MATHS

BOOKS - ML KHANNA

DEFINITE INTEGRAL

Example

1. Let $g(x) = \int_0^x f(t)dt$, where f is such that $\frac{1}{2} \leq f(t) \leq 1$ for $t \in [0, 1]$ and $0 \leq f(t) \leq \frac{1}{2}$ for $t \in [1, 2]$. then . $g(2)$ satisfies the inequality

A. $-\frac{3}{2} \leq g(2) < \frac{1}{2}$

B. $0 \leq g(2) < 2$

C. $\frac{1}{2} < g[2] \leq \frac{3}{2}$

D. $2 < g[2] < 4$

Answer:

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2. $\int_0^{\pi/2} \log \sin x = -\left(\frac{\pi}{2}\right)\log 2$

then: $\int_0^{\pi/2} \theta \cot \theta d\theta = \frac{\pi}{2}\log 2$

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3. $\int_0^{\frac{\pi}{2}} \log \sin x = -\left(\frac{\pi}{2}\right)\log 2$

Then: $\int_0^{\pi/2} \theta^2 \cos ec^2 \theta d\theta = \pi \log 2$

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Problemset 1 Multiple Choice Questions

1. $\int_0^{\pi} \log(1+\cos x)dx$=

A. $\pi \log \frac{1}{2}$

B. $\frac{\pi}{2} \log 2$

C. $-\pi \log 2$

D. None

Answer: A::C

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2. $\int_0^{\pi/2} x \cot x dx =$

A. $\frac{\pi}{2} \log 2$

B. $\pi \log 2$

C. $2\pi \log 2$

D. $-\frac{\pi}{2} \log 2$

Answer: A

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3. If $\int_0^{100\pi} \sqrt{1-\cos 2x}dx = 200k$, then k is equal to

A. $2\sqrt{2}$

B. π

C. $\sqrt{3}$

D. $\sqrt{2}$

Answer: D

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4. If $\int_0^{50\pi} \left(\sin^4 x + \cos^4 x\right)dx = k\int_0^{\pi/2} \left(\frac{3}{4} + \frac{1}{4}\cos 4x\right)dx$, then k=

A. 200

B. 100

C. 50

D. 25

Answer: B

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5. $\int_0^{4\pi} |\cos x| dx =$

A. 4

B. 8

C. 0

D. None of these

Answer: B

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6. $\int_0^{32\pi/3} \sqrt{1+\cos 2x} dx$

A. $20\sqrt{2}-\sqrt{\frac{3}{2}}$

B. $22\sqrt{2}+\sqrt{\frac{3}{2}}$

C. $22\sqrt{2}-\sqrt{\frac{3}{2}}$

D. None

Answer: C

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7. The value of $\int_0^2 \left|\cos\frac{\pi}{2}x\right| dx$ is

A. 2π

B. $\pi/2$

C. $3/4\pi$

D. $4/\pi$

Answer: D

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8. $I_0 = \int_0^{n\pi} f(|\cos x|)dx$ and $I_2 = \int_0^{5\pi} f|\cos x|dx$, then

A. $\frac{I_1}{I_2} = \frac{5}{n}$

B. $\frac{I_1}{I_2} = \frac{n}{5}$

C. $I_1 + I_2 = n + 5$

D. $I_1 - I_2 = n - 5$

Answer: B

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9. If $I_1 = \int_0^{3\pi} f\left(\cos^2 x\right)dx$ and $I_2 = \int_0^{\pi} f\left(\cos^2 x\right)dx$, then

A. $I_1 = I_2$

B. $I_1 = 2I_2$

C. $I_1 = 5I_2$

D. $I_1 = 3I_2$

Answer: D

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10. The value of $\int_a^{a+\pi/2} \left(\sin^4 x + \cos^4 x\right) dx$ is

A. independent of a

B. $a(\pi/2)^2$

C. $\frac{3\pi}{8}$

D. $\frac{3}{8}\pi a^2$

Answer: A::C

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11. If for every integer n, $\int_n^{n+1} f(x)dx = n^2$, then the value of $\int_{-2}^{4} f(x)dx$ is

A. 16

B. 14

C. 19

D. None of these

Answer: C

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12. If $\int_{-2}^{3} f(x)dx = 5$ and $\int_{1}^{3} [2 - f(x)]dx = 6$, then $\int_{-2}^{1} f(x)dx$=

A. -3

B. -7

C. 3

D. 7

Answer: D

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13. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4} [3 - f(x)]dx = 7$, then the value of $\int_{2}^{-1} f(x)$ is

A. 2

B. -3

C. -5

D. None of these

Answer: C

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14. The value of the integral $\Sigma_{r=1}^{n}\int_0^1 f(r-1+x)dx$ is

A. $\int_0^1 f(x)dx$

B. $\int_0^2 f(x)dx$

C. $\int_0^n f(x)dx$

D. $n\int_0^1 f(x)dx$

Answer: C

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15. The value of $\int_0^{100} e^{x-[x]}dx$ is

A. 100e

B. $100(e-1)$

C. $100(e+1)$

D. none

Answer: B

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16. If f(x) is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all non-zero x, then $\int_{\sin.\theta}^{\cos ec\theta} f(x)$ dx equals

A. $\sin\theta + \cos ec\theta$

B. $\sin^2\theta$

C. $\cos ec^2\theta$

D. 0

Answer: D

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17. If $2f(x) + 3f\left(\frac{1}{x}\right) = \frac{1}{x} - 2, x \neq 0$ then $\int_1^2 f(x)dx$=

A. $-\frac{2}{5}\log 2+\frac{1}{2}$

B. $-\frac{2}{5}\log 2-\frac{1}{2}$

C. $\frac{2}{5}\log 2+\frac{1}{2}$

D. none

Answer: A

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18. The value of the integral $\int_0^\infty \frac{x\log x}{(1+x^2)^2}dx$ is

A. 1

B. 0

C. 2

D. None of these

Answer: A

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19. $\int_0^1 \tan^{-1}\frac{2x-1}{\{1+x-x^2\}}dx=$

A. -1

B. 1

C. 0

D. none

Answer: C

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20. The value of $\int_{1/e}^{\tan x}\frac{t}{1+t^2}dt+\int_{1/e}^{\cot x}\frac{1}{t(1+t^2)}dt$ is

A. -1

B. 1

C. 0

D. None of these

Answer: B

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21. $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ equals

A. $\frac{16}{15}$

B. $\frac{32}{15}$

C. $\frac{8}{15}$

D. $\frac{5}{6}$

Answer: A

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22. If $\int_0^{\pi/2} \cos^m x \sin^m x dx = \lambda \int_0^{\pi/2} \sin^m x dx$, then λ=

A. 2^m

B. 2^{-m}

C. $\sqrt{2^m}$

D. none

Answer: B

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23. The value of $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is

A. 1

B. -1

C. 2

D. none

Answer: C

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24. If $\int_{-2}^{5} f(x)dx = 7.5^3 - 7(-2)^3$ then f(x) is equal to

A. $21x^2$

B. $3x^2$

C. $\frac{3}{2}x^4$

D. None of these

Answer: A

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25. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^4 \frac{2xe^{\sin x^2}}{x^2}dx = F(k) - F(1)$ then one of the possible value of k is

A. 4

B. 16

C. 2

D. none

Answer: B

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26. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}, x > 0.$ If $\int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx = F(k) - F(1)$ then one possible value of k is

