MATHS

BOOKS - ML KHANNA

INEQUALITIES

Problem Set 1 Multiple Choice Questions

1. If a,b,c are all +ive real numbers which one of the following holds good?

A. $(b+c)(c+a)(a+b) > 8abc$

or $(1-a)(1-b)(1-c) > 8abc$ if $a+b+c=1$

B. $(a+b+c)(bc+ca+ab) > 9abc$

or $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) > 9$

C. $\left(\frac{a}{e}+\frac{b}{f}+\frac{c}{g}\right)\left(\frac{e}{a}+\frac{f}{b}+\frac{g}{c}\right) > 9$

D. If $x_f > 0, (i = 1, 2, \dots\dots n)$,then

$$(x_1 + x_2 + \dots\dots\dots x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots\dots\dots\dots + \frac{1}{x_n}\right) \geq n^2$$

Answer: A::B::C::D

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2. If a,b,c are the sides of a triangle then which of the following hold good?

A. $a^2 + b^2 + c^2 > ab + bc + ca$

B. $\dfrac{a^2 + b^2 + c^2}{ab + bc + ca}$ lies between 1 and 2

C. $a^3 + b^3 + c^3 > 3abc$

D. $3(ab + bc + ca) \leq (a + b + c)^2 \leq 4(bc + ca + ab)$

Answer: A::B::C::D

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3. If $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca$ lies in the interval

A. $[1, 2]$

B. $\left[0, \frac{1}{2}\right)$

C. $\left[-\frac{1}{2}, 1\right]$

D. $[0, 1]$

Answer: C

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4. Which of the following hold good?

A. $b^2c^2 + c^2a^2 + a^2b^2 \geq abc(a + b + c)$

B. $\frac{bc}{a} + \frac{ca}{b} + \frac{ab}{c} \geq a + b + c$

C. $\frac{bc}{a^3} + \frac{ca}{b^3} + \frac{ab}{c^3} \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

D. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{1}{\sqrt{(bc)}} + \frac{1}{\sqrt{(ca)}} + \frac{1}{\sqrt{(ab)}}$

Answer: A::B::C::D

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5. Which of the following hold good?

A. $abcd > 81(s-a)(s-b)(s-c)(s-d)$where $3s = a+b+c+d$

B. $(s-a)(s-b)(s-c)(s-d) > 81$ abcd where

$s = a+b+c+d$

C. If $a+b+c=1,$ then

$$\frac{8}{27abc}, > \left\{\frac{1}{a}-1\right\}\left\{\frac{1}{b}=1\right\}\left\{\frac{1}{c}-1\right\} > 8$$

D. If $x+y+z=a$ then

$$\frac{8}{27}a^3 \geq (a-x)(a-y)(a-z) \geq 8xyz$$

Answer: A::B::C::D

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6. For $a > 0, b > 0, c > 0$, which of the following hold good?

A. $2(a^3 + b^3 + c^3) \geq bc(b + c) + ca(c + a) + ab(a + b)$

B. $\dfrac{a^3 + b^3 + c^3}{3} \geq \dfrac{a + b + c}{3} \cdot \dfrac{a^2 + b^2 + c^2}{3}$

C. $\dfrac{bc}{b + c} + \dfrac{ca}{c + a} + \dfrac{ab}{a + b} < \dfrac{1}{2}(a + b + c)$

D. $\dfrac{2}{b + c} + \dfrac{2}{c + a} + \dfrac{2}{a + b} < \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$

Answer: A::B::C::D

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7. Which of the following hold good?

If sum of any two of quantities x,y,z be together greater than the third then

A. $(x + y + z)^3 > 27(y + z - x)(z + x - y)(x + y - z)$

B. $xyz > (y + z - x)(z + x - y)(x + y - z)$

C. If n is a +ve intege then

$$\left(1+x^3\right)\left(1+y^3\right)\left(1+z^3\right) > (1+xyz)^3$$

D. $2\left(a^2+b^2+c^2\right)(a+b+c) > a^3+b^3+c^3+15abc$

Answer: A::B::C::D

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8. Which of the following hold good?

A. $a^4+b^4+c^4 > abc(a+b+c)$

B. $a^5+b^5+c^5+d^5 > abcd(a+b+c+d)$

C. $a^5+b^5+c^5 > abc(ab+bc+ca)$

D. $\dfrac{a^8+b^8+c^8}{a^3b^3c^3} > \dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}$

Answer: A::B::C::D

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9. Which of the following hold good? If n is a +ve integer then

A. $n^n > 1.3.5.\dots\dots\dots(2n-1)$

B. $2.4.6.\dots\dots 2n < (n+1)^n$

C. $(n!)^3 < n^n\left(\frac{n+1}{2}\right)^{2n}$

D. $[1^r + 2^r + 3^r + \dots\dots + n^r]^n > n^n.(n!)]^r$

Answer: A::B::C::D

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10. Which of the following hold good?

