



MATHS

BOOKS - ML KHANNA

CORRELATION AND REGRESSION

Example Solution

:

1. Find the coefficient of correlation between the values of x and y

<i>x</i> :	10	14	18	22	26	30
<i>y</i> :	18	12	24	6	30	36

Note : Here, values in x, increase by the interval of 4 and values of y

have the common factor 6.

2. Ten students got the following percentage of marks in

Mathematics and Statistics :

Students :	1	2	3	4	5	6	7	8	9	10
Marks in maths :	78	36	98	25	75	82	90	62	65	39
Marks in Statistics :	84	51	91	60	68	62	86	58	53	47

Calculate rank correlation coefficient.



3. Illustration for calculating ρ in case of repeated values. Below are given marks of ten students in Maths and Physics out of 100, in which some students have got similar ranks. We have to find out, how proficiency in either subject interacts in the matter of rank by calculating ρ .

Students :	1	2	3	4	5	6	7	8	9	10
Marks in Maths :	68	64	75	50	64	80	75	40	55	64
Marks in Physics :	62	58	68	45	81	60	68	48	50	70

Note : In Maths two students get 75 marks each and three students get 64 marks each and in Physics two get 68 marks each.

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Illustrative Examples

In a partially destroyed laboratory record of an analysis correlation data, the following results only are legible: variance of x=9, regression equations 8x-10y+66=0, 40x-18y=214.
 What are (a) the mean values of x and y, (b) the standard deviation of y and (c) the coefficient of correlation between x and y?

2. Find the variance of the variate u = ax + by in terms of the

variances and covariance of x and y.



- 3. The heights (x) and weights (y) of 1000 policemen are given :
- $ar{x}=68$ inches, $ar{y}=150kg, \sigma_x=2.5$ inches
- $\sigma_y=20kg, r=0.6$

Estimate from the above data

The height of a particular policeman whose weight is 200 kg

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4. The heights (x) and weights (y) of 1000 policemen are given :

$$ar{x}=68$$
 inches, $ar{y}=150kg, \sigma_x=2.5$ inches

$$\sigma_y=20kg, r=0.6$$

Estimate from the above data

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The weight of a particular policeman who is 5 ft. tall.

Question Problems On Correlation

1. If a linear relation exists between the variables x and y, then

|r| = 1.

Let the linear relation be y = a + bx.

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2. Calculate the correlation coefficient between the height of father and height of son from the given data :

Height of father64656667686970(in inches) X :Height of son66676568706872(in inches) Y :

Taking the variable X for the height of father and the variable Y for the height of son, and x as the deviation of X from its mean \overline{X} , (which is 67 here), and similarly y as the deviation of Y from its mean \overline{Y} , (which is 69 here) i.e. $x = X - \overline{X}$ and $y = Y - \overline{Y}$.

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3.

 $N = 50, \Sigma X = 75, \Sigma Y = 80, \Sigma X^2 = 130, \Sigma Y^2 = 140, \Sigma X Y = 120$

find the value of the correlation coefficient between the variables

X and Y.

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4. Given $r=0.8, \Sigma xy=60, \sigma_y=2.5$ and $\Sigma x^2=90$. Find the

number of items (x and y are deviations from arithmetic average)

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5. In two sets of variables X and Y with 10 observations each, the following data were observed :

$$\overline{X}=12$$
, S.D. of $X=3$

 $\overline{Y}=15$, S.D. of Y=4

Coefficients of correlation between X and Y = 0.5 However, on subsequent vertification it was found that one value of X(=15)and one value of Y(=13) were wrongly taken as 16 and 18. What is correct value of correlation coefficient?



Question Problems On Correlation State Of Nature Of The Following Correlations Positive Negative Or No Correlation

1. Sale of wollen garments and the day temperature,

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2. The colour of the Sari, and the intelligence of the lady who

wears it,

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3. Rainfall and yield of crop,

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4. Age of applicants for life insurance and the premium of insurance,

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5. Unemployment index and the purchasing power of the common
man.
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Question Fill In The Blanks
1. The Karl Pearson's coefficient of correlation between variables X
and Y is





5. For the following observations :

The correlation coefficient is equal to



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8. The sign of regression coefficient is As that of correlation

coefficient.

