MATHS

BOOKS - BHARATI BHAWAN MATHS (HINGLISH)

Properties and Application of definite Integrals

Example

1. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) 2π

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2. Evaluate: $\int_{\pi/4}^{\pi/4} \frac{x+\pi/4}{2-\cos 2x} dx$

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3. $\int_{-\frac{1}{\sqrt{3}}}^{-\frac{1}{\sqrt{3}}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$

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4. If $P = \int_0^{\infty} \frac{x^2}{1+x^4} dx; Q = \int_0^{\infty} \frac{x dx}{1+x^4} \text{and} R = \int_0^{\infty} \frac{dx}{1+x^4}$,

then prove that :

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5. Evaluate: $\int_{-1}^{3/2} |x \sin \pi x| dx$

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6. Given a function $f(x)$ such that It is integrable over every interval on the real line, and $f(t+x) = f(x)$, for every x and a real t. Then show that the integral $\int_a^{a+t} (x)dx$ is independent of $a\cdot$

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7. Given a function $f(x)$ such that It is integrable over every interval on the real line, and $f(t+x) = f(x)$, for every x and a real $t\cdot$ Then show that the integral $\int_a^{a+t} (x)dx$ is independent of $a\cdot$

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8. Prove that for any positive integer k, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + \quad + \cos(2k-1)x]\cdot$ Hence, prove that $\int_0^{\frac{\pi}{2}} \sin 2xk \cot x dx = \frac{\pi}{2}\cdot$

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9. Evaluate $\int_0^1 (tx + 1 - x)^n dx$, where n is a positive integer and t is a parameter independent of x . Hence , show that

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10. If $y = \int_1^{x^3} \frac{dt}{1 + t^4}$, find dy/dx.

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11. Prove that $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4 - x^2 - x^3}} \leq \frac{\pi}{4\sqrt{2}}$

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12. Let $f: \left[0, \frac{\pi}{2}\right] \rightarrow R$ be continuous and satisfy $f'(x) = \frac{1}{1 + \cos x}$ for all $x \in \left(0, \frac{\pi}{2}\right)$. If f(0)=3 then $f\left(\frac{\pi}{2}\right)$ has the value equal to :

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13. A cubic function f(x) vanishes at $x = -2$ and has relative maximum/minimum at $x = -1$ and $x = \frac{1}{3}$. If $\int_{-1}^{1} f(x)dx = \frac{14}{3}$, find the cubic function f(x).

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14. Let $a + b = 4, wherea < 2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx \in crerasesas(b - a) \in crerases.$

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15. Let $f(x) = \int_{-2}^{x} |t + 1| dt$, then

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16. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$ then show that every line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$

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17. If $\int_0^{\pi} x f(\sin x)dx = A \int_0^{\frac{\pi}{2}} f(\sin x)dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) 2π

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18. Evaluate: $\int_{\pi/4}^{\pi/4} \frac{x + \pi/4}{2 - \cos 2x} dx$

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19. $\int_{-\frac{1}{\sqrt{3}}}^{-\frac{1}{\sqrt{3}}} \frac{x^4}{1 - x^4} \cos^{-1}\left(\frac{2x}{1 + x^2}\right) dx$

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20. Find the value of $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

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21. Evaluate the following integral: $\int_0^{15} [x^2] dx$

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22. Given a function $f(x)$ such that It is integrable over every interval on the real line, and $f(t+x) = f(x)$, for every x and a real $t\cdot$ Then show that the integral $\int_a^{a+t} (x) dx$ is independent of $a\cdot$

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23. Given a function $f(x)$ such that It is integrable over every interval on the real line, and $f(t+x)=f(x)$, for every x and a real $t\cdot$ Then show that the integral $\int_a^{a+t}(x)dx$ is independent of $a\cdot$

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24. Prove that for any positive integer $k, \frac{\sin 2kx}{\sin x}=2[\cos x+\cos 3x+ \quad +\cos(2k-1)x]\cdot$ Hence, prove that $\int_0^{\frac{\pi}{2}}\sin 2xk\cot x dx=\frac{\pi}{2}\cdot$

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25. Evaluate $\int_0^1(tx+1-x)^n dx$, where n is a positive integer and t is a parameter independent of x . Hence , show that

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26. $\lim_{x \to \infty} \frac{\int_0^{2x} xe^{x^2} dx}{e^{4}x^{2}}$

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27. Prove that $\frac{\pi}{6} \leq \int_0^1 \frac{dx}{\sqrt{4 - x^2 - x^3}} \leq \frac{\pi}{4\sqrt{2}}$

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28. Let $f:\left[0, \frac{\pi}{2}\right] \to R$ be continuous and satisfy $f'(x) = \frac{1}{1 + \cos x}$ for all $x \in \left(0, \frac{\pi}{2}\right)$. If f(0)=3 then $f\left(\frac{\pi}{2}\right)$ has the value equal to :

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29. A cubic function $f(x)$ vanishes at $x = -2$ and has relative minimum/maximum at $x = -1 and x = \frac{1}{3}$ if $\int_{-1}^{1} f(x)dx = \frac{14}{3}$.