A. Show that unless $p = q = r$ or x=1

$$px^{q-r} + qx^{r-p} + rx^{p-q} > p + q + r$$

B. $\left[\frac{x^2+y^2+z^2}{x+y+z}\right]^{x+y+z} > x^x y^y z^z > \left[\frac{x+y+z}{3}\right]^{x+y+z}$

C. $\left[\frac{a^2+b^2}{(a+b)^{a+b}} > a^a b^b\right.$

D. $a^a b^b > \left(\frac{a+b}{2}\right)^{a+b} > a^b . b^a$

Answer: A::B::C::D

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11. If A,B,C be the angles of a triangles, then which of the following hold good?

A. $\cot A \cot B \cot C \leq \frac{1}{3\sqrt{3}}$

B. $\cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} \geq 3\sqrt{(3)}$

C. $\cos A + \cos B + \cos C < 3/2$

D. $\sin(A/2)\sin(B/2)\sin(C/2) \leq 1/8$

Answer: A::B::C::D

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12. If x,y,z are three +ive real numbers, then minimum value of $\frac{y+z}{x}+\frac{z+x}{y}+\frac{x+y}{z}$ is

A. 1

B. 2

C. 3

D. 6

Answer: D

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13. If A,B,C are the angles of a triangle such that C is an obtuse angle then

A. $\tan A \tan B < 1$

B. $\tan A \tan B > 1$

C. $\tan A \tan B = 1$

D. None

Answer: A

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14. If $y = 3^{x-1} + 3^{-x-1}$, then the least value of y is 2 6 $2/3$ $3/2$

A. 2

B. 6

C. $2/3$

D. None of these

Answer: C

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15. If $x \in R$, the solution ste of the equation $4^{-x+0.5} - 7.2^{-x} - 4 < 0$ is equal to

A. $\left(2, \frac{7}{2}\right)$

B. $(-2, \infty)$

C. $(2, \infty)$

D. $(-\infty, \infty)$

Answer: B

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16. The solution set of the inequality $x + 1 > \sqrt{x+3}$ is

A. $[x : -1 < x \leq -3]$

B. $[x : x > -1]$

C. $[x : -3 \leq x \leq -2]$

D. $[x : x > 1]$

Answer: D

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17. The solution set of the inequaltiy $||x| - 1| < 1 - x, \forall x \in R$ is equal to

A. $(0, \infty)$

B. $(-1, \infty)$

C. $(-1, 1)$

D. $(-\infty, 0)$

Answer: D

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18. If $x \in R$ an $m = x^2 / (x^4 - 2x^2 + 4)$, then m lies in the interval

A. $[0, 1/4]$

B. $[0, 1/3]$

C. $[0, 1/2]$

D. $[0, 1/5]$

Answer: C

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19. Let $S_1, S_2. \dots$, be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side S_1 is 10 cm, then for which of the following value of n is the area of S_n less than 1 sq cm ?

A. 7

B. 8

C. 9

D. 10

Answer: B::C::D

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20. If a and b are two positive quantities whose sum is λ then the minimum value of $\sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}$ is

A. $\lambda-\frac{1}{\lambda}$

B. $\lambda-\frac{2}{\lambda}$

C. $1+\frac{1}{\lambda}$

D. $1+\frac{2}{\lambda}$

Answer: D

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21. If $x = \log_5 3 + \log_7 5 + \log_9 7$,then x is $\geq$?

A. $\frac{3}{2}$

B. $\frac{1}{2^{1/3}}$

C. $\frac{3}{2^{1/3}}$

D. None

Answer: C

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22. If a, b, c are three non-zero numbers of the same sign, then the value of $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ lies in the interval

A. $[3, \infty)$

B. $(3, \infty)$

C. $[2, \infty)$

D. $(-\infty, 3)$

Answer: A

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23. Let $a, bandc$ be real numbers such that $a+2b+c=4$. Find the maximum value of $(ab+bc+ca)$.

A. 2

B. 4

C. 6

D. 8

Answer: B

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24. Let $a, b and c$ be real numbers such that $a + 2b + c = 4$. Find the maximum value of $(ab + bc + ca)$.

A. 2

B. 4

C. 6

D. 8

Answer: B

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25. The least value of $2\log_{100} a - \log_a(.0001), a > 1$ is

A. 2

B. 3

C. 4

D. None

Answer: C

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26. If $\alpha \in \left(0, \frac{\pi}{2}\right), then \sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to $2 \tan \alpha \ 1 \ 2 \sec^2 \alpha$

A. 2

B. $5/2$

C. $2 \tan \alpha$

D. $\sec \alpha$

Answer: C

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27. If $x_n > 1$ for all $n \in N$, then the minimum value of the expression

$\log_{x_2} x_1 + \log_{x_3} x_2 + \dots + \log_{x_n} x_{n-1} + \log_{x_1} x_n$ is

A. 1

B. 2

C. 0

D. None

Answer: D

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28. If $a - 1, a_2, , a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \; + a_{n-1} + 2a_n$ is $a_{n-1} + 2a_n$ is b. $(n+1)c^{1/n}$ $2nc^{1/n}$ $(n+1)(2c)^{1/n}$

A. $n(2c)^{1/n}$

B. $(n+1)c^{1/n}$

C. $2nc^{1/n}$

D. $(n+1)(2c)^{1/n}$

Answer: A

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29. Let $a, b and c$ be real numbers such that $a+2b+c=4$. Find the maximum value of $(ab+bc+ca)$.