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9. When one regression coefficient is positive, the other would also be
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10. The further the two regression lines cut each other, the
Be the degree of correlation.
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12. Lines of regression are If $r=0$ and they are If $r=\pm 1$.
O Watch Video Solution
13. If the regression coefficients of X on Y and Y on X are -0.4 , and -0.9 respectively, then the correlation coefficient
or and or respectively, then the correlation coefficient

between X and Y is

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Question State Whether The Following Are True Or False

1. The coefficient of correlation r between two variates X and Y will

be greater, if a larger sample is chosen.







2. There is no relationship between correlation coefficient and regression coefficient.



3. The regression lines of two independent variables are parallel

to each.

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4. Both regression coefficients must be less than unity
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5. Regression coefficients are independent of change of scale and origin.
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9. The regression line of X on Y is the line which is drawn such that the aggregate of the squares of the vertical deviations of the observations from the corresponding point on the line is minimum.

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10. For paired set of data, there would invariably be only one regression line.



Problem Set 1 Mcq

1. If U=3X+1, V=Y+2 and $r_{XY}=0.6$ then r_{UV} is

A.0.2

 ${\rm B.}-0.2$

 $\mathsf{C}.\,0.6$

D. none of these

Answer: C



2. If X and Y are random variables and a, b, c, d are constants such

that a
eq 0, c
eq 0 then

r(aX+b, cY+d) is

A. r(X, Y)

 $\operatorname{B.}\operatorname{acr}(X,Y)$

C.
$$rac{|ac|}{ac}r(X,Y)$$

D. none of these

Answer: C

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3. x_1 and x_2 are two variates with variances σ_1^2 and σ_2^2 respectively and r is the correlation between them. The value of a such that $x_1 + ax_2$ and $x_1 + \frac{\sigma_1}{\sigma_2}x_2$ are uncorrelated is

A.
$$-rac{\sigma_1}{\sigma_2}$$

B. $rrac{\sigma_2}{\sigma_1}$

 $\mathsf{C}. \sigma_1 \sigma_2$

D. none of these

Answer: A

4. Correlation coefficient r of two variables X and Y is positive

when

A. $\sigma_{X+Y} > \sigma_{X-Y}$

B. $\sigma_{X-Y} > \sigma_{X+Y}$

 $\mathsf{C}.\,\sigma_{X+Y}=0$

D. none of these

Answer: A

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5. The line of regression Y and X referred to $\overline{X}, \overline{Y}$ as origin is

A.
$$Y-\overline{Y}=rrac{\sigma_Y}{\sigma_X}ig(X-\overline{X}ig)$$

B.
$$Y=rrac{\sigma_Y}{\sigma_X}ig(X-\overline{X}ig)$$
C. $Y=rrac{\sigma_Y}{\sigma_X}X$

$$\sigma_X$$

D. none of these

Answer: C



6. Angle between the two lines of regression is given by

$$\begin{array}{l} \mathsf{A}.\tan^{-1} \Biggl\{ \frac{b_{YX} - \frac{1}{b_{XY}}}{1 + \frac{b_{XY}}{b_{YX}}} \Biggr\} \\ \mathsf{B}.\tan^{-1} \Biggl\{ \frac{b_{YX}b_{XY} - 1}{b_{YX} + b_{XY}} \Biggr\} \\ \mathsf{C}.\tan^{-1} \Biggl\{ \frac{b_{YX} - b_{XY}}{1 + b_{YX} \cdot b_{XY}} \Biggr\} \end{array}$$

D. none of these

Answer: B

7. If means $\overline{X}, \overline{Y}$ of the variates X and Y are eah zero and $\sigma_X^2 = \sigma_Y^2 = 1$ and $r = r_{XY} \neq 1$, the value of b such that X + Y and X + bY are uncorrelated is

A. b = 0

B. b = 1

 $\mathsf{C}.\,b=\ -1$

D. none of these

Answer: C



8. Two variates X and Y have zero means, the same variance σ^2 and

zero correlation. Then

$$U = X \cos \alpha + Y \sin \alpha, V = X \sin \alpha - Y \cos \alpha$$
 have correlation
A. -1
B.1
C. 0
D. $\frac{1}{2}$

Answer: C

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9. σ_X^2, σ_Y^2 and σ_{X-Y}^2 are the variances of X, Y and X-Y respectively, then r_{XY} is

A.
$$\frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_2\sigma_Y}$$

B.
$$\frac{\sigma_X^2 + \sigma_Y^2}{\sigma_X\sigma_Y}$$

C.
$$\frac{\sigma_X^2 - \sigma_Y^2}{\sigma_X^2 + \sigma_Y^2}$$

D.
$$\frac{\sigma_X\sigma_Y}{\sigma_{X-Y}}$$

Answer: A

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10. The correlation between X and a-X is

