MATHS

BOOKS - ML KHANNA

EXAMINATION PAPER -2013

Paper I Section 1 Only One Correct Answers

1. For $a > b > c > 0$ if the distance between (1,1) and the point of intersection of the lines ax + by +c=0 and bx + ay+c=0 is less than $2\sqrt{2}$ then

A. $a + b - c > 0$

B. $a - b + c < 0$

C. $a - b + c > 0$

D. $a + b - c < 0$

Answer: A

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2. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$

A. $4(\sqrt{2} - 1)$

B. $2\sqrt{2}(\sqrt{2} - 1)$

C. $2(\sqrt{2} + 1)$

D. $2\sqrt{2}(\sqrt{2} + 1)$

Answer: B

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3. The number of points in $(-\infty, \infty)$ for which $x^2 - x\sin x - \cos x = 0$, is

A. 6

B. 4

C. 2

D. 0

Answer: C

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4. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

A. $\frac{23}{25}$

B. $\frac{25}{23}$

C. $\frac{23}{24}$

D. $\frac{24}{23}$

Answer: B

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5. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at eact point (x,y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then, the equation of the curve is

A. $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

B. $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

C. $\sec\left(\frac{2y}{x}\right) = \log x + 2$

D. $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

Answer: A

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6. Let $f: \left[\frac{1}{2}, 1\right]\vec{R}$ (the set of all real numbers) be a positive, non-constant, and differentiable function such that $f'(x) < 2f9x\Big) and f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{\frac{1}{2}}^{1} f(x)dx$ lies in the interval $(2e-1, 2e)$ (b) $(3-1, 2e-1)$ $\left(\frac{e-1}{2}, e-1\right)$ (d) $\left(0, \frac{e-1}{2}\right)$

A. $(2e-1), 2e)$

B. $(e-1, 2e-1)$

C. $\left(\frac{e-1}{2}, e-1\right)$

D. $\left(0, \frac{e-1}{2}\right)$

Answer: D

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7. Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector.Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}$,$\overrightarrow{PQ}$and $\overrightarrow{PS}$ is 1) 5,2) 10,3$)154)$20`

A. 5

B. 20

C. 10

D. 30

Answer: C

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8. Perpendicular are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$. The feet of perpendiculars lie on the line.

A. $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$

B. $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

C. $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$

D. $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Answer: D

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9. Four person independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that he problem is solve correctly by at least one of them is

a. $\frac{235}{256}$

b. $\frac{21}{256}$

c. $\frac{3}{256}$

d. $\frac{253}{256}$

Answer - A

A. $\frac{235}{256}$

B. $\frac{21}{256}$

C. $\frac{3}{256}$

D. $\frac{253}{256}$

Answer: A

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10. Let complex numbers α and $\frac{1}{\overline{\alpha}}$ lies on circles $(x-x_0)^2+(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4x^2$, , respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$, then $|\alpha|$ is equal to:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{3}$

Answer: C

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Paper I Section 2 One Or More Option Correct

1. A line l passing through the origin is perpendicular to the lines $l_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, \infty<t<\infty, l_2:(3+s)\hat{i}+(3+2$

then the coordinates of the point on l_2 at a distance of $\sqrt{17}$ from the point of intersection of $l \& l_1$ is/are:

A. $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$

B. $(-1, -1, 0)$

C. $(1, 1, 1)$

D. $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

Answer: B::D

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2. Let $f(x) = x \sin \pi x, x > 0$ then for all natural number $n, f'(x)$ vanishes at

A. a unique ponit in the interval $\left(n, n + \frac{1}{2}\right)$

B. a unique point in the interval $(n + \frac{1}{2}, n + 1\Big)$

C. a unique point in the interval $(n, n + 1)$

D. two point in the interval $(n, n+1)$

Answer: B::C

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3. Let $S_n = \sum_{k=1}^{4n} (-1)\frac{k(k+1)}{2}k^2$. Then S_n can take value (s) 1056 b. 1088 c. 1120 d. 1332

A. 1056

B. 1088

C. 1120

D. 1332

Answer: A::D

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4. For 3×3 matrices M and N, which of the following statement(s) is/are not correct?