A. 2

B. 4

C. 6

D. 8

Answer: B

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Problem Set 1 True And False

1. If $a > 0, b > 0, c > 0$ prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$.

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2. If n is a positive integer then $2^n > 1 + n\sqrt{(2^{n-1})}$. True or False.

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3. If a,b,c are in HP, then prove that $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} > 4$.

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4. If $a > bandn$ is a positive integer, then prove that $a^n - b^n > n(ab)^{(n-1)/2}(a-b)$.

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5. If $a > b > 1$ then $a^n - b^n > n(a - b)$ for every +ive integer $n \geq 2$ .True or False.

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6. $\frac{2}{b+c} + \frac{2}{c+a} + \frac{2}{a+b} > \frac{9}{a+b+c}$

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7. If $S = a + b + c$ then prove that $\frac{S}{S-a} + \frac{S}{S-b} + \frac{S}{S-c} > \frac{9}{2}$ where a,b & c are distinct positive reals.

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8.

$$\frac{3}{b+c+d} + \frac{3}{c+d+a} + \frac{3}{d+a+b} + \frac{3}{a+b+c} > \frac{16}{a+b+c+d}$$

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9. $n(n+1)^3 < 8\left(1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3\right)$

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10. If a, b, c are in H.P. and they are distinct and positive then prove that $a^n + c^n > 2b^n$

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11. Sum of the nth powers of the m^{th} power of n even numbers is $> n(n+1)^m$ if $m > 1$.

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12. If a,b,c are real numbers such that $a^2 + b^2 + c^2 = 1$ then $ab + bc + ca > -\frac{1}{2}$.

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13. In any triangle the semi perimeter is greater than each of its sides.

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14. The sum of the cubes of the legs of a right angled triangle is less than the cube of the hypotenuse.

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15. Thesum of the hypotenuse and the altitude of a right angled triangle dropped on the hypotenuse exceeds the half perimeter of the triangle.

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16. The area of an arbitrary triangle is less than one fourth the square of its semi perimeter. True or False?

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17. If x,y,z are all positive and $x < y < z$, then $\frac{x^2}{z} < \frac{x^2 + y^2 + z^2}{x + y + z} < \frac{z^2}{x}$

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18. If $0 < \alpha_1 < \alpha_2 < \dots\dots\dots < \alpha_n < \frac{\pi}{2}$ then $\tan\alpha_1 < \frac{\sin\alpha_1 + \sin\alpha_2 + \dots\dots\dots + \sin\alpha_n}{\cos\alpha_1 + \cos\alpha_2 + \dots\dots\dots + \cos\alpha_n} < \tan\alpha_n$

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19. If $A + B + C = \pi$then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2} \geq 1.$

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20. Prove that $2\sin x + \tan x \geq 3x$, for all $x \in \left[0, \frac{\pi}{2}\right]$.

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21.

$(a_1^2 + a_2^2 + \dots a_n^2)(b_1^2 + b_2^2 + \dots + b_n^2) \geq (a_1b_1 + a_2b_2 + \dots + a_nb_n)^2$

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22. A.M. of the square root of products taken two together on n+ve quantities is less than A.M. of the n quantities itself.

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23. Given $n^4 < 10^n$ for a fixed positive integer $n \geq 2$, then $(n+1)^4 < 10^{n+1}$

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24. For real a,b and x

$$-\sqrt{(a^2+b^2)} \leq a\sin x + b\cos x \leq \sqrt{(a^2+b^2)}$$

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25. $\log_e 4 + \log_4 e > 2$

a. True b. False

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26. $4^{\sin^2 x} + 4^{\cos^2 x} \geq 4$ for all real x.

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27. If $y = \sin nx + \cos nx$ (x,n real0 then $-\sqrt{2} \leq y \leq \sqrt{2}$.

a. True b. False

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Problem Set 1 Fill In The Blanks

1. $\log_{(3x+7)}(2a^7+3) < , \forall a \in R$, then x lies in the interval ....

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2. Solve the inequality: $\log_{x-2}(2x-3) > \log_{x-2}(24-6x)$

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3. The solution set of the inequality

$\sqrt{6-x}\left(5^{x^2-7.2x+3.9}+25\sqrt{5}\right) \geq 0$ are ....

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4. Solve the following inequation .

(xv) $x^{(\log_{10} x)^2 - 3\log_{10} x + 1} > 1000$

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