A. 15

B. 16

C. 63

D. 64

Answer: D

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27. $\frac{1}{c}\int_{ac}^{bc} f\left(\frac{x}{c}\right)dx=$

A. $\frac{1}{c}\int_{a}^{b} f(x)dx$

B. $\int_{a}^{b} f(x)dx$

C. $c\int_{a}^{b} f(x)dx$

D. $\int_{ac^2}^{bc^2} f(x)dx$

Answer: B

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28. If $A = \int_0^1 \frac{dx}{\sqrt{1+x^4}}$ and $B = \frac{\pi}{4}$ then

A. A=B

B. A= 2B

C. $A < B$

D. $A > B$

Answer: D

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29. If $g(x) = \int_0^x \cos^4 \text{ t dt}$, then $g(x+\pi)$ equals

A. $g(x) + g(\pi)$

B. $g(x) - g(\pi)$

C. $g(x)g(\pi)$

D. $\frac{g(x)}{g(\pi)}$

Answer: A

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30. $\int_{-a}^{a} f(x)dx$ is equal to

A. $\int_0^a [f(x) + f(-x)]dx$

B. $\int_0^a [f(x) - f(-x)]dx$

C. $2\int_0^a f(x)dx$

D. 0

Answer: A

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31. $\int_{1/2}^{2} |\log_{10} x| dx =$

A. $\log_{10}(8/e)$

B. $\frac{1}{2}\log_{10}(8/e)$

C. $\log_{10}(2/e)$

D. None of these

Answer: B

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Problemset 1 True And False

1. If f(x) is periodic with period T, then $\int_{a+2T}^{b+2T} f(x)dx = \int_{a}^{b} f(x)dx$

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Problemset 2 Multiple Choice Questions

1. Prove that: $\int_{0}^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$

A. $\frac{\pi}{2}$

B. π

C. $\frac{\pi}{4}$

D. none

Answer: C

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2. $\int_0^a \frac{dx}{x+\sqrt{(a^2-x^2)}}=$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none

Answer: A

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3. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{(\cot x)}}{\sqrt{(\cot x)}+\sqrt{(\tan x)}}dx$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none

Answer: A

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4. $\int_0^{\pi/2} \frac{\sqrt{(\sin^3 . x)}dx}{\sqrt{(\sin^3 x)} + \sqrt{(\cos^3 x)}} =$

A. $\pi/4$

B. 1

C. 0

D. $\pi/2$

Answer: A

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5. The value of $\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x}$ is

A. 0

B. 1

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: D

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6. $\int_0^{\pi} \frac{dx}{1+\tan^4 x} =$

A. 0

B. $\pi/6$

C. $\pi/4$

D. $\pi/2$

Answer: D

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7. The value of the integral $\int_0^{\pi/2} \frac{\phi(x)}{\phi(x)+\phi\left(\frac{\pi}{2}-x\right)}dx$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none

Answer: A

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8. $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx$=

A. 0

B. $(a+b)\frac{\pi}{2}$

C. $a+b$

D. $(a+b)\frac{\pi}{4}$

Answer: D

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9. $\int_0^{\infty} \frac{xdx}{(1+x)(1+x^2)} =$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. none

Answer: A

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10. $\int_0^{\pi/4} \log(1+\tan x)dx = ?$

A. $\frac{1}{2}\pi \log 8$

B. $\frac{1}{8}\pi \log 2$

C. $\frac{1}{4}\pi \log 2$

D. None of these

Answer: B

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11. $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ is equal to

A. $\frac{\pi}{8}$

B. $\frac{\pi}{8}\log 2$

C. $\log 2$

D. None of these

Answer: B

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12. $\int_0^{\pi} \sin^n x . \cos^{2m+1} x dx$ is equal to

A. 0

B. $\int_0^{\pi} \cos^{2m-1} x dx$

C. $\frac{(2m+1)!}{n!}$

D. None of these

Answer: A::B

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13. $\int_0^{\pi/2} \frac{(\cos x - \sin x)}{(1+\sin x \cos x)} dx$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. 0

D. none

Answer: C

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14. $\int_0^{\pi/2} \frac{\cos 2x}{(\sin x + \cos x)^2} dx =$

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. none

Answer: A

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15. $\int_{1/2}^{2} \frac{1}{x} \cos ec^{101}\left(x - \frac{1}{x}\right) dx =$

A. $1/4$

B. 1

C. 0

D. none

Answer: C

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16. Given that $\int_0^{\pi/2} \sin^4 x \cos^2 x dx = \frac{\pi}{32}$, then $\int_0^{\pi/2} \cos^4 x \sin^2 x dx$=

A. $\frac{\pi}{32}$

B. $\frac{3\pi}{32}$

C. $\frac{\pi}{2}$

D. none

Answer: A

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17. The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

A. 2

B. $3/4$

C. 0

D. None of these

Answer: C

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18. $\int_0^{\pi/2} \frac{dx}{\sqrt{\tan x} - \sqrt{\cot x}} =$

A. $\pi/2$

B. $\pi/4$

C. 0

D. none

Answer: C

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19. The value of $\int_0^{\pi} \frac{2^{\sin x}\cos x}{s^{[\sin x]}}.dx$ when [.] denotes the greatest integer function is equal to

A. 0

B. $\frac{\pi}{\log 2}$

C. $\frac{2\pi}{\log 2}$

D. none

Answer: A

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20. The value of the integral $\int_0^{\pi/2} \sin 2x \log \tan x dx$ equals

A. 0

B. $\frac{\pi}{8}\log 2$

C. $\frac{\pi}{4}\log 2$

D. $\frac{\pi}{6}\log 2$

Answer: A

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21. $\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x dx, (n \in I)$=

A. π

B. 0

C. 1

D. None of these

Answer: B

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22. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx =$

A. $\frac{\pi^2}{8}$

B. $\frac{\pi^2}{16}$

C. $\frac{\pi^2}{4}$

D. $\frac{\pi^2}{32}$

Answer: C

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23. Prove that : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{(\pi)^2}{16}$

A. $\frac{\pi^2}{8}$

B. $\frac{\pi^2}{16}$

C. $\frac{\pi^2}{32}$

D. $\frac{\pi^2}{4}$

Answer: B

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24. $\int_{\pi}^{5\pi/4} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx =$

A. $\pi/4$

B. $\pi/2$

C. $3\pi/4$

D. π

Answer: A

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25. Prove that : $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx = \frac{\pi^2}{2ab}$

A. $\frac{\pi^2}{2ab}$

B. $\frac{\pi^2}{2a}$

C. $\frac{\pi^2}{2b}$

D. $\frac{\pi^2}{ab}$

Answer: A

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26. $\int_0^{\frac{\pi}{2}} \log \sin x = -\left(\frac{\pi}{2}\right)\log 2$

$\int_0^{\pi} x \log \sin x dx =$

A. $\frac{1}{2}\pi^2 \log 2$

B. $-\frac{1}{2}\pi^2 \log 2$

C. $\frac{1}{2}\pi^2 \log\left(\frac{1}{2}\right)$

D. $-\frac{1}{2}\pi^2 \log\left(\frac{1}{2}\right)$

Answer: B::C

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27. If

$$I_1 = \int_0^{\pi} x f\{\sin^3 x + \cos^2 x\} dx \text{ and } I_2 = \pi \int_0^{\pi/2} f(\sin^3 x + \cos^2 x\} dx$$

then

A. $I_1 = I_2$

B. $I_1 + I_2 = 0$

C. $I_1 = 2I_2$

D. none

Answer: A

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28. $\int_0^{\pi} x f(\sin x) dx =$