Find the cubic function $f(x)$.

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30. Let $a+b=4, wherea<2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx}>0$ for all $x,$ prove that $\int_0^a g(x)dx+\int_0^b g(x)dx \in crerasesas(b-a) \in crerases.$

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31. Let $f(x)=\int_{-2}^x |t+1|dt$, then

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32. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$ then show that every line $y=mx$ intersects the curve $y^2+\int_0^x f(t)dt=2$

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Exercise

1. Prove that $\int_0^{\frac{\pi}{2}} \frac{\cos\theta - \sin\theta}{1 + \sin\theta.\cos\theta} d\theta = 0.$

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2. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

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3. $\int_0^{\pi} \log(1 + \cos x) dx = -\pi(\log 2)$

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4. Prove that $\int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)} dx = a$

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5. Evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$

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6. Evaluate: $\int_0^{\pi/4} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$

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7. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$

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8. Evaluate $\int_0^{\pi} \frac{x dx}{1 + \cos \alpha \sin x}, 0 < \alpha < \pi.$

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9. $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$

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10. Evaluate $\int_0^{\pi} \frac{x\,dx}{1+\cos^2 x}$.

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11. $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$

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12. Prove that $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx = \frac{\pi}{2}(\log 2)$.

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13. $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

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14. Prove that: $\int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2$

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15. $\int_0^{\pi/2} \log(\tan x + \cot x) dx = \pi(\log 2)$

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16. $\int_0^{\infty} \frac{x \log x}{\left(1+x^2\right)^2} dx =$

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17. $\int_{\pi/6}^{\pi/3} \frac{1}{(1+\sqrt{\tan x})} dx = \frac{\pi}{12}$

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18. Q. $\int_0^{\pi} \left(e^{\cos^2 x} (\cos^3 (2n+1) x dx, n \in I\right.$

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19. $\int_0^{2\pi} \frac{\sin 2\theta}{a - b\cos 2\theta} d\theta =$

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20. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

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21. Prove that: $y=\int_{\frac{1}{8}}^{\sin^2 x}\sin^{-1}\sqrt{t}dt+\int_{\frac{1}{8}}^{\cos^2 x}\cos^{-1}, where 0 \leq x \leq \frac{\pi}{2}$, is the equation of a straight line parallel to the x-axis. Find the equation.

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22. Evaluate the following: $\int_0^{\frac{\pi}{2}}|\cos x-\sin x|dx$

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23. Evaluate the following: $\int_{-2}^{2}(|x|+|x-1|)dx$

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24. Prove that $\int_a^b\{[x]+[-x]\}dx=a-b$, where [x]is the greatest integer $\leq x$.

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25. Statement-I $\int_0^9 [\sqrt{x}]\,dx = 13$, Statement-II $\int_0^{n^2} [\sqrt{x}]\,dx = \frac{n(n-1)(4n+1)}{6}$, $n \in N$ (where [.] denotes greatest integer function) (1) Statement-I is true, Statement-II is true Statement-II is a correct explanation for Statement-I (2) Statement-I is true, Statement-II is true Statement-II is not a correct explanation for Statement-I, (3) Statement-I is true, Statement-II is false. (4) Statment-I is false, Statement-II is true.

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26. Show that: $\int_0^x [x]dx = [x]\frac{[x]-1}{2} + [x](x-[x])$, where $[x]$ denotes the integral part of x.

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27. The value of the integral $\int_0^{100\pi} \sqrt{1 - \cos 2x} \, dx$is

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28. Evaluate the following : $\int_0^{\pi} \frac{dx}{1 + \sin x}$

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29. Evaluate the following: $\int_0^{10\pi} |\cos x| dx$

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30. Evaluate $\int_0^{n\pi + t} (|\cos x| + |\sin x|) dx,$ where $n \varepsilon N$ and $t \varepsilon [0, \pi/2]$.

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31. Show that: $\int_a^b f(x)dx = \int_{a+c}^{b+c} f(x-c)dx$ and hence show that $\int_0^\pi \sin^{100} x \cos^{99} x dx = 0$

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32. If $f(x)$ is a continuous function defined on $[0, \ 2a]$. Then prove that $\int_0^{2a} f(x)dx = \int_0^a \{f(x) + (2a - x)\}dx$

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33. If $f(t)$ is an odd function, then $\varphi(x) = \int_a^x f(t)dx$ is an even function.

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34. It is known that $f(x)$ is an odd function in the interval $\left[\frac{p}{2}, \frac{p}{2}\right]$ and has a period p, Prove that $\int_q^x (t)dt$ is also periodic function with the same period.

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35. If $f(a+x) = f(x)$, then prove that $\int_a^{na} f(x)dx = (n-1)\int_0^a f(x)dx$ where $a > 0$ and $n \in N$.