A. $N^T MN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric.

B. $MN - NM$ is skew symmetric for all symmetric matrices M and N

C. MN is symmetric for all symmetric matrices M and N

D. (adj. M) (adj. N)= adj. (MN) for all intertiable matrices M and N

Answer: C::D

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5. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. The lengths of the sides of the rectangular sheet are :

A. 24

B. 32

C. 45

D. 50

Answer: A::C

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Paper I Section 3 Integer Value Correct Type

1. Consider the set of eight vector $V = \left\{a\hat{i} + b\hat{j} + c\hat{k}; a, bc \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V is 2^p ways. Then p is______.

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2. Of the three independent event E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. If the probavvility p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). Then, $\frac{\text{probability of occurrence of } E_1}{\text{probability of occurrence of } E_3}$ is equal to

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3. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio $5:10:14$. Then, n is equal to :

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4. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20$ is equal to

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5. A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the point P and Q. Let the tangets to the ellipse at P and Q meet at the point R. If $\triangle(h) =$ area of the triangle PQR,

$$\triangle_1 = \max_{\frac{1}{2} \leq h \leq 1} \triangle(h) \text{ and } \triangle_2 = \max_{\frac{1}{2} \leq h \leq 1} \triangle(h)$$

Then $\frac{8}{\sqrt{5}} \triangle_1 - 8\triangle_2 =$

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Paper Ii Section 1 One Or More Option Correct Type

1. Let ω be a complex cube root of unity with $\omega \neq 1 and P = [p_{ij}]$ be a $n \times n$ matrix withe $p_{ij} = \omega^{i+j}$. Then $p^2 \neq O, whe \cap =$ a.57 b. 55 c. 58 d. 56

A. 50

B. 55

C. 56

D. 58

Answer: B::C::D

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2. The function f(x)=2|x|+|x+2|-||x+2|-2|x|| has a local minimum or a local maximum at x equal to:

A. -2

B. $-\frac{2}{3}$

C. 2

D. $\frac{2}{3}$

Answer: A::B

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3. Let $\omega = \frac{\sqrt{3+i}}{2}$ and $P = \{W^n : n1, 2, 3.........\}$ further $H_1 = \left\{z \in C : Re(z) > \frac{1}{2}\right\}$
and $H_2 = \left\{z \in C : Re(z) < -\frac{1}{2}\right\}$, where C is the set of all complex numbers. If $z \in P \cap H_2$ and 0 represents the origin then $\angle z_1 0 z_2 =$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C::D

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4. If $3^x = 4^{x-1}$, then x =

A. $\frac{2\log_3 2}{2\log_3 2 - 1}$

B. $\dfrac{2}{2-\log_2 3}$

C. $\dfrac{1}{1-\log_4 3}$

D. $\dfrac{2\log_2 3}{2\log_2 3-1}$

Answer: A::B::C

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5. Two lines $L_1: x=5, \dfrac{y}{3-\alpha}=\dfrac{z}{-2}$ and $L_2: x=\alpha, \dfrac{y}{-1}=\dfrac{z}{2-\alpha}$ are coplanar. Then, α can take value(s)

A. 1

B. 2

C. 3

D. 4

Answer: A::D

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6. In a triangle PQR, P is the largest angle and $\cos P = 1/3$. Further the incircle of the triangle touches the sides PQ. QR and PR at N, L and M, respectively, such that the length of PN, QL, and RM are consecutive even integers. Then possible length (s) of the side(s) of the triangle is (are)

A. 16

B. 18

C. 24

D. 22

Answer: B::D

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7. For $a \in R$ (the set of all real numbers), $a \neq -1)$,
$$(\lim)_{n \overrightarrow{\infty}} \left(\frac{1^a + 2^a + \quad + n^a}{(n+1)^{a-a}[(na+1) + (na+2) + \&(na+n)]} = \frac{1}{60.}\right.$$
Then $a =$ 5 (b) 7 (c) $\frac{-15}{2}$ (d) $\frac{-17}{2}$