A. $\pi \int_0^{\pi} f(\sin x) dx$

B. $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$

C. $\pi\int_0^{\pi/2} f(\cos x)dx$

D. $\pi\int_0^{\pi} f(\cos x)dx$

Answer: C

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29. Evaluate $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$

A. $\pi\log\frac{1}{2}$

B. $\pi\log 2$

C. $\frac{\pi}{2}\log 2$

D. none

Answer: B

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30. $\int_0^{\pi/2} [2\log\sin x - \log\sin 2x]dx =$

A. $\pi\log 2$

B. $-\pi\log 2$

C. $\frac{\pi}{2}\log\frac{1}{2}$

D. $-\frac{\pi}{2}\log\frac{1}{2}$

Answer: C

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31. If $\int_0^{\pi} xf(\sin x)dx = k\int_0^{\pi/2} f(\sin x)dx$ then the value of k is

A. 2

B. 1

C. π

D. 0

Answer: C

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32. For $n > 0 \int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx = \ldots.$

A. π

B. π^2

C. $\pi / 2$

D. 2π

Answer: B

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33. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$ is equal to

A. $\pi / 2$

B. $\sqrt{2}\log\left(\sqrt{2}+1\right)$

C. $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

D. None of these

Answer: C

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34. The value of the integral $\int_0^1 x(1-x)^n dx$ is

A. $\frac{1}{n+1}+\frac{1}{n+2}$

B. $\frac{1}{(n+1)(n+2)}$

C. $\frac{1}{n+2}-\frac{1}{n+1}$

D. $2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

Answer: B

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35. If $\int_0^1 x^m(1-x)^n dx = R\int_0^1 x^n(1-x)^m dx$, then

A. R=1

B. $R = -1$

C. $R = 1/2$

D. None of these

Answer: A

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36. If $I = \int_0^1 \frac{e^t}{1+t} dt$, then $p = \int_0^1 e^t \log(1+t) dt$=

A. I

B. 2I

C. $e\log 2 - I$

D. none

Answer: C

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37. $\int_0^{\pi/2n} \frac{dx}{1+\cot^n nx}$ is equal to

A. 0

B. $\frac{\pi}{4n}$

C. $\frac{\pi}{2n}$

D. $\frac{\pi}{2}$

Answer: B

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38. The value of the integral $\int_0^1 \cot^{-1}\left(1-x+x^2\right)dx$, is

A. $\pi - \log 2$

B. $\frac{\pi}{2} - \log 2$

C. $\pi + \log 2$

D. $\frac{\pi}{2} + \log 2$

Answer: B

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39. $\int_0^1 \tan^{-1}\left(1 - x + x^2\right) dx =$

A. $\log 2$

B. $\log \frac{1}{2}$

C. $\pi \log 2$

D. $\frac{\pi}{2} \log \frac{1}{2}$

Answer: A

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40. $\int_0^{\pi/2} \frac{\cos x dx}{1+\cos x+\sin x} =$

A. $\frac{\pi}{4}+\log 2$

B. $\frac{\pi}{4}+\frac{1}{2}\log 2$

C. $\pi-\frac{1}{2}\log 2$

D. none

Answer: C

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41. Let $I=\int_0^{\pi/2} \frac{dx}{1+\sin x}$, then $\int_0^{\pi} \frac{x^2\cos x}{(1+\sin x)^2}dx$

A. $-\pi^2$

B. $\pi^2-2\pi I$

C. $2\pi I$

D. $2\pi I-\pi^2$

Answer: D

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42. $a_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx$, then $a_2 - a_1, a_3 - a_2, a_4 - a_3$ are in

A. A.P

B. G.P

C. H.P

D. none

Answer: C

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43. If f(x) and g(x) are continuous functions satisfying $f(x) = f(a - x)$ and $g(x) + g(a - x) = 2$, then $\int_0^a f(x)g(x)dx$ is equal to

A. $\int_0^a g(x)dx$

B. $\int_0^a f(x)dx$

C. 0

D. None of these

Answer: B

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44. If f (x) is monotonic differentiable function on [a,b] then $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f^{-1}(x)dx$=

A. $bf(a) - af(b)$

B. $bf(b) - af(a)$

C. $f(a) + f(b)$

D. cannot be found

Answer: B

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45. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}I$

B. 2I

C. 3I

D. 6I

Answer: C

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46. If $\lim_{t \to a} \frac{\int_a^t f(t)dt - \frac{t-a}{2}(f(t) - f(a))}{(t-a)^3} = 0$, then maximum degree of $f(x)$ is:

A. 4

B. 3

C. 2

D. 1

Answer: D

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47. If $f(y) = e^y, g(y) = y, y > 0$ and $F(t) = \int_0^t f(t-y)g(y)dy,$ then F(t)=

A. $1 - e^{-t}(1+t)$

B. $e^t - (1+t)$

C. te^t

D. te^{-t}

Answer: B

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Problemset 2 Fill In The Blanks

1. If $I_n = \int_1^e \log^n x dx$ and $I_n = a + bI_{n-1}$ then a= .... and b= .....

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Problem Set 3 Multiple Choice Questions

1. The value of $\int_{-\pi/2}^{\pi/2} \log\left\{\frac{2-\sin\theta}{2+\sin\theta}\right\} d\theta$

A. 0

B. 1

C. 2

D. None of these

Answer: A

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2. The integral $\int_{-1/2}^{1/2}\left([x]+\ln\left(\frac{1+x}{1-x}\right)\right)dx$ equals

A. $-\frac{1}{2}$

B. 0

C. 1

D. $2\ln\left(\frac{1}{2}\right)$

Answer: A

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3. If $\int_{-\pi/3}^{\pi/3}\left[\frac{a}{3}|\tan x|+\frac{b\tan x}{1+\sec x}+c\right]dx=0$, where a, b, c are constants, then c=

A. $-\frac{a}{\pi}\log 2$

B. $\frac{2a}{\pi}\log 2$

C. $a\log 2$

D. $\frac{a}{\pi}\log 2$

Answer: A

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4. $f: R \to R, g: R \to R$ are continuous functions. The value of integral $\int_{-\pi/2}^{\pi/2}[f(x)+f(-x)][g(x)-g(-x)]dx$ is

A. π

B. 1

C. -1

D. 0

Answer: D

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5. $\int_{-1}^{1} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$

A. 2

B. 1

C. -1

D. 0

Answer: A

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6. $\int_{-1}^{1} \frac{x^2 \sin^{-1}[x]}{\sqrt{(1-x^2)}} dx =$

A. 1

B. 2

C. 0

D. none

Answer: C

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7. $\int_{-\pi/2}^{\pi/2} \sin(|x|)dx=$

A. 0

B. 1

C. 2

D. none

Answer: C

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8. $\int_{-\pi}^{\pi} \left(x^3 + x\cos x + \tan^5 x + 2\right) =$

A. 4π

B. 2π

C. π

D. none

Answer: A

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9. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2} dx$ is

A. π

B. π^2

C. 0

D. none

Answer: B

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10. $\int_{-3\pi/2}^{-\pi/2} \left\{(x+\pi)^3 + \cos^2(x+3\pi)\right\} dx$, is

A. $\frac{\pi^4}{32} = \frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4} - 1$

D. $\frac{\pi^4}{32}$

Answer: B

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11. Evaluate: $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x - \pi} dx$

A. $\frac{8}{\pi^2}$

B. $\frac{2}{\pi}$

C. $\frac{1}{\pi^2}$

D. none

Answer: A

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12. Evaluate the following definite integral:

$$\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7 + 3x^6 - 10x^5 - 7x^3 - 12x^2 + x + 1}{x^2 + 2} dx$$

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13. The value of $\int_{-2}^{2} (ax^3 + bx + c) dx$ depends on which following ?