A.
$$-1$$

B. $-\frac{1}{2}$
C. $-\frac{1}{4}$

D. 0

Answer: A



11. The two variates X and Y are uncorrelated and have standard deviations σ_X and σ_Y respectively, the correlation coefficient between X + Y and X - Y is

A.
$$\frac{\sigma_X \sigma_Y}{\sigma_X^2 + \sigma_Y^2}$$

B.
$$\frac{1}{2} \left(\frac{1}{\sigma_X} + \frac{1}{\sigma_Y} \right)$$

C.
$$\frac{\sigma_X^2 - \sigma_Y^2}{\sigma_X^2 + \sigma_Y^2}$$

D. none of these

Answer: C



12. X and Y are two correlated variables with the same standard deviation and the correlation coefficient r, the correlation coefficient between X and X + Y is

A.
$$\frac{r}{2}$$

B. $\sqrt{\frac{1-r}{2}}$
C. $\sqrt{\frac{1+r}{2}}$

D. 0

Answer: C

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13. \bar{x} is the arithmetic mean of n independent variates $x_1, x_2, x_3, \ldots, x_n$ each of standard deviation σ , then variance (\bar{x}) is

A.
$$\frac{\sigma^2}{n}$$

B. $\frac{n\sigma^2}{2}$
C. $\frac{(n+1)\sigma^2}{3}$

D. none of these

Answer: A



14. A computer while calculating r_{xy} from 25 pairs of observations

obtained the following constants

$$n=25, \Sigma x=125, \Sigma x^2=650, \Sigma y=100,$$

$$\Sigma y^2 = 460, \Sigma xy = 508$$

A recheck showed that he had copied down two pairs (6, 14), (8, 6) while the correct values were (6, 12), (6, 8). The correct value of the correlation coefficient is

A.
$$\frac{2}{3}$$

B. $\frac{3}{4}$
C. $-\frac{1}{2}$

D. 1

Answer: A



15. The coefficient of correlation between X and Y is 0.6. Their covariance is 4.8 var (x)=9. Find the value of σ_y .

A.
$$\frac{8}{9}$$

B. $\frac{5}{8}$
C. $\frac{8}{3}$

D. none of these

Answer: C

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16. The coefficients of rank correlation between marks in Mathematics and marks in Physics obtained by a certain group of students is 0.8. If the sum of the squars of the difference in the rank is given to be 33, then find the number of students in the group.

A. 30

B. 11

C. 10

D. 15

Answer: C

17. In two sets of variables x and y with 50 observations each, the following data were observed $\bar{x} = 10, \sigma_x = 3, \bar{y} = 6, \sigma_y = 2, r = 0.3$ But on subsequent verification, it was found that one value of x(=9) and one value of y(=7) were found inacurate and hence weeded out. The value of r with remaining 49 pairs of value is

 $\mathsf{A.}\,0.3$

 $\mathsf{B.}\,0.2$

C. - 0.1

 $\mathsf{D.}\,0.6$

Answer: A

18. Two random variables have the least squares regression lines 3x + 2y - 26 = 0 and 6x + y - 31 = 0 correlation coefficient r_{xy} is

A. -0.5B. $\frac{1}{2}$ C. $\frac{1}{4}$

D. none of these

Answer: A



19. The lines of regression of y on x and x on y are respectively y = x and 4x - y - 3 = 0 and the second moment about the

origin for x is 2, variance of y is

A. 5

B. 4

C. 9

D. none of these

Answer: B

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20. The regression lines of x on y and y on x are x = 4y + 5 and y = kx + 4 respectively. The limits of k are given by

A.
$$0 < k < rac{1}{4}$$

B. $-1 < k < rac{1}{2}$

C. -1 < k < 1

D. none of these

Answer: A

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21. For two variables x and y, the two regression lines are x+2y-5=0, 2x+3y-8=0 and variance (x)=12. Then σ_y is

A. 4

B. 5

C. 2

D. none of these

Answer: C



22. For 10 observations on price (x) and supply (y) the following data were obtained (in appropriate units)

 $\Sigma x = 130, \Sigma y = 220, \Sigma x^2 = 2288, \Sigma y^2 = 5506, \Sigma x y = 3467$