A. 5

B. 7

C. $\frac{-15}{2}$

D. $-\frac{17}{2}$

Answer: B::D

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8. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are)

A. $x^2 + y^2 - 6x + 8y + 9 = 0$

B. $x^2 + y^2 - 6x + 7y + 9 = 0$

C. $x^2 + y^2 - 6x - 8y + 9 = 0$

D. $x^2 + y^2 - 6x - 7y + 9 = 0$

Answer: A::C

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Paper Ii Section 2 Paragraph Type

1. Let $f:[0,1] \rightarrow R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0)=f(1)=0$ and $satiies$ f\'\'(x)-2f\'(x)+f(x) ge e^x, x in [0,1]$Whichofthefollow \in gistruef$ or 0 lt x lt 1?(A)0 lt f(x) lt oo(B)-1/2 lt f(x) lt 1/2(C)-1/4 lt f(x) lt 1(D)-oo lt f(x) lt 0`

A. $0f(x)<\infty$

B. $-\frac{1}{2}<f(x)<\frac{1}{2}$

C. $-\frac{1}{4}<f(x)<1$

D. $-\infty<f(x)<0$

Answer: D

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2. Let $f:[0, 1] \rightarrow R$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0)=f(1)=0$ and $satiies$ f\'\'(x)-2f\'(x)+f(x) ge e^x, x in [0,1]$Ifthefunction$e^(-x)f(x) $as\sum esits \min i\mu m \in the \int erval$[0,1]$at$x=1/4 $, whichofthefollow \in gistrue? (A)$f\'(x) lt f(x), 1/4 lt x lt 3/4(B)f\'(x) gt f(x), $0<x<\frac{1}{4}$ (C) $f'(x)<f(x), 0<x<\frac{1}{4}$ (D) $f'(x)<f(x), \frac{3}{4}<x<1$

A. $f'(x)<f(x), \frac{1}{4}<x<\frac{3}{4}$

B. $f'(x)>f(x), 0<x<\frac{1}{4}$

C. $f'(x)<f(x), 0<x<\frac{1}{4}$

D. $f'(x)<f(x), \frac{3}{4}<x<1$

Answer: C

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3. 'Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0$''

Length of chord PQ is

A. $7a$

B. $5a$

C. $2a$

D. $3a$

Answer: B

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4. Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a, a > 0$.

If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

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5. 'Let $S = S_1 \cap S_2 \cap S_3$ where $S_1\{z \in C : |z| < 4\}, S_2 = \left\{z \in C : Im\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$ and $S_3 = \{z \in C : Re(z) > 0\}$"

Area of S=

A. $\frac{10\pi}{3}$

B. $\frac{20\pi}{3}$

C. $\frac{16\pi}{3}$

D. $\frac{32\pi}{3}$

Answer: B

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6. 'Let $S = S_1 \cap S_2 \cap S_3$ where $S_1\{z \in C : |z| < 4\}, S_2 = \left\{z \in C : Im\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$ and

$S_3 = \{z \in C : Re(z) > 0\}$"

$\frac{\min}{z \in C} |1 - 3i - z| =$

A. $\frac{2 - \sqrt{3}}{2}$

B. $\frac{2 + \sqrt{3}}{2}$

C. $\frac{3 - \sqrt{3}}{2}$

D. $\frac{3 + \sqrt{3}}{2}$

Answer: C

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7. A box B_1 contains 1 white ball, 3 red balls, and 2 black balls. An- other box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls, and 5 black balls.

If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same color is

A. $\frac{82}{648}$

B. $\frac{90}{648}$

C. $\frac{558}{648}$

D. $\frac{566}{648}$

Answer: A

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8. A box B_1 contains 1 white ball, 3 red balls, and 2 black balls. An- other box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls, and 5 black balls.