A. b

B. c

C. a

D. a& b

Answer: B

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14. If $f(x) = ax^2 + bx + c$ such that f(0)=2 $f'(0) = -3, f''(0) = 4,$ then $\int_{-1}^{1} f(x)dx$=

A. -3

B. $16/3$

C. 0

D. none

Answer: B

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15. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x(\sin x + \cos x)dx=$

A. $2/15$

B. $4/15$

C. $6/15$

D. $8/15$

Answer: B

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16. The value of the integral $\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right)dx$

A. 0

B. $1/2$

C. $-1/2$

D. None of these

Answer: A

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17. The integral value of $\int_{-2}^{0}\left[x^3+3x^2+3x+3+(x+1)\cos(x+1)\right]dx$ is

A. 2

B. 4

C. 0

D. 8

Answer: B

Watch Video Solution

18. The value of the integral $\int_{-\pi/4}^{\pi/4} \frac{1}{\sin^4 x} dx$ is

A. $-8/3$

B. $3/2$

C. $8/3$

D. None of these

Answer: A

Watch Video Solution

19. The value of the integral

$$\int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1} \right)^2 + \left(\frac{x-1}{x+1} \right)^2 - 2 \right\} dx \text{ is}$$

A. $\log\left(\frac{4}{3}\right)$

B. $4\log\left(\frac{3}{4}\right)$

C. $4\log\left(\frac{4}{3}\right)$

D. $\log\left(\frac{3}{4}\right)$

Answer: C

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20. The value of the integral $\int_{-1}^{1} \log\left(x + \sqrt{x^2+1}\right) dx$ is

A. 0

B. log 2

C. log (1//2)

D. None of these

Answer: A

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21. The value of $\int_{-\pi/2}^{\pi/2} \sin\left\{\log\left(x+\sqrt{x^2+1}\right\}dx$ is

A. 1

B. -1

C. 0

D. None of these

Answer: C

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22. $\int_{\log 1/2}^{\log 2} \sin\left\{\frac{e^x-1}{e^x+1}dx\right.$=

A. $\cos\frac{1}{3}$

B. $\sin\frac{1}{2}$

C. 2cos 2

D. 0

Answer: C

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23. The value of $\int_{-1/2}^{1/2} \left| x \cos\left(\frac{\pi x}{2}\right) \right| dx$ is

A. $\frac{1}{\pi}\left(\pi\sqrt{2} + 4\sqrt{2} - 8\right)$

B. $\frac{1}{\pi^2}\left[\pi\sqrt{2} + 4\sqrt{2} - 8\right]$

C. $\frac{1}{\pi^2}\left(\pi\sqrt{2} + 4\sqrt{2} + 8\right)$

D. none

Answer: B

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24. The function $F(x) = \int_0^x \log\left(t + \sqrt{1 + t^2}\right) dt$ is

A. an even function

B. an odd function

C. a periodic function

D. None of these

Answer: A

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25. The function $F(x) = \int_0^{\pi} \log \frac{(1-x)}{(1+x)} dx$ is a function which is

A. even

B. odd

C. periodic

D. None of these

Answer: A

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26. The antiderivative of every odd function is an

A. even function

B. odd function

C. neither even nor odd

D. none

Answer: A

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27. If $n \in N$, then $\int_{-n}^{n} (-1)^{[x]} dx$ equals

A. 2n

B. n

C. n^2

D. 0

Answer: D

Watch Video Solution

28. $\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx=$

A. 0

B. 1

C. -1

D. None of these

Answer: A

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29. The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x dx$ is

A. 0

B. $\pi - \frac{\pi^3}{3}$

C. $2\pi - \pi^3$

D. $\frac{\pi}{2} - 2\pi^3$

Answer: A

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30. $\int_{-1}^{1} \frac{\sin x - x^2}{3 - |x|} =$

A. 0

B. $2\int_{1}^{0} \frac{\sin x}{3 - |x|} dx$

C. $2\int_{0}^{1} \frac{-x^2}{3 - |x|} dx$

D. $2\int_{0}^{1} \frac{\sin x - x^2}{3 - |x|} - dx$

Answer: C

Watch Video Solution

31. If $f(x)+f(Y)=f(x+y)$ and $\int_0^3 f(x)dx=\lambda$, then $\int_{-3}^3 f(x)dx=$

A. $-2k$

B. 2k

C. 0

D. k/2

Answer: C

Watch Video Solution

32. $I=\int_{-\pi/3}^{\pi/3}\frac{x\sin x}{\cos^2 x}dx$ is equal to

A. $\left(\frac{\pi}{3}-\log\tan\frac{3\pi}{2}\right)$

B. $2\left(\frac{2\pi}{3}-\log\tan\frac{5\pi}{12}\right)$

C. $3\left(\frac{\pi}{2}-\log\sin\frac{5\pi}{12}\right)$

D. None of these

Answer: B

Watch Video Solution

33. The value of the integral $\int_{-1}^{1} \sin^{11} x \quad \mathrm{dx}$ is

A. $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3}$

B. $\frac{10}{11} \cdot \frac{8}{9} \cdot \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot \frac{\pi}{2}$

C. 1

D. 0

Answer: D

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34. The value of $\int_{-1}^{1} \sin^3 x \cos^2 x dx$ is

A. $\frac{4}{15}$

B. $\frac{2}{15}$

C. 0

D. none

Answer: C

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35. If f is an odd function, then $I = \int_{-a}^{a} \frac{f(\sin\theta)}{f(\cos\theta) + f(\sin^2\theta)} =$

A. 0

B. $\pi/2$

C. 2

D. None of these

Answer: A

Watch Video Solution

36. If $f(x) = \begin{cases} e^{\cos x} \sin x & \text{for } |x| \leq 2 \\ 2 & \text{otherwise} \end{cases}$ then $\int_{-2}^{3} f(x)dx$=

A. 0

B. 1

C. 2

D. 3

Answer: C

Watch Video Solution

37. $\int_{-1}^{1} \frac{x^2 + \sin x}{1 + x^2} dx =$

A. 2

B. 0

C. $2 - \frac{\pi}{2}$

D. $\frac{\pi}{2}$

Answer: C

Watch Video Solution

38. The value of $\int_{-a}^{a}\left(\cos^{-1}x-\sin^{-1}\sqrt{1-x^2}\right)dx$ is $(a>0)$ there $\int_{0}^{a}\cos^{-1}xdx=A$ is

A. $\pi a-A$

B. $\pi a+A$

C. $\pi a-2A$

D. $\pi a+2A$

Answer: C

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Problemset 3 True And False

1.

$$\int_0^{\pi/2} f(\sin 2x)\sin x\,dx = \int_0^{\pi/2} f(\sin 2x)\cos x\,dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x$$

Watch Video Solution

Problem Set 3 Fill In The Blanks

1. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & 1 & 2 \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2} f(x)dx$ is equal to

A. 0

B. $\pi/2$

C. π

D. none

Answer:

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Problem Set 4 Multiple Choice Questions