Estimate of supply when the price is 16 units is

A. 24.02

B.26.03

C.25.04

D. none of these

Answer: C

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23. If two lines of Regression are respectively y=ax+b and x= $\alpha y + \beta$. If the two variables have the same mean, then b/β =

A.
$$\frac{1-a}{1-\alpha}$$
B.
$$\frac{1+a}{1+\alpha}$$
C.
$$\frac{1-\alpha}{1-\alpha}$$
D.
$$\frac{1+\alpha}{1+\alpha}$$

Answer: A



24. $r_{xy} < 0$, according as

A.
$$\sigma_{x+y} > \sigma_{x-y}$$

B.
$$\sigma_{x+y} < \sigma_{x-y}$$

C. $\sigma_{x+y} > 0$

D. none of these

Answer: B

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25. The correlation between two variables x and y is given to be r. The values of x and y change such that cov (x, y) remains unaltered and var (x) and var (y) are changed to four times their original values. The new correlation coefficient is changed to

A. 4r

B. 16r

C. r/16

D. r/4

Answer: C::D



26. If U = aX + b, and V = -cY + d where a and c positive. If r(U, V) = 0.6, then r(X, Y) is equal to

A. 0.6

B. - 0.6

C.
$$0.6 \frac{c}{a}$$

D. $0.6 \frac{a}{c}$

Answer: B

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27. Let X and Y be two variables with the same variance and let U = X + Y, V = X - Y then cov (U, V) is equal to

A. cov (X, Y)

B. 0

C. 1

D. -1

Answer: B

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28. If Z=aX+bY and r is the correlation coefficient between X and Y, then σ_Z^2 is equal to

A. $2ab, r\sigma_X\sigma_Y$

B.
$$a^2\sigma_X^2+b^2\sigma_Y^2-2br\sigma_X\sigma_Y$$

C.
$$a^2\sigma_X^2+b^2\sigma_Y^2+2abr\sigma_X\sigma_Y$$

D. none of these

Answer: C

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29. If X and Y are two independent variables with $\sigma_X^2 = 36, \sigma_Y^2 = 16$ then correlation coefficient between U = X + Y and V = X - Y is

A.
$$\frac{5}{13}$$

B. $-\frac{5}{13}$
C. $\frac{4}{9}$

D. none of these

Answer: A

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30. The coefficient of correlation is independent of

A. change of origin only

B. change of scale only

C. both changes of scale and origin

D. neither change of scale nor change of origin

Answer: C



31. If both the regression coefficients b_{YX} and b_{XY} are positive,

then

A.
$$rac{1}{b_{YX}} + rac{1}{b_{XY}} > rac{2}{r}$$

B. $rac{1}{b_{YX}} + rac{1}{b_{XY}} < rac{2}{r}$
C. $rac{1}{b_{YX}} + rac{1}{b_{XY}} < rac{2}{r}$

D. none of these

Answer: A

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32. If the slopes of the line of regression of Y and X and of X and Y are 30° and 60° respectively, then r (X,Y) is

$$A. -1$$

B. 1

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: C



33. If the two lines of regression are 3x + y = 15 and x + 4y = 3, then value of x when y = 3 is

A.-4

B.-8

C. 4

D. none of these

Answer: C Watch Video Solution

34. The coefficient of correlation between random variables X and Y is 0.28, covariance between X and Y is 7.6 and the variance of X is

9, then the standard deviation of Y series is

A. 9.8

 $B.\,10.1$

C. 9.05

 $D.\,10.05$

Answer: C

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35. Linear relation between the variables is given by the equation ax + by + c = 0 such that ab > 0. Then r(x, y) is given by

A. 1

B. - 1

C. 0

D. any number lying between -1 and 1

Answer: B

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36. The lines of regression of y on x is $a_1x + b_1y + c_1 = 0$ and

that of x on y is $a_2x + b_2y + c_2 = 0$, then

A. $a_1b_2 \leq a_2b_1$

 $\mathsf{B.}\,a_1a_2\leq b_1b_2$

 $\mathsf{C}.\,a_2b_1\leq a_1b_2$

D. none of these

Answer: A

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37. If $\Sigma x = 55, \Sigma y = 74, \Sigma x y = 411, n = 10$, then covariance

between x and y is

 $\mathsf{A}.\,0.5$

B. - 0.5

 $\mathsf{C.}-0.4$

 $\mathsf{D}.\,0.4$

Answer: D



Problem Set 1 True False

1. x_1, x_2 and x_3 are three variates with means $\overline{x_1}, \overline{x_2}, \overline{x_3}$ respectively, each variate occurring equal number of times and having correlation coefficients r_{12} between x_1 and x_2 etc. Then $r_{12} + r_{23} + r_{13} \geq -3/2$