If 2 balls are drawn (without replecement) from a randomly selected box and one of the balls is white and the other ball is red the probability that these 2 balls are drawn from box B_2 is

A. $\frac{116}{181}$

B. $\frac{126}{181}$

C. $\frac{65}{181}$

D. $\frac{55}{181}$

Answer: D

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Paper Ii Section 3 Matching List Type

1. Match list-I and list-II and select the answer using the code given below the lists.

	List-I	List-II	
P	Volume of the parallelopiped determined by vectors **a**, **b** and **c** is 2. Then the volume of the parallelopiped determined by vector 2 (**a** × **b**), 3 (**b** × **c**) and (**c** × **a**) is	1	100
Q	Volume of the parallelopied determine by vectors **a**, **b** and **c** is 5. Then the volume of the parallelopiped determined by vector 3 (**a** + **b**), (**b** + **c**) and 2 (**c** + **a**) is	2	30
R	Area of a triangle with adjacent sides determined by vectors **a** and **b** is 20. Then the area of the triangle with adjacent sides determined by vectors (2**a** + 3**b**) and (**a** − **b**) is	3	24
S	Area of a parallelogram with adjacent sides determined by vectors **a** and **b** is 30. Then the area of a parallelogram with adjacent sides determined by vector (**a** + **b**) and **a** is	4	60

A. $\begin{array}{cccc} P & Q & R & S \\ 4 & 2 & 3 & 1 \end{array}$

B. $\begin{array}{cccc} P & Q & R & S \\ 2 & 3 & 1 & 4 \end{array}$

C. $\begin{array}{cccc} P & Q & R & S \\ 3 & 4 & 1 & 2 \end{array}$

D. $\begin{array}{cccc} P & Q & R & S \\ 1 & 4 & 3 & 2 \end{array}$

Answer: C

2. Consider the lines

$L_1: \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2: \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1: 7x + y + 2z = 3$, $P_2: 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicualr to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists.

	List I		List II
P.	a =	1.	13
Q.	b =	2.	-3
R.	c =	3.	1
S.	d =	4.	-2

A. $\begin{matrix} P & Q & R & S \\ 3 & 2 & 4 & 1 \end{matrix}$

B. $\begin{matrix} P & Q & R & S \\ 1 & 3 & 4 & 2 \end{matrix}$

C. $\begin{matrix} P & Q & R & S \\ 3 & 2 & 1 & 4 \end{matrix}$

D. $\begin{matrix} P & Q & R & S \\ 2 & 4 & 1 & 3 \end{matrix}$

Answer: A

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3. Match list-I and list-II and select the answer using the code given below the lists.

	List-I		List-II
P	$\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{1/2}$ takes value	1	$\frac{1}{2}\sqrt{\frac{5}{3}}$
Q	If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x-y}{2}$ is	2	$\sqrt{2}$
R	If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is	3	$\frac{1}{3}$
S	if $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}(x\sqrt{6})\right)$, $x \neq 0$, then possible value of x is	4	1

A. $\begin{matrix} P & Q & R & S \\ 4 & 3 & 1 & 2 \end{matrix}$

B. $\begin{matrix} P & Q & R & S \\ 4 & 3 & 2 & 1 \end{matrix}$

C. | P | Q | R | S |
|---|---|---|---|
| 3 | 4 | 2 | 1 |

D. | P | Q | R | S |
|---|---|---|---|
| 3 | 4 | 1 | 2 |

Answer: B

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4. line L:y=mx+3 meets y–axis at E(0, 3) and the are of the parabola $y^2 = 16x, 0 \leq y \leq 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List II and select the correct answer using the code given below the lists :

List I	**List II**
P. m =	**1**. $\frac{1}{2}$
Q. Maximum area of ΔEFG is	**2**. 4
R. y_0=	**3**. 2
S. y_1=	**4**. 1

Codes :

A. $\begin{matrix} P & Q & R & S \\ 4 & 1 & 2 & 3 \end{matrix}$

B. $\begin{matrix} P & Q & R & S \\ 3 & 4 & 1 & 2 \end{matrix}$

C. $\begin{matrix} P & Q & R & S \\ 1 & 3 & 2 & 4 \end{matrix}$

D. $\begin{matrix} P & Q & R & S \\ 1 & 3 & 4 & 2 \end{matrix}$

Answer: A

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