1. The value of integral $\int_{-\pi}^{\pi}(\cos ax - \sin bx)^2 dx$, where (a and b are integers), is

A. $-\pi$

B. 0

C. π

D. 2π

Answer: D

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2. For any integer n, the integral $I = \int_0^{\pi} \cos^4 x \cos^5(2n+1)x dx$ has the value

A. π

B. 1

C. 0

D. None of these

Answer: C

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3. The value of $\int_0^{2\pi} \cos^{99} x dx$ is

A. 1

B. -1

C. 99

D. 0

Answer: D

Watch Video Solution

4. $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx=$

A. $\frac{b-a}{2}$

B. $\frac{a-b}{2}$

C. a/2

D. b/2

Answer: A

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5. $\int_1^5 \frac{\sqrt{x}}{\sqrt{(6-x)}+\sqrt{x}}dx=$

A. 1

B. $3/2$

C. 2

D. $5/2$

Answer: C

Watch Video Solution

6. $\int_3^6 \frac{\sqrt{x}}{\sqrt{(9-x)}+\sqrt{x}}dx=$

A. $3/2$

B. 2

C. 1

D. $1/2$

Answer: A

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7. If $f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

A. $\frac{a+b}{2}\int_a^b f(b-x)dx$

B. $\frac{a+b}{2}\int_a^b f(x)dx$

C. $\frac{b-a}{2}\int_a^b f(x)dx$

D. None of these

Answer: B

Watch Video Solution

8. If $\int_a^b \frac{x^n}{x^4+(16-x)^n}dx=6$, then

A. $a=4, b=12, n\in R$

B. $a=2, b=14, n\in R$

C. $a=-4, b=20, n\in R$

D. $a = 2, b = 8, n \in R$

Answer: B

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9. If $f(3 - x) = f(x)$, then $\int_1^2 x f(x) dx$ is equal to

A. $\frac{3}{2}\int_1^2 f(2 - x) dx$

B. $\frac{3}{2}\int_1^2 f(x) dx$

C. $\frac{1}{2}\int_1^2 f(x) dx$

D. None of these

Answer: B

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10. For any $t \in R$ and f be a continuous function

Let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} xf(x(2-x))dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f(x(2-x))dx$

then $\frac{I_1}{I_2}$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

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11. If f(x) is an integrable function in $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2 x f(2\sin 2x)dx$ and $I_2 = \int_{\pi/6}^{\pi/3} \cos ec^2 x f(2s$

, then:

A. $I_1 = 2I_2$

B. $2I_1 = I_2$

C. $I_1 = I_2$

D. none

Answer: C

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12. Let f be a positive function. Let $I_1 \int_{1-k}^{k} x.\, f\{x(1-x)\}dx, I_2 = \int_{1-k}^{k} f\{x(1-x)\}dx$ where $2k-1>0$, then I_1 / I_2 is

A. 2

B. k

C. $1/2$

D. 1

Answer: C

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13. If $f(x) = \frac{e^x}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x$

, then the value of $\frac{I_2}{I_1}$ is

A. 2

B. -3

C. -1

D. 1

Answer: A

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14. The value of $\int_{1/n}^{(an-1)/n} \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$ is equal to

A. $\frac{a}{2}$

B. $\frac{1}{2n}(na+2)$

C. $\frac{na-2}{2n}$

D. None of these

Answer: C

Watch Video Solution

15. If [x] stands for the greatest integer function, the value of $\int_4^{10} \frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$ is

A. 0

B. 1

C. 3

D. None of these

Answer: C

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16. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to

A. 2

B. -2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: A

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17. $\int_{-\pi/2}^{\pi/2} \frac{\cos x dx}{1+e^x} =$

A. 1

B. 2

C. log 2

D. none

Answer: A

Watch Video Solution

18. $\int_0^{\pi} \frac{dx}{1+2^{\tan x}} =$

A. 0

B. $\pi/4$

C. $\pi/2$

D. π

Answer: C

Watch Video Solution

19. $\int_{-\pi/2}^{\pi/2} \frac{\pi^{\sin x}}{1+\pi^{\sin x}} dx =$

A. 0

B. $\pi/4$

C. $\pi/2$

D. π

Answer: C

Watch Video Solution

20. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$ is equal to

A. 0

B. 1

C. $-\pi/2$

D. $\pi/2$

Answer: D

Watch Video Solution

21. $\int_0^{\pi} \frac{dx}{1+4^{\cos x}} =$

A. 0

B. $\pi/2$

C. π

D. 2π

Answer: B

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22. The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} \mathrm{dx}$, where $a > 0$, is -

A. π

B. $a\pi$

C. $\frac{\pi}{2}$

D. 2π

Answer: C

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Problem Set 4 True And False

1. $\int_0^{\pi} x f(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = \pi\int_0^{\pi/2} f(\sin x)dx$

Watch Video Solution

2. The value of the integral $\int_0^{\infty} \frac{x\log x}{(1+x^2)^2}dxis$ 0 (b) log 7 (c) 5 log 13 (d) none of these

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3. $\int_a^b \frac{\sin(x-a)-\cos(x-a)}{\sin(b-x)-\cos(b-x)}dx = \int_a^b \frac{\sin(b-x)-\cos(b-x)}{\sin(x-a)-\cos(x-a)}dx$

True or False?

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Problem Set 4 Fill In The Blanks

1. $\int_2^4 x\sqrt{6-x}dx$= ....

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Problem Set 5 Multiple Choice Questions

1. If $F(x) = \frac{1}{x^2}\int_4^x \left[4t^2 - 2F'(t)\right]dt$, then F'(4) equals

A. 32

B. $\frac{32}{3}$

C. $\frac{32}{9}$

D. None of these

Answer: C

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2. The value of $Lt_{x \to 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$ is

A. 1

B. 2

C. 3

D. none

Answer: A

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3. $\lim_{x \to 0} \frac{\int_0^{x^2} (\sin \sqrt{t} dt)}{x^3}$ is equal to

A. $1/3$

B. 1

C. $2/3$

D. None of these

Answer: C

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4. $Lt_{x \to 0} \int_0^x \frac{(\sin^2 4t + t^2) dt}{x^3} =$

A. 0

B. $\frac{4}{3}$

C. $\frac{17}{3}$

D. ∞

Answer: C

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5. If $f(x) = \int_{x^2}^{x^4} \sin \sqrt{t} dt$, then f'(x) equals

A. $\sin x^2 - \sin x$

B. $4x^3 \sin x^2 - 2x \sin x$

C. $x^4 \sin x^2 - x \sin x$

D. None of these

Answer: B

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6. Let $f(x) = \int_1^x \sqrt{2 - t^2} dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are

A. ± 1

B. $\pm \frac{1}{\sqrt{2}}$

C. $\pm \frac{1}{2}$

D. 0 and 1

Answer: A

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7. $\lim_{x \to 0} \frac{1}{x}\left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt\right]$ is equal to

A. $e^{\sin^2 y}$

B. $\sin 2y e^{\sin^2 y}$

C. 0

D. None of these

Answer: A

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8. The equation of tangent to the curve $y = \int_{x^2}^{x^3} \frac{dt}{\sqrt{1+t^2}}$ at x=1 is equal to

A. $x\sqrt{3} - y + \left(\sqrt{3}+1\right) = 0$

B. $x\sqrt{3} - y + 1 = 0$

C. $x - y\sqrt{2} - 1 = 0$

D. None of these

Answer: C

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9. If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{\sqrt{1+9y^2}}$