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2. If the correlation coefficient between the random variables X and Y be negative, then which of the following statements is true or false

The correlation coefficient between -X and -Y is positive

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3. If the correlation coefficient between the random variables X and Y be negative, then which of the following statements is true or false

E(XY) < E(X)E(Y)

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4. If the correlation coefficient between the random variables X and Y be negative, then which of the following statements is true or false

The regression coefficient of X on Y and that of Y on X are both

negative



5. If the correlation coefficient between the random variables X and Y be negative, then which of the following statements is true or false

The regression of Y on X is linear and the line is downward sloping

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6. If the correlation coefficient between X and Y is r, then correlation coefficient between aX and bY is equal to

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Problem Set 1 Fill In The Blanks

1. In case of perfect correlation between x and y, the number of

regression lines is





8. If the two lines of regression are 3x + 2y = 26 and 6x + y = 31, then the coefficient of correlation between x and y is

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9. If 2x + 3y - 1 = 0 is the regression line of y on x, then b_{yx} is

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10. If
$$b_{xy} = -rac{3}{2}, b_{yx} = -rac{1}{6}$$
, then value of r is

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Problem Set 1 True And False

1. Regression coefficients are independent of change of scale and

origin.

Watch Video Solution 2. If r is negative both the variables are decreasing. Watch Video Solution **3.** Two random variables are such that $\sigma_{X+Y} > \sigma_{X-Y}$, then r (X, Y) is positive. Watch Video Solution **4.** If $U = \frac{X-a}{b}$ and $V = \frac{Y-b}{k}$ then $b_{UV} = b_{XY}$

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5. If one of the two regression coefficients is negative, then the

variables are negatively correlated.

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6. If r(X, Y) = r, then r(aX, bY) is also equal to r.

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7. The regression coefficient of y on x is $\sigma_x \, / \, \sigma_y$

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8. If the correlation coefficient between the random variables X and Y is negative, then E(XY)>E(X)E(Y)



- A. $r \geq 1$
- $\mathsf{B.}\,r\leq 1$
- C. $|r| \leq 1$

D. $|r| \geq 1$

Answer: C



Answer: C

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3. The variances x and y represent height in cm and weight in gm

respectively. The correlation between x and y has the unit

A. gm

B. cm

C. gm-cm

D. none of these

Answer: D

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4. The coefficient of correlation for the following data will be

approximately

x:	2	4	5	6	3	6	8	10
y:	5	6	6	8	4	8	12	15

A. 90

B. 96

 $C.\,0.90$

D. none of these

Answer: B

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E If	x :	3	4	8	6	2	1
J. II	y:	5	3	9	6	9	2

The coefficient of correrlation will be approximately

 $\mathsf{A.}\,0.49$

B. - 0.49

 $\mathsf{C.}\,0.40$

 $\mathsf{D.}-0.40$

Answer: A

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6. For the data

The Karl Pearson coefficient is

A.
$$\frac{63}{\sqrt{94 \times 66}}$$
B. 63

$$\mathsf{C}.\,\frac{63}{\sqrt{94}}$$

D. none of these

Answer: A

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7. Two numbers within the bracket denote the rank of 10 students

in

(1, 10), (2, 9), (3, 8), (4, 7), (5, 6), (6, 5), (7, 4), (8, 3), (9, 2), (10, 1)

two

subjects

:

. Then rank correlation coefficient is

class

A. 0

а

of

 $\mathsf{B.}-1$

C. 1

D.0.5

Answer: B



8. If the lines of regression are 3x + 12y = 19 and 3y + 9x = 46

then r will be

A. 0.289

B. - 0.289

C.0.209

D.0.409

Answer: B





. Find the regression coefficient of y on x.

 $\mathsf{A.}-3.1$

B.-3.2

C. - 3.3

D. - 3.4

Answer: C

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10. If
$$\bar{x} = 0, \bar{y} = 0, \Sigma x_i y_i = 24, \sigma_x = 3, \sigma_y = 4, \text{ and } n = 10,$$

then the coefficient of correlation is

A.0.1

 $\mathsf{B.}\,0.2$

 $\mathsf{C}.\,0.3$

D. none of these

Answer: B

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