$, find all possible vlaues of b,c,d.

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**150.** The values of $xyz$ is $\frac{15}{2}$ or $\frac{18}{5}$ according as the series $a, x, y, z, b$ is an $AP$ or $HP$. Find the values of $a \& b$ assuming them to be positive integer.

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**151.** In a $A. P$ & an $H. P$ have the same first term, the same last term & the same number of terms; prove that the product of the $r^{th}$ term from the beginning in one series & the $r^{th}$ term from the end in the other is independent of $r$.

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**152.** $l, m, n$ are three numbers in G.P., $a$ is the first term of an A.P., whose $lth, mth$ and $nth$ terms are in H.P. If d is the common difference of the A.P.. prove that $a = (m+1)d$.

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**153.** Three unequal numbers are in HP and their squares are in AP. Show that the numbers are in the ratio $1-\sqrt{3}: -2: 1+\sqrt{3}$ and $1+\sqrt{3}: -2: 1-\sqrt{3}$.

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**154.** Insert 12 arithmetic means between $-3.5$ and $-42.5$.

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**155.** Insert 6 geometric means between 27 and $\frac{1}{81}$.

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**156.** Find the 5th harmonic mean when $n$ harmonic means are inserted between 1 and 2.

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**157.** If n A.Ms are inserted between two numbers, prove that the sum of the means equidistant from the beginning and the end is constant.

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**158.** Prove that the product $n$ geometric means between two quantities is equal to the nth power of $a$ geometric mean of those two quantities.

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**159.** If n arithmetic means xare inserted between 1 and 31 such that the $7^{th}$ mean $:$ the $(n+1)^{th}$ mean= $5:9$ then prove that n=14

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**160.** The sum of two numbers is $\frac{13}{6}$. An even number of arithmetic means are being inserted between them and sum exceeds their number by $1.$ find the number of means inserted.

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**161.** If a, b, c, d are positive numbers in HP then prove that $ad > bc$.

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162. If a, b, c are in AP or GP ot HP where $a > 0, b > 0, c > 0$ then prove that $b^2 >$ or = or $< ac$.

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163. If $a > 0, b > 0$ and $c > 0$ prove that (1984, 2M) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$

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164. If $a > 0, b > 0, c > 0$ prove that $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2}$.

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**165.** If $a, b, c$ are unequal and positive then show that $\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} < \frac{1}{2}(a+b+c)$

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**166.** If $x > 0, y > 0, z > 0,$ prove that $x^{\log y - \log z} + y^{\log z - \log x} + z^{\log x - \log y} \geq 3.$

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**167.** If $s = a + b + c + d$ where a, b, c, d are positive unequal numbers then prove that $(s-a)(s-b)(s-c)(s-d) \geq 81abcd.$

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**168.** If a, b, c are natural numbers, prove that $\left(\frac{a^2+b^2+c^2}{a+b+c}\right)^{a+b+c} \geq a^a b^b c^c \geq \left(\frac{a+b+c}{3}\right)^{a+b+c}.$

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**169.** Prove that $1^1 \cdot 2^2 \cdot 3^3 .... n^n \leq \left(\frac{2n+1}{3}\right)^{\frac{n(n+1)}{2}}$.

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**170.** Prove that $\left(\frac{n+1}{2}\right)^n > n!$

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**171.** Prove that the product of the first $n$ odd natural numbers cannot exceed $n^n$.

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**172.** If s be the sum of n positive unequal quantities a, b, c then prove the inequality ,

$\frac{s}{s-a}+\frac{s}{s-b}+\frac{s}{s-c}+\ldots\ldots>\frac{n^2}{n-1}(n\geq 2)$.

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**173.** If $a_i>0, i=1,2,3,\ldots,n$ then prove that $\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_4}+\ldots+\frac{a_{n-1}}{a_n}+\frac{a_n}{a_1}\geq n$.

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**174.** If $a\neq b$ and $a, b, x, y$ are positive rational such that $x+y=1$ then prove that $a^x b^y<ax+by$.

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**175.** If x, y, z be three sides of a triangle, prove that $xyz\geq(y+z-x)(z+x-y)(x+y-z)$.

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**176.** If $a$,$b$,$c$ are positive real numbers and sides of a triangle, then prove that

$(a+b+c)^3 \geq 27(a+b-c)(b+c-a)(c+a-b)$.

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**177.** In a triangle ABC, prove that

$\left(1+\frac{b-c}{a}\right)^a \cdot \left(1+\frac{c-a}{b}\right)^b \cdot \left(1+\frac{a-b}{c}\right)^c \leq 1.$

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**178.** If a+b+c=abc,a,b and c $\in R^+$, prove that $a+b+c \geq 3\sqrt{3}$.