B. $\sqrt{1+9y^2}$

C. $\left(1+9y^2\right)$

D. $\frac{1}{1+9y^2}$

Answer: B

Watch Video Solution

10. If $f(x)=\int_x^{x^2}\frac{dt}{1+t^3}$, then f'(2)=

A. $-\frac{56}{585}$

B. $-\frac{29}{585}$

C. $\frac{101}{585}$

D. none

Answer: B

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11. If $f(x) = \int_{1/x^2}^{2} \cos\sqrt{t}dt$ then f'(1) is equal to

A. cos 1

B. 2cos 1

C. 4 cos 1

D. None of these

Answer: B

Watch Video Solution

12. If $\int_{\sin x}^{1} t^2 f(t)dt = 1 - \sin x, x \in \left(0, \frac{\pi}{2}\right)$ then $f\left(\frac{1}{\sqrt{3}}\right)$ equal to

A. 3

B. $1/3$

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: A

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13. If $f(x) = \int_{x^2}^{x^3} \frac{dt}{\log t}, x > 0$ then

A. $f'(x) = -\frac{1}{6\log x}$

B. f is an increasing function

C. f has minimum at x=1

D. f is an increasing function on $[1, \infty]$

Answer: B::D

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14. Let $f:(0,\infty) \in R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1+x)$ then f (4) equals

A. $\frac{5}{4}$

B. 7

C. 4

D. 2

Answer: C

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15. If $\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$, then f(4/25)=

A. $-2/5$

B. $-5/2$

C. 1

D. $5/2$

Answer: A

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16. The integral $\int_0^2 \frac{|x+2|}{x+2} dx$ is equal to

A. 1

B. 3

C. 0

D. -1

Answer: B

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17. $\int_{-3}^{3} \frac{x-4}{(|x-4|)} dx =$

A. 0

B. 6

C. -6

D. none

Answer: C

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18. The value of

$$\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}dt, \text{ is}$$

A. $\pi/2$

B. 1

C. $\pi/4$

D. None of these

Answer: C

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19. If $\int_{\pi/3}^{x} \sqrt{3-2\sin^2 u}du + \int_{0}^{y} \cos t dt = 0$, then $\frac{dy}{dx}$ is equal to

A. $\frac{\sqrt{(4-3\sin^2 x)}}{\cos y}$

B. $\frac{-\sqrt{(3-2\sin^2 x)}}{\cos y}$

C. $\sqrt{(3-2\sin^2 x)} + \cos y$

D. None of these

Answer: B

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20. The points of extremum of the function $F(x) = \int_{1}^{x} e^{-t^2}(1-t^2)dt$ are

A. x=0

B. x=1

C. x=1/2

D. $x = -1$

Answer: B::D

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21. The points of extemum of $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$ are

A. x=1

B. $x = -1$

C. x=0

D. $x = -2$

Answer: A::B::C::D

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22. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$, then the valeu of f(1) is

A. $\frac{1}{2}$

B. 0

C. 1

D. $-\frac{1}{2}$

Answer: A

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23. If $\int_0^t \frac{bx\cos 4x - a\sin 4x}{x^2}dx = \frac{a\sin 4t}{t} - 1$, where $0 < t < \frac{\pi}{4}$, then the value of a, b are equal to

A. $\frac{1}{4}, 1$

B. $-1, 4$

C. 2, 2

D. 2, 4

Answer: A

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24. $Lt_{x \to 0} \frac{\int_0^{x^2} \left(\tan^{-1} t\right)^2 dt}{\int_0^{x^2} \left(\sin \sqrt{t} dt\right)} =$

A. -1

B. $-1/2$

C. 0

D. 1

Answer: C

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25. The value of the integral $\int_a^b \frac{|x|}{x} dx, a < b$ is

A. $b - a$

B. $a - b$

C. $b + a$

D. $|b| - |a|$

Answer: D

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26. If $a < 0 < b$, then $\int_a^b x|x|dx =$

A. $\frac{1}{2}(a^2 + b^2)$

B. $\frac{1}{3}(b^2 - a^2)$

C. $\frac{1}{3}(a^3 + b^3)$

D. None

Answer: C

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27. $\int_{-1}^{1} |1-x| dx$ is equal to

A. -2

B. 0

C. 2

D. 4

Answer: C

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28. $\int_{-2}^{2} |x(x-1)| dx =$

A. $\frac{16}{3}$

B. $\frac{17}{3}$

C. $\frac{11}{3}$

D. $\frac{13}{3}$

Answer: B

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29. $\int_0^4 |x-1|dx=$

A. 1

B. 2

C. 3

D. None of these

Answer: D

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30. The vaue of $\int_{-1}^{2} \frac{|x|}{x} dx$ is

A. 2

B. 1

C. 3

D. none

Answer: B

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31. $\int_0^2 |(1-x)|dx=$

A. 1

B. 2

C. 4

D. none

Answer: A

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32. $\int_0^4 \{|x-1|+|x-3|dx\}$=

A. 5

B. 8

C. 10

D. none

Answer: C

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33. The value of $\int_{-3}^{3} |x|dx$ is

A. 3

B. 6

C. 9

D. none

Answer: C

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34. What is the value of $\int_{-1}^{1} x|x|dx$

A. 1

B. 2

C. 3

D. none

Answer: D

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35. $\int_{-1}^{1} \{|x-1|+|x|\}dx$ is equal to

A. 1

B. 2

C. 3

D. none

Answer: C

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36. The value of $\int_{-2}^{2} \left|\left(1-x^2\right)\right| dx$ is

A. 2

B. 4

C. 0

D. none

Answer: B

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37. The value of $\int_{-2}^{3} |1 - x^2| dx$ is

A. $\frac{1}{3}$

B. $\frac{14}{3}$

C. $\frac{7}{3}$

D. $\frac{28}{3}$

Answer: D

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38. $\int_{0}^{1} |\sin 2\pi x| dx$ id equal to

A. 0

B. $-\frac{1}{\pi}$

C. $\frac{1}{\pi}$

D. $\frac{2}{\pi}$

Answer: D

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39. The value of the integral $\int_0^{\pi/2} |\sin x - \cos x| dx$, is

A. 0

B. $2\left(\sqrt{2}-1\right)$

C. $2\sqrt{2}$

D. $2\left(\sqrt{2}+1\right)$

Answer: B

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40. Evaluate $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx.$

A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. 0

Answer: C

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41. $\int_{-\pi/2}^{\pi/2} \sqrt{\frac{1-\cos 2x}{2}} dx =$

A. 0

B. 2

C. $1/2$

D. none

Answer: B

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42. $\int_0^{\pi/3} |\tan x - 1| dx =$

A. $\frac{\pi}{2} + \frac{1}{2}\log 2$

B. $\frac{\pi + \log 8}{6}$

C. $\log 2 - \frac{\pi}{3}$

D. None of these

Answer: D

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43. The value of $\int_{\pi}^{2\pi} [2\sin x] dx,$ where [] repreents the greatest integer function, is

A. $-\frac{5\pi}{3}$

B. $-\pi$

C. $\frac{5\pi}{3}$

D. -2π

Answer: A

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44. $\int_0^{\pi}[2\sin x]dx=$

A. $\frac{2\pi}{3}$

B. $-\frac{5\pi}{3}$

C. $-\pi$

D. -2π

Answer: A

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45. The value of $\int_0^{\pi/2} \frac{\cos 3x + 1}{2\cos x - 1} dx$ is