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36. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

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37. Let a, b, c be non-zero real numbers such that ;

$$\int_0^1 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx = \int_0^2 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$$

then the quadratic equation $ax^2+bx+c=0$ has -

A. a)no root in(0,2)

B. b)at least one root in(1,2)

C. c)at least one root in(0,1)

D. d)two imaginary roots

Answer:

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38. If f(X) is continuous and $n \in N$then the value of

$\int_{-2}^{2} \{f(x) - f(-x)\}x^{2n} dx$dx is

A. $nf(2)$

B. $2n$

C. 0

D. none of these

Answer: C

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39. An extremum value of $y = \int_0^x (t-1)(t-2)dt$ is : $\frac{5}{6}$ (b) $\frac{2}{3}$ (c) 1 (d) 2

A. a)maximum at x=1

B. b)maximum at x=2

C. c)minimum at x=1

D. d)minimum at x=2

Answer:

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40. An AC voltage is given by $E = E_0 \frac{\sin(2\pi t)}{T}$. Then the mean value of voltage calculated over time interval of T/2 seconds

A. a)E_0

B. b)0

C. c)$\frac{E_0}{2\pi}$

D. d)none of these

Answer:

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41. If $g(x) = \int_0^x \cos^4 t dt$, then $g(x+\pi)$ equals $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ $g(x)g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

A. a)$g(x)+g(\pi)$

B. b)$g(x) - g(\pi)$

C. c)$g(x)g(\pi)$

D. d)$\frac{g(x)}{g(\pi)}$

Answer:

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42. The value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

A. a)$2\pi^2$

B. b)$\frac{\pi^2}{2}$

C. c)π^2

D. d)0

Answer:

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43. State whether the statements are true or false.If $y = \int_0^x f(t)dt. Then \frac{dy}{dt} = f(t).$

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44. State whether the statements are true or false. $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\sin x} - \sqrt{\cos x}} = 0$

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45. If $\int_0^a \{f(x) + f(-x)\}dx = \int_{-a}^a \phi(x)dx$then $\phi(x)$=_____.

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46. Evaluate: $\int_0^\pi \frac{\cos x}{(1 + \sin x)^2} dx$

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47. The value $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^2 \sin^{-1} x dx$=____.

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48. If $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$, then A is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) 2π

A. a)1/2

B. b)2n

C. c) $\frac{\pi}{2}$

D. d)none of these

Answer:

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49. The value of $\int_0^{\frac{\pi}{2}} \sin 2x . \log \tan x dx$ is

A. 0

B. π

C. $\frac{\pi}{2}$

D. none of these

Answer: A

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50. If $\int_0^1 \frac{e^t}{t+1} dt = a$, then $\int_{b-1}^{b} \frac{e^{-t}}{t-b-1} dt$=

A. a)ae^{-b}

B. b)$-ae^{-b}$

C. c)$-be^{-a}$

D. d)none of these

Answer:

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51. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin 2x}{1+\cos^3 x} dx$

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52. If f is a continuous function on the interval $[a, b]$ and there exists some $c \in (a, b)$ then prove that $\int_a^b f(x)dx = f(c)(b-a)$.

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53. If $l_n = \int_1^e (\log x)^n dx, \text{then} l_n + n l_{n-1}$ equal to

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54. If f(x)=$|x-21|+|x-1|$then evaluate $\int_{-2}^{2} f(x)dx.$

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55. If $f(x)=f(4-x), g(x)+g(4-x)=3$ and $\int_{0}^{4} f(x)dx=2,$ then : $\int_{0}^{4} f(x)g(x)dx=$

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56. It is known that $f(x)$ is an odd function and has a period p. Prove that $\int_{a}^{x} f(t)dt$ is also periodic function with the same period.

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57. The value of $\int_0^2 x^{[x^2+1]}(dx)$, where $[x]$ is the greatest integer less than or equal to x is

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58. Let $f(x)=[b^2+(a-1)b+2]x-\int(\sin^2 x+\cos^4 x)dx$ be an increasing function of $x\in R$ and $b\in R$, then a can take value(s)

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59. For $x>0, let f(x)=\int_1^x \frac{\log t}{1+t}dt\cdot$ Find the function $f(x)+f\left(\frac{1}{x}\right)$ and find the value of $f(e)+f\left(\frac{1}{e}\right)\cdot$

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60. If $\int_0^1 \frac{\sin t}{1+t} dt = \alpha$, them the value of the integral $\int_{4\pi-2}^{4\pi} \frac{\sin\frac{t}{2}}{4\pi+2-t} dt$ in terms of α is given by

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61. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x+\cos x} dx$

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62. The point of extremum of $f(x)=\int_0^x (t-2)^2(t-1)dt$ is a

A. maximum at x=1

B. maximum at x=2

C. minimum at x=1

D. minimum at x=2

Answer: C

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