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**179.** If $a+2b+3c=1$ and $a>0, b>0, c>0$ and the greatest value of $a^3b^2c$ is $\frac{1}{k}$ then $(k-5180)$ is

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**180.** If $4a+3b+2c=5$ and $a>0, b>0, c>0$ then the greatest value of $a^2b^3c^4$ will be ?

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**181.** If $a+b+c=1$ and $a>0, b>0, c>0$ then prove that $ab^2c^3\leq\frac{1}{432}$.

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**182.** Find the greatest value of the product of three positive numbers if the sum of their products taking two at a time is 12.

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**183.** Find the sum of n terms of the series $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5.........$ .

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**184.** Sum to n terms :$1 \cdot 3^2 + 3 \cdot 5^2 + 5 \cdot 7^2 + \ldots$

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**185.** Find the sum of the series $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + 4 \cdot (n-3) + \ldots + (n-1) \cdot 2 + , \ \cdot 1$ also, find the coefficient of $x^{n-1}$ in th cxpansion of $\left(1 + 2x + 3x^2 + \ldots .nx^{n-1}\right)^2$.

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**186.** Sum the series : $(1 \cdot 2) + (1 \cdot 2 + 2 \cdot 3) + (1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4) + \ldots$ to 12 brackets.

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**187.** If $a_1, a_2, a_3, \ldots, a_n$ be in AP whose common difference is $d$ then prove that $\sum_{i=1}^{n} a_i a_{i+1} = n\left\{a_1^2 + na_1 d + \frac{n^2 - 1}{3} d^2\right\}$.

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**188.** Find the sum of $n$ terms of the series $1 \cdot 2 \cdot 3 + 2 \cdot 4 \cdot 6 + 3 \cdot 6 \cdot 9 + \ldots$

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**189.** Find the sum of n terms of the series $1 + 9 + 24 + 46 + 75 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

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**190.** Find the nth term and sum to n tems of the following series: 3+6+11+18+...

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**191.** If $t_n = \sum_1^n n$, find $S_n = \sum_1^n t_n$.

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**192.** Find the sum of n terms of the series $1^2 + 4^2 + 7^2 + \dots\dots\dots$

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**193.** Find the sum of the series $2 + 5 + 14 + 41 + 122$ +.... up to $n$ terms and hence evaluate $S_a$.

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**194.** The sum to $n$ terms of series

$1+\left(\frac{1}{2}+\frac{1}{2^2}\right)+1+\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right)+$ is

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**195.** Find the sum to n terms of the series

$5+11+19+29+41+\ldots\ldots$

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**196.** The sum of first 9 terms of the series

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots\ldots..$ is

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**197.** Find the sum to n terms of the series

$$\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+\cdots$$

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**198.** Find $\sum n^3$ from n=1 to n=100.

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**199.** Evaluate $\sum_{n=1}^{n}\left\{\sum_{n=1}^{n}(2^n+3n)\right\}$.

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**200.** Sum : $1\cdot 1+2\cdot 3+4\cdot 5+8\cdot 7+\ldots$ to $n$ terms.

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**201.** $\forall n \in N, 1+2x+3x^2+ \dot{} + nx^{n-1} = (x \in R, x \neq 1)$

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**202.** $1-\frac{4}{2}+\frac{7}{2^2}-\frac{10}{2^3}+$

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**203.** Sum to infinite terms : $1 \cdot 2 + 2 \cdot 3x + 3 \cdot 4 \cdot x^2 + \ldots, (|x| < 1)$.

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**204.** Evaluate : $\sum_{r=1}^{\infty} r^2 x^{r-1}, (|x| < 1)$.

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**205.** Evaluate : $\sum_{n=1}^{15} (2n+1)2^n$.

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**206.** If $f(x) = x + 3x^2 + 5x^4 + 7x^8 + \dots$ to $n$ terms then find the value of $f'(1)$.

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**207.** If $g(x) = 1 + 3x + 5x^2 + 7x^3 + \dots$ to $n$ terms then find $\int_0^1 g(x)dx$.

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**208.** Show that

$$\frac{n-1}{n+1} + 3\left(\frac{n-1}{n+1}\right)^2 + 5\left(\frac{n-1}{n+1}\right)^3 + \dots + \infty = \sum_{r=1}^{n-1} r.$$

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**209.** Find the sum to n terms of the series $1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots$

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**210.** $\left(x^2+\frac{1}{x^2}+2\right)+\left(x^4+\frac{1}{x^4}+5\right)+\left(x^6+\frac{1}{x^6}+8\right)+$

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**211.** Sum to infinite terms : $\frac{1}{5}-\frac{2}{7^2}+\frac{3}{5^3}-\frac{4}{7^4}+\ldots.$

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**212.** Sum to $n$ terms : $0.4+0.44+0.444+\ldots.$

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**213.** Find the sum to $n$ terms of the series $5+55+555+\ldots$

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**214.** Natural numbers are divided into groups in the following way: $1, (2, 3), (4, 5, 6), (7, 8, 9, 10),$ Show that the sum of the numbers in the nth group is $\left(n\frac{n^2+1}{2}\right.$

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**215.** Prove that the sum of the numbers in any group in the following is the square of an odd integer: $(1), (1, 3, 5), (1, 3, 5, 7, 9), \ldots.$

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**216.** Find the sum of the n terms of the sequence

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots$$

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**217.** Sum to n terms of the series $\frac{1}{(1+x)(1+2x)}+\frac{1}{(1+2x)(1+3x)}+\frac{1}{(1+3x)(1+4x)}+$ .........