A. 2

B. 1

C. $1/2$

D. 0

Answer: B

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46. $\int_a^b [|x-a| + |x-b|] dx =$

A. $\frac{(b-a)^2}{2}$

B. $\frac{b^2 - a^2}{2}$

C. $\frac{a^3 - b^3}{2}$

D. $(b-a)^2$

Answer: D

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47. If $f(x)=|2^x-1|+|x-1|$, then $\int_{-2}^{2} f(x)dx$=

A. $5-\frac{9}{4}\log 2$

B. $5+\frac{9}{4\log 2}$

C. $-5-\frac{9}{4}\log 2$

D. none

Answer: B

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48. Evaluate: $\int_{-1}^{1}\{x+|x|\}dx$

A. 1

B. $1/2$

C. 0

D. 2

Answer: A

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49. The value of $\int_1^2 [2x^2 - 3]\,dx$ is ([.] denotes the greatest integer function)

A. 4

B. $\frac{\sqrt{3}}{2} + \sqrt{2} + \sqrt{3} - 10$

C. $9 - \left\{\sqrt{\frac{3}{2}} + \sqrt{2} + \sqrt{\frac{5}{2}} + \sqrt{3} + \sqrt{\frac{7}{2}}\right\}$

D. $15 - \sqrt{\frac{3}{2}} - \sqrt{2} - \sqrt{\frac{5}{2}} - \sqrt{3} - \sqrt{\frac{7}{2}} - \sqrt{\frac{9}{2}}$

Answer: C

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50. The value of the integral $\int_{-1}^{1}(x-[2x])$dx,is

A. 0

B. 1

C. 2

D. 4

Answer: B

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51. $\int_{0}^{3/2}\left[x^{2}\right]dx=$

A. $2+\sqrt{2}$

B. $2-\sqrt{2}$

C. $3/2$

D. 3

Answer: B

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52. Evaluate :

$$\int_0^2 [x^2]\,dx$$

A. $2-\sqrt{2}$

B. $2+\sqrt{2}$

C. $\sqrt{2}-1$

D. $-\sqrt{2}-\sqrt{3}+5$

Answer: D

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53. $\int_0^3 \left[x^3 - 3x^2 + 2x\right] dx =$

A. $\frac{9}{4}$

B. $\frac{7}{4}$

C. $\frac{3}{4}$

D. none

Answer: A

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54. The value of $\int_{-\pi/2}^{199\pi/2} \sqrt{(1+\cos 2x)}\, dx$ is

A. $50\sqrt{2}$

B. $100\sqrt{2}$

C. $150\sqrt{2}$

D. $200\sqrt{2}$

Answer: D

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55. The expression $\frac{\int_0^n [x]dx}{\int_0^n \{x\}dx}$ a where [x] and [x] are integrala and fractional parts of x and $n \in N$ is equal to

A. $\frac{1}{n-1}$

B. $\frac{1}{n}$

C. n

D. $n-1$

Answer: D

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56. $\int_0^{n^2} [\sqrt{x}]\,dx =$

A. $\frac{n}{6}(n+1)[4n+1]$

B. $\frac{n}{6}(n-1)(4n+1)$

C. $\frac{n}{6}(n-1)(4n-1)$

D. none

Answer: B

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57. The value of $\int_0^{[x]} \{x-[x]\}dx$ is

A. [x]

B. 2[x]

C. $\frac{1}{2[x]}$

D. $\frac{1}{2}[x]$

Answer: D

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58. $\int_3^6 2[x]dx$ is equal to

A. 12

B. 30

C. 18

D. 24

Answer: D

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59. If [.] denotes the greatest integer function, then $\int_0^\infty \left[2e^{-x}\right]dx =$

A. 0

B. e^2

C. $2/e$

D. log 2

Answer: D

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60. Evaluate $\int_1^{e^6} \left[\frac{\log x}{3}\right] dx$, where [.] denotes the greatest integer function.

A. 0

B. $e^6 - e^3$

C. $e^6 + e^3$

D. $e^3 - e^6$

Answer: B

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61. The value of the integral $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$ is

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. 3

D. 5

Answer: B

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62. $\int_{1/e}^{e} |\log x| dx =$

A. $2\left(1 - \frac{1}{e}\right)$

B. $2\left(\frac{1}{e} - 1\right)$

C. $2 - \frac{1}{e}$

D. $2+\frac{1}{e}$

Answer: A

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63. If [x] denotes the greatest integer function then $\int_{0.5}^{4.5}[x]dx+\int_{-1}^{1}|x|dx$ is equal to

A. 9

B. 8

C. 7

D. 6

Answer: A

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64. $\int_0^5 [x]dx$= ....., where [x] denotes the greatest integer function

A. 10

B. 8

C. 6

D. 4

Answer: A

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65. The value of the integral $\int_0^2 x[x]dx$ is where [x] is greatest integer function.

A. $7/2$

B. $3/2$

C. $5/2$

D. None of these

Answer: B

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66. $\int_{-1}^{3}\{|x-1|+[x]\}dx$ with usual notations is

A. 3

B. 4

C. 5

D. 6

Answer: D

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67. The value of $\sum_{n=1}^{1000} \int_{n-1}^{n} e^{x-[x]}\,dx$, where $[x]$ is the greatest integer function, is (A) $\frac{e^{1000}-1}{1000}$ (B) $\frac{e-1}{1000}$ (C) $\frac{e^{1000}-1}{e-1}$ (D) $1000(e-1)$

A. $\frac{e-1}{1000}$

B. $1000(e-1)$

C. $\frac{e^{1000}-1}{e-1}$

D. $\frac{e^{1000}-1}{1000}$

Answer: B

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68. $\int_{0}^{2\pi} e^{\cos x} \cos x(\sin x)dx=$

A. 1

B. π

C. 2π

D. none

Answer: C

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69. $\int_0^{\pi/3} [\sqrt{3}\tan x]\,dx$, where [.] denotes the greatest integer function is

A. $\frac{\pi}{2} - \tan^{-2}\frac{2}{\sqrt{3}}$

B. $\frac{5\pi}{6} - \tan^{-1}\frac{2}{\sqrt{3}}$

C. $\frac{5\pi}{6}$

D. none

Answer: A

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Problem Set 5 Fill In The Blanks

1. If $f(t) = \int_{t^2}^{t^3} \frac{1}{\log x} dx$, then f'(t)= ....

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2. Find the value of $\int_{-1}^{\frac{3}{2}} |x \sin \pi x| dx$

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3. The value of $\int_{-1/2}^{1/2} \left| x \cos \frac{1}{2} \pi x \right| dx$ is .....

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4. $f(x) = |2^x - 1| + |x - 1|$, then $\int_{-2}^{2} f(x) dx$= .....

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Problem Set 6 Multiple Choice Questions

1. $\lim_{n \to \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{2n} \right] =$

A. log 2

B. $\log \frac{1}{2}$

C. $\frac{1}{2}$

D. ∞

Answer: A

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2. $\lim_{n \to \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{6n} \right] =$

A. log 4

B. log 6

C. log 2

D. none

Answer: B

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3. $\lim_{n \to \infty} \left[\frac{\sqrt{n+1} + \sqrt{n+2} + ... + \sqrt{2n}}{n\sqrt{(n)}} \right]$

A. $\frac{2}{3}\left(2\sqrt{2} - 1\right)$

B. $\frac{1}{3}\left(2\sqrt{2} - 1\right)$

C. $3\left(2\sqrt{2} - 1\right)$

D. none

Answer: A

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4.

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{(n^2-1)}}+\frac{1}{\sqrt{(n^2-2^2)}}+\ldots.+\frac{1}{\sqrt{\left(\left[n^2-(n-1)^2\right]\right)}}\right]=$$

....

A. 0

B. $\frac{\pi}{2}$

C. π

D. none

Answer: B

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5. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\left(\frac{n+r}{n-r}\right)}$ is :

A. $\frac{\pi}{2}$

B. $\frac{\pi}{2}+1$

C. π

D. none

Answer: B

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6. The value of $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots+\frac{n}{n^2+(n-1)^2}\right]$ is :

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: D

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7. $\lim_{n \to \infty} \left(\frac{1}{1+n^3} + \frac{4}{8+n^3} + ... + \frac{r^2}{r^3+n^3} + ... + \frac{1}{2n} \right)$

A. $\frac{1}{2}\log 2$

B. $\frac{1}{3}\log 2$

C. $\log \frac{1}{2}$

D. none

Answer: B

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8. $\lim_{n \to \infty} \sum_{r=1}^{n} \left(\frac{r^3}{r^4+n^4} \right)$ equals to-

A. $\frac{1}{4}\log 2$

B. $\frac{1}{4}\log 2$

C. log 2

D. none

Answer: B

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9. $\lim_{n \to \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \ldots + \frac{1}{8n} \right]$ is equal to

A. $1/8$

B. $3/8$

C. $5/8$

D. none

Answer: B

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10. v20.1

A. e

B. 1/e

C. 2e

D. $-e$

Answer: B

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11. Evaluate: $\left(\lim\right)_{n\to\infty}\left(\frac{(n+1)(n+2)(n+n)^{\frac{1}{n}}}{n}\right.$

A. 2e

B. e

C. 2/e

D. 4/e

Answer: D

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12. The value of $\lim_{n \to \infty} \left[\frac{(2n)!}{n!n^n}\right]^{1/n}$ is equal to

A. 4e

B. e/4

C. 4/e

D. None of these

Answer: C

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13. $\lim_{n \to \infty} \left[\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots . + \frac{n}{1-n^2}\right]$ is equal to-

A. 0

B. 2

C. $1 + e^{-1}$

D. None of these

Answer: D

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14. $\lim_{n\to\infty} \frac{1}{n}\left[\tan\frac{\pi}{4n} + \tan\frac{2\pi}{4n} + \ldots + \tan\frac{n\pi}{4n}\right] =$

A. $\frac{1}{\pi}\log 2$

B. $\frac{2}{\pi}\log 2$

C. $\frac{4}{\pi}\log 2$

D. none

Answer: B

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15. The value of $\lim_{n\to\infty} \frac{1}{n}\left[\sec^2\frac{\pi}{4n} + \sec^2\frac{2\pi}{4n} + \ldots. + \sec^2\frac{n\pi}{4n}\right]$ is

A. $\frac{4}{\pi}$

B. $\frac{2}{\pi}$

C. $\frac{1}{\pi}$

D. none

Answer: A

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16. Evaluate: $(\lim)_{n\overrightarrow{\infty}}\left[\frac{1}{n^2}\frac{\sec^2 1}{n^2}+2/n^2\frac{\sec^2 4}{n^2}+\ +\frac{1}{n}\sec^2 1\right]$

A. tan 1

B. $\frac{1}{2}\tan 1$

C. $\frac{7}{2}\sec$

D. $\frac{1}{2}\cos ec1$

Answer: B

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17. $\lim_{n \to \infty} \frac{1^{99} + 2^{99} + \ldots + n^{99}}{n^{100}} =$

A. $\frac{99}{100}$

B. $\frac{1}{100}$

C. $\frac{1}{99}$

D. $\frac{1}{101}$

Answer: A

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18. $Lt_{n \to \infty} \frac{1 + 2^4 + 3^4 + \ldots + n^4}{n^5} - Lt_{n \to \infty} \frac{1 + 2^3 + 3^3 + \ldots + n^3}{n^5}$ is (A) $\frac{1}{5}$ (B) $\frac{1}{30}$ (C) 0 (D) $\frac{1}{4}$

A. $\frac{1}{30}$

B. 0

C. $\frac{1}{4}$

D. $\frac{1}{5}$

Answer: B

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19. $Lt_{n\to\infty}\frac{1+2^4+3^4+\ldots+n^4}{n^5}-Lt_{n\to\infty}\frac{1+2^3+3^3+\ldots+n^3}{n^5}$ is (A) $\frac{1}{5}$ (B) $\frac{1}{30}$ (C) 0 (D) $\frac{1}{4}$

A. $\frac{1}{4}$

B. $\frac{1}{5}$

C. $\frac{1}{20}$

D. 0

Answer: D

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20. $Lt_{n\to\infty}\dfrac{2^k+4^k+6^k+\ldots+(2n)^k}{n^{k+1}}, k\neq -1, =$

A. $\dfrac{2^k}{k+1}$

B. $\dfrac{1}{k+1}$

C. 2^k

D. none

Answer: B

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21. $\lim_{n\to\infty}\dfrac{1}{2}\sum_{r=+1}^{2n}\dfrac{r}{\sqrt{n^2+r^2}}$ equals

A. $1+\sqrt{5}$

B. $-1+\sqrt{5}$

C. $-1+\sqrt{2}$

D. $1+\sqrt{2}$

Answer: A

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22. $Lt_{n\to\infty}\Sigma_{r=1}^{n-1}\frac{\pi}{n}\sin\left(\frac{r\pi}{n}\right)=$

A. 1

B. 2

C. 3

D. 4

Answer: B

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23. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{r/n}$ is

A. e

B. $e-1$

C. $1-e$

D. $e+1$

Answer: B

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24.

$$Lt_{n\to\infty}\left[\frac{1}{n^2}\sin\left(\frac{1+n^2}{n^2}\right)+\frac{2}{n^2}\sin\left(\frac{4+n^2}{n^2}\right)+\frac{3}{n^2}\sin\left(\frac{9+n^2}{n^2}\right)+\ldots\frac{2}{n}\right.$$

=

A. $\cos 2 \sin 3$

B. $\sin 2 \cos 3$

C. $\sin 2 \sin 3$

D. $\cos 2 \cos 3$

Answer: C

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25. Given that

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\log(r+n)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right),$$

$$\lim_{x\to\infty}\frac{1}{x^k}\left[(n+1)^k(n+2)^k\ldots..(n+n)^k\right]^{1/n}, \text{ is}$$

A. $\frac{4\lambda}{e}$

B. $\left(\frac{4}{3}\right)^{1/\lambda}$

C. $\left(\frac{4}{e}\right)^{\lambda}$

D. $\left(\frac{e}{4}\right)^{\lambda}$

Answer: C

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26. Let $S_n=\sum_{k=1}^{n}\frac{n}{n^2+nk+k^2}$ and $T_n=\sum_{k=0}^{n-1}\frac{n}{n^2+nk+k^2}$ for $n=1,1,2,3\ldots,$ they

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: A::D

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Problem Set 6 Fill In The Blanks

1. The value of $\lim_{n \to \infty} \left[\left(1 + \frac{1}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right) \ldots \left(1 + \frac{n^2}{n^2}\right) \right]^{\frac{1}{n}}$

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Miscellaneous Questions Assertion Reason

1. Statement-1: The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$

Statement-2: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

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