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**218.** The sum of the series : $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5\cdot 6}+\frac{1}{5\cdot 6\cdot 7\cdot 8}+\ldots+\infty$ is

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**219.** Sum to n terms of the series $\frac{1}{5!}+\frac{1}{6!}+\frac{2!}{7!}+\frac{3!}{8!}+\ldots\ldots$ is

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**220.** $\left(\frac{1}{x+1}+\frac{1}{x+5}\right)=\left(\frac{1}{x+2}+\frac{1}{x+4}\right)$:

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221. Evaluate $\sum_{n=1}^{n} \frac{n^2+n-1}{(n+2)!}$.

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222. Evaluate $\sum_{r=1}^{n} (rr!) =$

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223. If $a_1, a_2, a_3, \ldots, a_n$ are in AP and $a_i \neq (2k-1)\frac{\pi}{2}$ for all $i$, find the sum

$\sec a_1 \cdot \sec a_2 + \sec a_2 \cdot \sec a_3 + \sec a_3 \cdot \sec a_4 + \ldots + \sec a_{n-1} \cdot \sec a_n$.

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224. Sum of the first $n$ terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} +$ is equal to $2^n - n - 1$ b. $1 - 2^{-n}$ c. $n + 2^{-n} - 1$ d. $2^n + 1$

A. $2^n - n - 1$

B. $1 - 2^{-n}$

C. $n + 2^{-n} - 1$

D. $2^n - 1$.

**Answer:**

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**225.** Sum of infinite series $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \ldots\ldots\infty$ is

A. $\frac{1}{3}$

B. 3

C. $\frac{1}{4}$

D. $\infty$.

**Answer:**

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**226.** If first and $(2n-1)^t h$ terms of an AP, GP. and HP. are equal and their nth terms are a, b, c respectively, then (a) a=b=c (b)a+c=b (c) a>b>c and $ac-b^2=0$ (d) none of these

A. $a=b=c$

B. $a \geq b \geq c$

C. $a+c=b$

D. $ac-b^2=0.$

**Answer:**

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**227.** If $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between" a and b, then find the value of n.

A. 0

B. $\frac{1}{2}$

C. 1

D. $-1$

**Answer:** $-\frac{1}{2}$.

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**228.** If $H$ is the harmonic mean between $P and Q$ then find the value of $H/P+H/Q$.

A. 2

B. $\frac{PQ}{P+Q}$

C. $\frac{P+Q}{PQ}$

D. None of these.

**Answer:**

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**229.** If $x > 0$ then the minimum value of $x^3 + \frac{1}{x^3}$ is

A. 3

B. 0

C. 2

D. $\frac{1}{2}$

**Answer: 1**

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**230.** Evaluate: $\int \frac{x^3}{(x-1)(x-2)} dx$

A. a positive integer

B. divisible by $n$

C. equal to $n + \frac{1}{n}$

D. never less than $n$.

**Answer:**

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**231.** If $\sum_{k=1}^{n}\left(\sum_{m=1}^{k} m^2\right) = an^4 + bn^3 + cn^2 + dn + e$ then -

A. $a = \frac{1}{12}$

B. $b = \frac{1}{6}$

C. $d = \frac{1}{6}$

D. $e = 0.$

**Answer:**

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**232.** If a,b,c are in H.P., then the value of

$$\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$$ is

A. True

B.

C.

D.

**Answer:**

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**233.** For $x=\log_2 5$, the number $\log_{10}(2^x-1)$ is the AM of $\log_{10} 2$ and $\log_{10}(2^x+3)$.

A. True

B. False

C. Can not determine

D. False and these are in GP

**Answer: A**

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**234.** If x is AM of $\frac{\tan \pi}{9}$ and $\frac{\tan(5\pi)}{18}$ and y is AM of $tha\frac{\pi}{9}$ and $\frac{\tan(7\pi)}{18}$, then

A. True

B.

C.

D.

**Answer:**

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**235.** The sum of $\frac{1}{10}+\frac{23}{10^3}+\frac{23}{10^5}+\frac{23}{10^7}+....$ as a recurring decimal fraction is

A. $0.1\overline{23}$

B. $0.\overline{123}$

C. $0.12\overline{3}$

D. $0.1234$

**Answer: A**

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